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MANUAL

of

GOTHIC CONSTRUCTION

by

G. ORTENBURGER

Third Edition

Rewritten by

E. SCHREMMER

Professor at the Baltic Polytechnic School at Riga

Volume I

With 822 illustrations in text and on inserted plates

Leipzig

1920

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Translated by M. Clifford Ryder, C. E.

Emeritus Professor of Architecture

University of Illinois

Urbana, Ill.

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Preface to First Edition.

In the first decades of the 19 th century, the generally approved and even now extremely valuable in many respects, Manual of Rural Architecture by Gilly, Wolfram, etc., designates as the three principal requirements of every structure, utility, durability and beauty. Yet to the latter has been previously assigned a secondary place, and this has been explained as allowable so far as it did not oppose the two first qualities.

This extremely rational conception is only erroneous, so far as it permits the possibility of a conflict between beauty and its preferred sisters, and even this error is based on the general idea of esthetic beauty then prevailing. Therefore it was generally adopted, particularly being made an article of faith of the state architectural officials, and had extremely injurious consequences, because the greater number of the latter were far removed in existing circumstances from examining the causes of that conflict, and to remove them from the path as far as possible, extendind further the liberty contained in this idea, even they themselves judged ugliness as allowable. Men did not express it directly, but proceeded in this wayto the conception, and indeed to the "beautiful architecture," to which ordinary architectural practice was opposed. The former almost exclusively belonged to the splendid buildings erected in the great cities, the palaces, theatres and museums, but to the latter belonged the far prodominant number of utility structures, rural and communal buildings, indeed even the smaller churches (Note). That such a conception, by which an entire category of works had to remain bare and plain, whereby the great multitude of men in life could only see ugly buildings, and in all manufactures the dominant influence of the architecture could only bring to view unbeautiful objects, could not be conclusive.

(Note). The design and execution of such a one would even now be generally regarded as the lowest and least artistic treatment of the required problem. Thus there exist highly esteemed street and hydraulic engineers, who carelessly state, that although architecture proper lies far from them, yet that they are qualified to make designs for a church, a school, in brief for any building occurring in their official circle of work.

We have already designated the former conception of beauty as the cause of the unsuitable ness blamed. But it was the same for the antique architecture, when it was assumed as the basis, that its forms must serve as ideal for all time, and no doubt could exist as to their baing standard. And still one was conscious of a certain endeavor to copy these forms, and be repeated measurings and by the introduction of models of parts, men were prepared, not only to reproduce them, but even to correct them where possible, --- how could their use then lead to errors?

But the Romans could already combine Grecian columnar architecture by a certain compulsion with the arrangedent of their construction, when they transferred the porticos to the wall masses of their structures, they succeeded only by the royal magnificence of their execution in combining the opposing ideas; yet they had to proceed to a transformation of the detail forms, and then the position of later men was far worse. The organism of their structures required by the possibility of use opposed the employment of Greek forms of detail in a far higher degree, than was the case for the Romans. Splendor was lacking, a transformation of those forms was attempted by the architects of the Renaissance, was then opposed to the former written law of mudules and parts, but further to the fact that the latest past offered a much less usable material, than that inherited by the 15 th and 16 th centuries. Men were thus able to add antique beauty only in a far more powerful way than was possible to the Romans, i.e., they built in the conventional way, well or badly, and added just as much beauty, as the conditions and the possibility of use of the proper building would permit.

This beauty or rather the means of producing it, is the same for every species of building; for the church, the palace and the museum, down to the druggist's shop in the smallest provincial city, are the same fronts, columnar orders, cornices, window caps and architraves, which the consecrated service had to use; the purpose there came under consideration not the least. Just as little could the material find any attention, when marble for which the forms employed were important, and whose qualities had caused their origination, was not at hand. Whether men saw themselves obliged to construct the different

parts of that beautiful architecture of stone, gypsum or of wood, that could affect little as the form remained the same.

But since the beauty for which men strove in such manner had no relation whatever to the purpose of the building, then must it oppose suitability, since the nature of the materials found no consideration, and thus the durability would be endangered. There was not alone the possibility but almost the necessity of such a conflict.

In what is said above, we have desired only to indicate the usual point of view of the building officials and architects with a tendency to the antique. That there were highly gifted artists in this and perhaps still are, which our description does not fit, to whom it has been given to breathe a new life into those dead and foreign forms, adapting them to modern requirements, should not be denied. But how few is their number, how small their influence on the great multitude of architects.

The inadequacy of Grecian-Roman architecture then also led early in the thirties to a medley the system of infringing license. With this began from the period of different style periods of the middle ages to borrow certain alterations, certain forms of details, and to combine them with the antique-like conception of the whole, and wished by the piquant appearance of such innovations, allowed themselves to extend farther the circle of their license.

But men either abandoned first the foundation of Grecian architecture, or declared the style of one or another mediaeval period suitable for further development or requiring this, and strove to seek this development in refining the detail forms according to Greek principles.

It is in the nature of the matter, that these endeavors lacking a unity of principle, proceeding entirely from the individual views of each person, must scatter in an endless multitude of separating directions, from which finally developed the idea that the present art period must be regarded as a transient condition, in which the new Messiah was to be expected. And t therein from the very first this idea was opposed to the views of those , which were based on the everlasting of the once born, whose church is founded, will permeate all conditions, and has found an artistic form, whose diversity is as eternal as themselves.

It might matter little whether this artistic form is seen in the style of the 13 or of the 12 th centuries, in the Gothic, Romanesque or the Early Christian. But we might assume that if Romanesque art did not suffice for the grand cathhedral buildings of the 13 th century, it still less suited the multiform problems of the present time, but that no problem can be conceived, for whose solution the means cannot be developed from the principle of Gothic construction.

To the absolute conception of beauty of the antique tendency described above is then contrasted the beauty of Gothic architecturein this, that it is suited to every problem, that it builds for each in its inmost being, and as a necessary result represents a well understood durability and suitability. Accordingly it shapes itself in every detail as the form best adapted to its structural purpose, but for the whole as best serving the purpose of the work and bringing the conception into the clearest expression.

Thus the beauty of the cathedral differs from that of the parish church, the beauty of the ecclesiastical structure in general is other than that of the secular. And for each of the greatly varied problems of secular architecture again is a speial character suited therefor. But the unity in the endless diversity of its solutions will result from the connection of e each with the structure of the church. Then the cathedral includes within itself the entire development of forms of all other works, just as the spirit which dwells beneath its vaults permeates all interiors. This very connection with the church, this predominance of a religious character in Gothic art, men have gladly made a reproach to it, and therefore also declared it could not serve for secular purposes. But with the same right would an individual be blamed, whose religion permeates his outward behavior, or him that fulfils his religious duties might be held unfit for worldly affairs. The palace, city hall or house indeed differ from the church, but prayer is offered in all, that serve the purposed sanctioned by religion, and therefore their exterior would contradict the inmost germ of their being, if Their derivation from the form of the church were not brought into view.

It may be that the not rare horror of the ecclesiastical

character of Gothic secular buildingsis to be ascrited in part to the account of the many recent exaggerations, in which the forms of the ecclesiastical style are emplcyed similarly, as the building officials with a taste for the antique changed t their columnar orders; its deeper cause lies in a certain offosition to everything decided and characteristic, to all treatof form beyond the flattest elegance. The striving for this modern elegance, the entire lack of all earnest endeavor has reached its climax in the great multitude of the different structures connected with railways. (Note). These almost have the appearance as if men desired here to attain by steam power the often promised style of the future.

Note. But of what artistic treatment these would be capable, especially Eisenlohr has shown in the buildings of the Baden railway. Yet in contrast to those works is the recently sought empty elegance, that seldom or never comes to light, and apparently is generally refused to Germans.

As now so many tendencies become apparent on the whole, the drift to Gothic architecture has increased in a series of years. Finally men have become wearied by allowing themselves to use antique or contemporary conceptions, with which so many building officials are so extremely liberal. Churches and secular works rise everywhere, which by the return from the previously traveled paths give evidence of a more or less happy but always honorable endeavor.

According to what is said above, it is now the purpose of our plates to show the different constructions of Gothic art, the formation of the details for their different functions according to the peculiarities of the materials, later to explain the works od the middle ages, and to illustrate their combination in an entirety. The buildings on which we depend and from which we have taken the different figures, are almost entirely those which we know by examination. A sharp separation of the national peculiarities can less be given, since these were naturally developed in the middle ages, as men studied and utilized the results obtained by other nations, so far as they were available. When we now follow the same way in practice, we must fear the loss of the special tinge, that the Gothic style assumed in the different countries, that it once had, yet the conditions of the materials and climate still remain.

We have so arranged the sequence of the different sections as far as possible, that each one only assumes those preceding it, and also in the figures is made the transition from the simpler to the more complex forms.

The clearness of our book will perhaps conduce to the development of the parts from the whole. But we believe that first of all is easy use to be cared for in a book, which must contain the instruction necessary for the beginner.

Cassel. Dec. 22. 1858.

G. Ungewitter.

Preface to the Second Edition.

After the death of the early departed and deserving author, whom A. Reichensperger has so fully treated in his Biography:- "George Gottlieb Ungewitter and his works as architect;" the publishers believed it to be a duty for them to introduce the necessary new edition of his "Manual of Gothic Construction" by a few commendatory words to the circle of those interested in it. The need of a manual like this is sufficiently recognized, its excellence is fully guaranteed by the ability and fame of the author, and both are sufficiently justified by the need for a new edition. We are thereby relieved from any further recommendation of the work, and so that the labors and creations of the author may receive just appreciation, we restrict ourselves to his buildings in north Germany and to his activity in the literary domain. His entire thoughts and acts culminated in the endeavor to arouse in our time the feeling for mediaeval art, to cherish serve it was also the predominating purpose in this, like all his works. It will be a guide for all practising artists and architects, that will lead them with a sure hand through the apparent confusion of Gothic forms and constructions, and as such may this new edition also enjoy a favorable reception.

Leipzig. Jan. 1875.

The Publishers.

Preface to the Ninth Unwritten Edition.

George Gordon Underwriter's Manual of Certain Encumbrances
like no other of his works. It called to throw light on the d
the number of predecessors that have made real use of it are
not in proportion to the worth of its contents and to the im-
portance of the matter.

Some mostly external circumstances have contrived to make the
study of the work in part somewhat difficult. The illustrations
were inconveniently separated from the text on large plates.
The book itself was rather dark in arrangement and colouring.
certain many platitudes inserted for the benefit of beginn-
ers in the earlier decades. Since it is not now the object of
anyone to study or consult it a Manual is said to be sup-
plied.

This text and the need of additions for this time required
its rewriting. The procedure decided on that the publishers
should be criticised for [illegible]. Illustrations should
be, where possible or plans. In addition [illegible]
to the text. and that clarity should be increased in every pos-
sible.

The plans [illegible] should be, wherever possible,
be entirely obeyed. The considerable enlargement of certain
offset by reductions. and so reverence towards the author such
[illegible] it should make the reader held to be a
first duty.

Additions and extensions extend towards the entire work. En-
largement of the bureaus of vaults and the thickness of walls
and of buttresses.

The recent advance in physics --- particularly acoustic physics
--- applied to vaults and buttresses seemed urgent. the more
[illegible] the reinforcing [illegible] in the drawings of t
almost entirely omitted, or where these were treated. Incorrect-
ly with basal assumptions. many diverge from the reality.

Preface to the third rewritten edition.

George Gottlob Undewitter's Manual of Gothic Construction11 like no other of his works, is called to throw light on the d depths of mediaeval architecture, and still it has shown that the number of professionals that have made real use of it are not in proportion to the worth of its contents and to the importance of the matter.

Some mostly external circumstances have combined to make the study of the work in caft somewhat difficult. The illustrations were inconveniently separated from the text on large plates, the text itself was rather dark in arrangement and subdivision, containing many platitudes inserted for the benefit of beginners in the earlier decades. Since it is not now the affair of every man to acquire his knowledge in a toilsome way, most convince themselves that collected in a manual is gold to be acquired.

This fact and the need of additions for this time required its rewriting. The programme decided on that the publishers demanded, that without passing beyond the former limits there should be extensions and additions, explanations should be abbreviated, figures should be placed in more intimate relation to the text, and that clarity should be increased in every respect.

The first requirement to retain the former range, could not be entirely obeyed. The considerable enlargement of certain sections and the addition of about 400 figures could not be offset by reductions, and so reverence toward the author must suffer thereby; but to retain this the reviser held to be a first duty.

Additions and extensions extend through the entire work. Entirely rewritten is the section on vaults and now is inserted one on abutments, and there are added tables relating to the magnitude of the thrusts of vaults and the thickness of walls and of buttresses.

The recent advance in statics --- especially graphic statics --- applied to vaults and buttresses seemed urgent, the more because the scientific investigation just in the domains of t the complex constructions here coming in consideration has been almost entirely omitted, or where these were treated, frequently with basal assumptions, they differ from the reality.

It could not be the intention of this work to conduct a far reaching theoretical treatment, for architects would be little benefited thereby. But indeed there are sufficiently treated and are established first of all basal principles, which without tedious calculations and without much mathematical science permit a correct judgment of the effect of forces, to train a again the modern architect to feel the stresses in his structures, just as the old master could in his unfortunately lost directness of invention. With these points of view the reviser hopes to have so treated the statical explanations inserted, that they will not make more difficult the unfathomable and manysided Gothic construction, but simplify it.

Yet the theorists among professionals, to whom the new seems represented with too little science, may take it into friendly consideration, that it was here first to write for the practitioner. We have little lack of theorists for theorists, but there is ever still wanting a correct connection of theory and practice; indeed for more than nine tenths of persons engaged in building, the theoretical results of the last decades have passed over without influencing them.

In the sequence of the material appeared necessary some changes. Before the introduction of mouldings and tracery. Ungewitter himself wrote in his preface, that perhaps the developgent of the parts from the whole would be required for clearness, but he took into account the instruction of the beginner. Since in the essentially changed education of the beginner came into consideration less in the earlier sense, if these two sections were removed to a later place, and yet other changes were made. These are now preceded by the still required vaults with the buttresses and supports, and after them follows the treatment of the church in plan and elevation with the addition of the different details of construction.

To increase the clarity of the work an endeavor has beey made in all directions to subdivide the materials more, and a twofold size of type separates the continuous text from the insertions and explanations; there are added on the side margins a brief statement of the contents, and the illustrations will further serve as a speaking guide.

The illustrations are but partly inserted in the text, but in much greater number are on plates of the size of the book,

bound in at proper places and not folded. In the addition of new illustrated examples, more would have been added if the exceeding the limits had not required otherwise.

An extension of the Manual to the less fully treated domain of brick construction, and to secular architecture was not possible within the former compass, yet it is designed to add these sections to the work in separate volumes.

Here the undersigned must express in the highest veneration his thanks to his old master and instructor, C. W. Hase in Hanover, for counsel and direction, and who enabled him to make this revision.

Riga. May. 1889.

K. Mohrmann.

MANUAL OF GOTHIC CONSTRUCTION.

I. Vaults.

1. Development of the Art of Vaulting from Roman to Gothic.

Vaults of the Romans.

Forms of Vaults in Roman Architecture.

Roman art knew but two basal forms in this vaulting.

1. The tunnel vault in form of a half cylinder (Fig. 1).
2. The dome in form of a hemisphere (Fig. 2).

Both originated from the same generating curve, the semicircle, one by moving it sidewise, the other by its rotation.

The tunnel vault covers rectangular rooms, the dome those w with a circular plan. But the rich development of plan was not satisfied by the circle and the simple rectangle, and men required these interiors with many variations. But a freer development of plan also demands freer forms of covering, which finds its expression in variations of these two forms of vaults.

Particularly important are the forms from the tunnel vault. When in plan two interiors of different widths intersect at right angles, then the vault of the smaller extends into the larger, and there results the form of the Welsh groin vault. (Fig. 3).

If there intersect in this way two rectangular interiors of equal widths, the intersection of their vaults forms the regular shape of the cross vault (Fig. 4).

After men were led in such manner to the latter very important form, they made use of it; they continued the vault of the rectangular room with cross tunnel vaults arranged beside each other, to obtain elevated semicircular wall surfaces along the longitudinal walls, which were very desirable here for the arrangement of openings (Fig. 5). They attained a continued series of cross vaults over a long interior. (Basilica of Maxentius, gallery of palace on the Palatine).

There now remained only one step, the arrangement in both longitudinal and transverse directions, and the problem was solved of covering a wide interior uniformly vaulted on separate supports (Fig. 6). This solution is found in the Baths and the inner rooms of the Coliseum.

A similar extension was experienced by the use of the dome. If on the ground plan of the domed interior a wall is built on a chord, it cuts each dome above in the form of a semicircle.

(Since every intersection of a dome by a plane is a circular arc). Now if such walls are joined in the plan in the form of any polygon inscribed in the circle, each wall ends at top in the form of a semicircle. But the part of the dome remaining between the walls receives the character of an independent form of ceiling, made possible by covering a polygonal room, as an octagon, square, etc., that is termed a pendentive dome (Figs. 7, 12). The pendentive dome first appeared in the later time and chiefly in the eastern half of the empire.(For example, Cisterns of Constantine at Byzantium, Tomb of Galla Placidia at Ravenna).

As a further variation of the dome the half dome for covers niches and the hip dome are to be mentioned. The latter is also termed a cloister dome and can be derived from the tunnel vault just as well as from the dome.(Examples, Temple of Minerva Medica at Rome. Fig. 8).

The before mentioned forms include about all that the Roman art of vaulting created. They are mentioned here to show in what a direct way they were derived from the forms of rooms, without the needs of construction having much to do with them. The last circumstance is explained by the Roman technics.

Method of construction of Roman vaults.

Construction of vaults in courses of cut stone or of bricks was indeed employed by the Romans, but for the covering of architectural interiors concrete was the ever increasing method. Often the execution of this was far removed from our present methods. Men constructed a centering according to the form of the vault, laid thereon a covering of thin slabs of stone fitted close together and then laid on the concrete in horizontal layers, just as done for any other masonry (Fig. 9). That this held was naturally due first to the intimate adhesion of the great masses of mortar. But once left to the latter, within fixed limits there could be given to the centering a nearly optional form, so that the shape of the vault could be named without paying attention to its architectural appearance, which resulted from the use of the semicircular form.

For covering very large rooms of great span, such a simple procedure as the preceding would not suffice, and here appeared the practical sense of the Romans in the introduction of carefully developed methods of execution. To these belong the iso-

isolated pottery vaults, but especially the cast cellular vaults. For the latter was first placed on the centering the usual large and thin bricks, then a network skeleton, whose cells w were later filled with concrete (Fig. 10).

The importance of the brick skeleton is that it permits convenient and safe construction. When at the completion of the vault it is covered by mortar, its chief function has been fulfilled; the vault then acts like any other concrete vault as a unit by the mass held by the mortar. The thickness of such a vault at the crown usually varies between 1.2 and 3 metres.

From tunnel vaults the enclosed brick framework generally f forms rectangular panels, from which may be derived a certain structural justification for the formation of coffers. On the contrary for domes the ribs usually take the form of a relieving arch lying in the surface of the vault on another arch. Prominent examples of this kind are offered by the Pantheon, Temple of Minerva Medica, the numerous remains of other buildings at Rome, Tivoli, etc. Details are given in the important work of Choisy; Art of building by the ancient Romans.

The technics of the Romans were very developed and very manifold. But in this respect, they always pursued the same aim; whether a structure was of cut stone, bricks or concrete, it was always their endeavor to make vaults and walls a single rigidly connected mass, that by the piling of great masses should form an almost indestructible work.

Vaults of Early Christian period.

The Art in the West.

Early Christian art in the west existed in the first centuries almost exclusively on their inheritance from the Romans, particularly in regard to technics. But traditions were gradually effaced more and more, there was on one hand a decadence in good execution, but on the other the impulse led to pursuing new aims. In two directions under a partial influence of t the East a certain transformation cannot be denied, men often consciously strove to reduce the masses of the walls, and they usually wandered from the old technics of concrete. Thus in t the vaults more than formerly is found a coursed arrangement of the stones in the vaults, on which was cast mortar from above. The direction of these courses is quite varied, as generally the Early Christian period shows many groping and inst-

have received heretofore.

We can not follow the constantly increasing endeavor to repres - ent Early Christian art as absolutely the end of the antique or the last stage of the ruin of Roman art. An abrupt stagnation that men formerly say indeed did not exist, but the slow growth of a new spirit is undeniable. Even in the devastation and the impoverishment of Roman forms is felt a conscious transition to the new path. One cannot demand from those centuries a splendid advance, they afforded no place for the senile decay and worn out ancient art; but they gathered all their endeavors to one point, and this is the first basal condition for the growth of the flower of a new style. What the temple was for the Greeks, the basilica became for the middle ages and indeed to have laid the foundation for its form is the part of the E Early Christian period; the whole living structure was reserved for the active power of a youthful and fresh people.

Art in the East.

More rapidly than in western Europe was completed the development in the East. While in the West was accepted with great preference the little changed old low roofed basilica, in the Byzantine empire a stronger acceptance of the central building afforded opportunity for new forms. Two important acquisitions appeared in the art of vaulting, the dome on pendentives and the raised cross vault. Both are clearly expressed in their entire shapes. Indeed they had earlier precursors, but they appear to have first attained their independent development in the time of Justinian.

The dome on pendentives brings a very bold idea into execution, a completely formed dome erected on four piers (Fig. 11). The plan of the dome lies entirely within the square of the piers, and therefore is not directly supported at any single point, but rather must the entire load be transmitted to the piers by the pendentives and cross arches. The pendentives form parts cut from a greater spherical surface, whose diameter corresponds to the diagonal of the bay of the vault.

For covering a square room men had three solutions:- 1, the new pendentive dome; 2, the segmental dome or Bohemian vault (Fig. 12); 3, the raised cross vault (Fig. 13).

Method of construction of Byzantine vaults.

Byzantine technics differ substantially from Roman, is not

angles downward back. The domes of S. Maria church at Venice -- perhaps unintentionally --- show the converse variation.

Tunnel vaults were usually turned in transverse or vertical ring courses, until they were closed, the bricks must be held by cementing to the previous ring course. To make this easier lined in the way shown in Figs. 12 h and 12 i, or executed in conical rings (Fig. 12 k). The transverse courses frequently, yet a change in the arrangement of courses is observed during of this kind were already erected by the Egyptians under the 12 th dynasty (Lepsius, Denkmaeler in Egypt. I. Pl. 80).

The cross vaults of the Byzantines like the tunnel vaults of the Byzantines) and believes, that man did not once need centions and were set Brosband (Fig. 18). Choisy first threw light on the Byzantine mode of construction (Art of Building among

acquainted with Roman concrete, employs stones with thick mortar joints, and uses in the vaults in extended measure freehand masonry without centering. Already under the Romans the art of the East had gone its own way, much had remained from Greek art, and the technics in the provinces were in much closer relation to that transmitted from the ancient flowering of art in Persia, Assyria and Egypt. There had men learned the primitive and also primitive invention of freehand vaulting of high domed as well as of tunnel vaulted rooms.

Where they had to do without cut stone or heavy rubble, the East Romans also certainly preferred the centering, but as soon after Augustus burned bricks came into honor everywhere, the door was widely opened to freehand vaulting.

Domes were almost always vaulted in horizontal rings and were stable after completion. To prevent the bricks from slipping in the upper courses, the beds were made more horizontal there, (Fig. 11 a), unless the upper part was not simply made pointed or conical (Persia, Arabia). The pendentives are not corbelled, but are built in vaulting courses, which further is equally s statical. The generating line of the dome often varies from the circle, since gen by foresight kept the pendentives in the four angles somewhat back. The domes of S. Mark's church at Venice -- perhaps unintentionally --- show the converse variation.

Tunnel vaults were usually turned in transverse or vertical ring courses. until they were closed, the bricks must be held by cementing to the previous ring course. To make this easier and to prevent the yielding of the course at top, it was often inclined in the way shown in Figs. 12 h and 12 i, or executed in conical rings (Fig. 12 k). The tranrverse courses frequently began at a height where horizontal courses became unsuitable, yet a change in the arrangement of courses is observed according to circumstances at the time. Very rational tunnel vaults of this kind were already erected by the Egyptians under the 19 th dynasty (Lepsius, Monuments in Egypt. I, Pl. 89).

The cross vaults of the Byzantines like the tunnel vaults r received endwise transverse courses, that alternate in the groins and were set freehand (Fig. 18). Choisy first threw light on the Byzantine mode of construction (Art of Building among the Byzantines) and believes, that men did not once need centerings under the groins of cross vaults. So far we cannot fol-

follow them for static reasons -- at least in regard to large

follow them for static reasons -- at least in regard to large cross vaults.

The groins of cross vaults nearly always have the form of a circular arc, somewhat less than a semicircle. Each course forms a circular arc, whose middle point lies on the horizontal axis x x, whose curve is easily executed by a string fastened at p. With this form of groin and courses the crown must take the recurved form occurring in Fig. 18, which the vault actually shows, that men certainly sought also to avoid in many examples, for instance by flatter wall arches (elliptical groins likewise produced a different crown). The groins of Byzantine crossvaults project very strongly beneath, while almost disappearing above. If the rise so far increases that the groin becomes a semicircle, then with the same mode of construction it passes for itself from the cross vault into the pendentive dome.

The pendentive dome exhibits ring courses (Fig. 12 a), transverse courses like the cross vault (Fig. 12 b), oblique courses (Fig. 12 c), and also finally an alternation of the two forms (Figs 12 d and 12 e). Such a change is also frequently carried over into the spandrel of the dome.

The technics were transmitted to the Byzantines from earlier peoples, during the Roman period they were evermore adopted by them, but under their rule the architecture became less determinative. Only when Byzantium acquired its importance as an independent centre of a great Christian empire, particularly after the 6 th century, then did the mode of building pass into a more decided expression. The conditions of construction claimed a certain leadership more surely felt than before. B But that helped to smooth the way for the endeavors of the following middle ages aiming at the same point.

While the West fixed the ground type of the church, the East loosened the compulsion of the architectural form in favor of a greater domination by the construction.

At the point of contact between East and West did not fail brisk trade relations, the Byzantine rule in Ravenna and Venice, the call of Greek architects and workmen to the courts of the West, and finally later the impressions brought home by the crusaders and pilgrims, maintained a sufficient intellectual exchange.

Introduction of the vault in the Romanesque Basilica.

In the Romanesque basilica fell the problems we stated, to
ed numerous structures of the Byzantine works, the ever rec-
urring construction by time supplied a motive sufficient to re-
quire vaulting. In all parts of the structure, where they con-
ld be easily executed, it quickly became the rule; thus it al-
ways stepped over the case, that was covered by a belt done.
and likewise is always found in the crypt. That really vaulti-
ed cases vaults with the old Roman treatment.

It was also easy to cover the side aisles, and therefore they
created aligned nice vaults for a long time. While the extend-
ed room of the structure, the middle staff was still to be a
to look the most incorrect position or mediaval art. These
1. The vaulting of the middle staff.

The traditions of Rome entered the new blooming art like a continuous chain, its fall freed Byzantium, and where one of the old cords came to an end, there the master knotted one of his own strong spinning, until at last an entirely new fabric appeared beneath a new hand skilled in art. By this picture can be represented the worth of Romanesque art, whose first problems reach the climax in vaulting after Byzantine precedents in a new monumental conception, the Roman basilica formerly covered by timbers.

Introduction of the vault in the Romanesque Basilica.

Introduction of the vault in the Romanesque basilica.

To the Romanesque basilica fell the problems as stated, to give the basilica a massive covering. Besides the distinguished monumental appearance of the Byzantine works, the ever recurring destruction by fire supplied a motive sufficient to require vaulting. In all parts of the structure, where they could be easily executed, it quickly became the rule; thus it always appears over the apse, that was covered by a half dome, and likewise is always found in the crypt, that mostly retained cross vaults with the old Roman treatment.

It was also easy to cover the side aisles, and therefore they appeared alread with vaults for a long time, while the principal room of the structure, the middle aisle was still to be s satisfied with a wooden ceiling. As the forms of the vaults of the side aisles occur the longitudinal tunnel with and without compartments, the transverse tunnel and the cross vaults.

So far had the vaulting been easily completed, but there remained two places in the plan of the church, whose mastery was to form the most important problem of mediaeval art. These were:-

1. The vaulting of the middle aisle.
2. The vaulting of the choir aisle.

The latter enrichment of the choir appeared after the 11 th century.

The attainment of both of these aims formed the climax of all architectural endeavors from the end of the 11 th to the beginning of the 13 th centuries. At this time the ancient art centres of Rome and Byzantium had almost entirely receded, the centre of gravity had passed to the Northwest into the youthfully fresh peoples in modern Germany, France and England. There arose a world competition for obtaining the perfect, from

which Gothic architecture finally came forth as a magnificent result and a rapid blossoming of a conscious victory.

Different attempts in vaulting the middle aisle.

a. The longitudinal tunnel vault.

As the first fruits of vaulting the middle aisle appeared t the tunnel vault nearly everywhere, and it is found from Spain and Italy to Scandinavia (Church at Ringsaker), and it became most commonly dominant in Southeast Prince --- aside from from the three chapels in Ireland. Its introduction was natural, since it was known from the Roman works and was the most naturally added to the rectangular interior, but it carried in itself its unconquerable defect. The impossibility of lighting it satisfactorily in a plan with three aisles, its cavern-like a appearance and the difficulty of stiffening it did not allow it to succeed in a development fulfilling the purpose in spite of all experiments.

The thrust was very skilfully received by high half tunnel vaults constructed over the side aisles, but the difficulty in lighting was increased thereby.

b. The dome on pendentives.

On the other hand men preferred a good side introduction of the light by larrying the middle aisle high up --- as in Burgundy, and then the thrust again made itself strongly marked. Even if they opposed this by the introduction of the pointed form of the tunnel and successfully by buttresses, there yet remained the oppressive effect of the tunnel in the interior. They finally sought to lessen this by inserting dividing and strengthening cross arches, --- but the tunnel vault never led to real satisfaction.

Therefore men pursued other difficult solutions. Among them especially appeared the placing of fully developed pendentive domes beside each other as employed in Southwest France. The transfer of the dome there is also explained by the animated trade relations of those provinces, especially the cities of Limoges and Perigueux, with Venice, Byzantine at that time. Recently the influence of Venice is doubted on many sides. (Dehio & Von Bezold, Church Architecture of the West, p. 339). The Church of S. Front at Perigueux with its five mighty domes arranged in cross form and supported by heavy pointed cross arches assumes a place of honor among these buildings. (The

other mostly cruciform plans at Angoulême, Cognac, Lincoln, etc.

The diameter of the domes always indicating a central point.

acite of outer bearing, and therefore contributed as little as the tunnel vault.

for more alive showed itself the restrengthening tendentive dome. (vide. 14). It was erected in many places, in France particularly at Avenay, Reims and Clermont, in Germany at Paderborn and Speyer, as in other places. It is mostly found over the intersection but also frequently over rooms of vaulted bays. There a ly. Many such cases are known as a particularly good example. cularly it one considers that it occurs itself to a rectangular tax just as conveniently as to a square one. Since further ritcous a century, one one should not wonder that it was long retained in many places, when the cross vault had already become generally familiar. It was frequently [illegible] the rule that only an ornament as most a settlement, but are not the actually supporting parts.

d. Arrangement of transverse tunnel vaults.

Before turning to the last and final solution, there is still to be mentioned a not extended experiment executed in the Church of S. Sebastian at Ingelheim. It is a combination of transverse tunnel vaults, as were constantly found over aisles. The [illegible] turnouse, that be desire exposed for more central restriction.

Cross vaults over middle aisle.

aerial end of the beginning, a cross vault erected after these

domes of S. Front are marked by plain construction in cut stone and by horizontal courses in the pendentives). The principle of actual arrangement beside each other appears even more clrarly in the Abbey Church of Fontevrault, as well as in many other mostly single-aiseld plans at Angouleme, Cognac, Limoges,etc.

The transfer of the domes always indicating a central point, to churches progressive repetition was unnatural and forced in spite of other beauties, and therefore contributed as little as the tunnel vault.

c. Rectangular pendentive dome.

Far more alive showed itself the rectangular pendentive dome. (Fig. 14). It was erected in many places, in France preferably in Anjou, Maine and Touraine, in Germany at Paderborn and Knechsteden as in other places. It is mostly found over the intersection but also frequently over rows of vaulted bays. These d domes join at the four enclosing sides so simply and organically, that this must be regarded as a preferable solution, particularly if one considers that it adapts itself to a rectangular bay just as conveniently as to a square one. Since further on account of the spherical form is the construction possible without a centering, and one should not wonder that it was long retained in some places, when the cross vault had already become generally dominant. It even frequently adopted the ribs of the cross vault without abandoning the spherical surface, and the ribs then merely form an ornament as most a stiffening, b but are not the actually supporting parts.

d. Arrangement of transverse tunnel vaults.

Before passing to the last and final solution, there is still to be mentioned a not extended experiment expressed in the Church of S. Saturnin at Toulouse. It is a result of transverse tunnel vaults, as more commonly found over side aisles. Its transfer to the middle aisle shows a desire to obtain more light, without considering the effect of the interior was so disturbing, that no desire appeared for more general repetition.

Cross vaults over middle aisle.

The sole solution remaining was the cross vault. It appears surprising nearly everywhere, that this otherwise so frequently employed form was so long avoided for the middle aisle; still this had its weighty reason. Aside from the nature of materials and of the technics, a cross vault erected after Roman

art in this height and space required an abutting mass, which the middle walls of the basilica could not giva, and at the same time with their uniform thickness, they were adapted to receive a load concentrated at certain points. If men would create the abutment by a continuous increase in thi thickness of the wall or by a pier, then was lost again the long desired spacious combination of the three aisles. There might also be made many dangerous experiments, until they succeeded step by step in reducing the thrust of the vault and making the abutment more resistant without piling up masses. The aim was completely attained only when the Gothic vault and buttress system were fully proposed.

Besides these hindrances in the construction, there were difficulties opposed to the introduction of the cross vault in the arrangement of the plan. The middle ailse is wider than the s side aisles, and therefore as in Fig. 15 I always appear rectngular bays in a continuous subdivision into bays, either in t the middle or at the sides. But the Roman cross vault was unsuited to cover a rectangular bay, and also the transformation of the cross vault in the Romanesque period produced difficulties, if the lengths of the sides were quite different. Hence men sought to arrange square bays as much as possible for the three aisles, and came to the arrangement II, where each two bays in the side aisles correspond to a larger one in the middle aisle. This ground plan was the typical one for the vaulted Romanesque basilica in Germany. There alternated heavily loaded main piers with those less loaded and intermediate. Men sought to connect the latter with the principal vault by the use of the hexapartite vault, particularly in Northern France, about the middle of the 12 th century. Thereby originated the arrangement III, but which was dropped again at the beginning of the 13 th century. In Germany the hexapartite vault was especially erected on the Rhine, then at Bremen, Maulbronn, Limburg-o-L, but was little employed in general, and as soon as the development of the cross vault permitted the covering of rectangular bays, men generally to the most natural solution I, which soon became the general rule for the Gothic ground plans of churches.

Transformation of the cross vault for the rectagular plan.

Experiments with the now favored subdivision into bays for

the middle aisle usually never stopped. Moreover since in covering the side aisles and no less in the arrangement of monastic and secular buildings men came again to the rectangular plan of a room, a transformation of the Roman cross vault only calculated for square bays could not at once exhibit durability. Men adhered indeed so long as possible to Roman traditions, but were compelled by necessity to abandon them more and more.

All essential solutions might be successively placed beside each other, than in general the old method of construction afforded for the vault of a middle aisle with transverse rectangular divisions. Since in the Romanesque period the cross vault was erected on a complete centering, it is useful for a clearer understanding to consider not the vault itself, but the form of its centering.

If not strictly proved, it is still probable that for a series of cross vaults the Romans first constructed a continuous centering of planks under the longitudinal tunnel vault (Fig. 16), and then first placed the centerings of the separate cross tunnel vaults upon thes, exactly as one proceeds today in vaulting small compartments in cellar vaults.

These technics were still employed in the first Romanesque period as proved by Schäfer (Centralblatt der Bauverwaltigen, 1885) very fully by the observation, that on such rows of vaults is lacking at one side a transverse tunnel vault, and that usually the ridges of two opposite tunnel vaults do not exactly meet.

Longitudinal tunnel vaults with side compartments.

It is now assumed, that the middle aisle of a basilica with rectangular division of the plan is to be vaulted. The inner longitudinal walls are already carried up high and attached to to them are the semicircular side arches. There is now to be established the centering for the main tunnel vault, which has the form of one half of a semicircular cylinder. On the continuous lagging planks of this half tunnel will be rested those of the cross tunnels, which are laid horizontally from the side arches. (See plan and cross section I in Fig. 17). These can form only low side compartments but not cross vaults.

It was only too natural for men to attempt to carry the side compartments higher by increasing the lengths of the planks. The highest point m of the section was then higher, but it could

The semicircular longitudinal vault is to be changed into a lower elliptical vault, as it more frequently occurred on the side aisles to equalize the heights of the crowns, Fig. 18, but this could not actually come in consideration for the middle aisle; the thrust would thereby be much increased; when choosing it was just here a main question. For more worthy of note and especially in Germany with semicircular forms, to which we therefore return.

If the principal tunnel vault remains round, then must the new rise or ribs arch or by the entire change in the curvature of the formerly cylindrical surfaces of the compartments.

of the vault.

The semicircular side arch was cast already changed by wattling the until its crown had the same height with the tunnel vault (Fig. 20 1). If the lateral pieces were laid perpendicular to it against the surfaces of the tunnel vault, they formed a [illegible] cross vault. It no such form as christina must be termed a formed a cross vault, still it did not correspond to the idea [illegible] or it by ten, for the more fluor (see plan) one

never be at the middle of the vault, at the highest point being at the contact n of the tangent g h. (See plan of section II in Fig. 17). A cross vault then did not originate in this manner.

Accordingly with round-arched tunnel and semicircular side arches after the Roman mode might be produced a vault with side compartments, but never a cross vault. Alterations would be made, that might extend to the main tunnel vault or the transverse compartments.

The semicircular longitudinal vault is to be changed into a lower elliptical vault, as it more frequently occurred on the side aisles to equalize the heights of the crowns, Fig. 18, b but this could not actually come in consideration for the middle aisle; the thrust would thereby be much increased; when opposing it was just here a main question. Far more worthy of consideration is the form of the pointed tunnel vault, which at the end of the 11 th century and in the 12 th century appeared in many places on account of its favorable structural qualities. Combined with larger side compartments this allowed a section like a cross vault over a rectangle (Fig. 19). It is possible that the pointed tunnel vault was a link deserving consideration in the course of the development of the Gothic vault, but before it could afford opportunity in import- and rebuildings, there were continued experiments in other places and especially in Germany with semicircular forms, to which we therefore return.

If the principal tunnel vault remains round, then must the transverse compartments be changed, either by introducing a new form of side arch or by the entire change in the curvature of the formerly cylindrical surfaces of the compartments.

Elevation of the side arches to the height of the middle of the vault.

The semicircular side arch was most simply changed by stilting it until its crown had the same height with the tunnel vault (Fig. 20 I). If the lagging planks were laid perpendicular to it against the surface of the tunnel vault, they formed a half cylinder, that intersected the great half cylinder in cross form. If an arch form so obtained must be termed must be termed a cross vault, still it did not correspond to the idea formed of it by men, for the groin lines (see plan) did not

lie over the diagonals, but modern forms recurved lines in a
true plan (see plan). Such more little adaptations to the one.
and particularly must screen needs result in a variation.

Men could not be contented with this vault, but must strive
for greater grandiose. To storm them, they could not have
them as a technical result or the intersection of the surf-
aces, but must stand from them, first fixing them and thereby
accentuating the form of the curvatures. But this meant an
important change in the tendency of vaulting, for in place of
the surface became the importance of the line. It was easily
clear the error lines. It was then only a question of time
(see Chapter Four: Modern Architecture) to re-
nounce the ancient mode of leaning. But they could not do so
like so far, and at the beginning were perhaps satisfied to ad-
apt the diagonal groin above on the plan leading of the grid-
shaped diagonally). We assume this for the time.

[illegible] clearly, based the possibility for the beginning
[illegible] thereby altered the way for the introduction of other forms
of access, particularly of the pointed arch.

Elliptical form of arches that are seldom built nowadays
with some attention, but are mostly changed into horizontal
ellipses and also frequently are vertical; in the side aisle
of S. Mary at Dortmund both appear at the same time, and in
[illegible] of the Dortmund Church at Paderborn the longitu-
dinal [illegible] a similar [illegible].
(The groin arches in the latter are quite irregularly curved).

It instead of the inconvenient elliptical line arch and al-
[illegible] plans be laid from this to the enclosed diagonal
[illegible]

lie over the diagonals, but rather formed recurved lines in t the plan (see plan), that were little attractive to the eye, and particularly must appear undesirable in a painting.

Men could not be contented with this vault, but must strive for regular groin lines. To obtain them, they could not take them as an accidental result of the intersection of the surfaces, but must start from them, first fixing them and thereby determining the form of the compartments. But this meant an important change in the technics of vaulting, for in place of the surface became the prominence of the line. If men would first fix the groin lines, it was then only a question of time when they passed to place under them diagonal centres, and abandoned the ancient mode of lagging. But they could not go quite so far, and at the beginning were perhaps satisfied to sketch the diagonal groin above on the plank lagging of the principal tunnel vault.(Perhaps by plumbind down from the line stretched diagonally). We assume this for the time.

Where lagging planks are laid from the groin line drawn in this manner to the wall surface, there result here side arches with the form of a vertical half ellipse (Fig. 20 II). The use of such elliptical arch curves is to be considered so far, as they already formed the transition from the semicircular form, and thereby cleared the way for the introduction of other forms of arches, particularly of the pointed arch.

Elliptical form of arches that are seldom built mathematically correct are very frequently found in Romanesque works with some attention, but are mostly changed into horizontal ellipses and also frequently are vertical; in the side aisle of S. Mary at Dortmund both appear at the same time, and in a side aisle of the district Church at Paderborn the longitudinal tunnel vault shows a stilted ellipse as cross section. (The groin angles in the latter are quite irregularly curved).

If instead of the inconvenient elliptical side arch and stilted semicircle is retained (which was perhaps already arranged on the previously erected walls), then could also the lagging planks be laid from this to the sketched diagonal groins (Fig. 20 III). Then the planks are no longer perpendicular to the side arch, and the surface of the compartment no longer remains a regular cylinder. Men had thus freed themselves from the cylindrical surface, but when this once occurred,

nothing more stood in the way of giving the side arch any other form desired, for example that of the pointed arch, far preferable for other reasons (Fig. 20, IV).

It is evident that by means of high side arches could be produced a rectangular cross vault with regular diagonal intersection. Men were led by this to abandon the semicircular arch or to forsake the cylindrical surface of the compartment.

Rectangular cross vault with low side arches. Elevated t transverse compartments. Swelled compartments.

Men must give up entirely the old form of compartments, if they desired not to use high side arches, but in the original and always preferred case in the Romanesque period, particularly in Germany, to turn to simple semicircles for side arches, littel or not at all stilted. Assuming that the side arches of a vault to be constructed were already built, and the tunnel vault inserted as a half cylinder, and further the groin lines were drawn on the plank centering, since they desired to have an exactly regular intersection of the compartmentr (Fig. 21). Then was to be laid the laggingfor the side compartments. That the end could not be attained in the usual way is already shown in Figs. 17, I, II. The planks abut against the longitudinal tunnel vault without reaching the crown. Yet if a connection of the side arch with the diagonal groins is forced by the lagging, than a part of these must penetrate through the surface of the tunnel vault, as shown by the hatched area onthe plan in Fig. 21. The lagging of the principal tunnel vault thus can no longer extend through, and what is more important, the lagging of the transverse compartments forms a surface, which appears to hang down like a trough; but this form is ugly and is structurally doubtful. (See the different views in Fig. 21. The trough must be filled, whether by a layer of earth or an added centering or by both together. (A different direction of the lagging planks already aids in filling the trough). This addition can be carried high like a swelling, that was even necessary to a certain extent, if the groin is to be everywhere recognized as projecting beneath. In this manner is formed a domical swelled surface of the compartment. If this swelling had been once employed for both rising transverse compartments a b o and c d o, it was natural that on account of similarity it should be carried to the two others, aco and bdo.

ellipse to the semicircle.

It was first assumed for the long vaults, that in the longitudinal compartments; but this was not long before a longer form, just as the rising cross tunnel vaults had already invented, but which must appear as a difficult matter to the semicircle. It must have had also a very great influence, that apparently when very poorly understood the laying out of elliptical centering arches, and already for this reason favored different forms. Frequently instead of the ellipse itself was resorted to the stilted semicircle, which the Byzantines employed entirely for their stilted cross vaults.

The Romanesque cross vault with stilting and swelling.

directions (Fig. 22), that is to be reached as the final result of the Romanesque treatment of vaulting. It has the same advantages as for the square bay as for the rectangular, and therefore is employed for both. The Saxons consciously worked forward in its development, the first being the difficulty

Independence of the diagonal groin. Transition from the ellipse to the semicircle.

It was first assumed for the last vault, that in the longitudinal direction of the middle aisle were still inserted true cylindrical compartments; but this has now become without porpose. For these vaults were already established supporting centering arches beneath the groins, and the lagging rested on t the latter above the always existing cross arches, thus they extended through. But also the destre to give them the form of a continuous cylinder also ceased, and they could as well take a freer form, just as the rising cross tunnel vaults had. In other words cross and groin arches could be just as independent of each other in the longitudinal direction, as the side and groin arches already were in the transverse direction. I.e., the groin arches could now be shaped just as desired. This was again an important advance. The flat elliptical shape of the groin arch was a defect in Roman cross vaults, which the Byzantines had already invented, but which must appear as a difficulty in the middle aisle of the Romanesque basilica. Therefore men would so readily adopt no variation from the ancient vaults, as just this freeing from the strongly thrusting and also statically objectionable form of the ellipse, that was usually replaced by the semicircle.

For an elongated rectangular bay the transition to the semicircle made no very great progress, since the ellipse was here very near the semicircle. It must have had also a very great influence, that apparently men very poorly understood the laying out of elliptical centering arches, and already for this reason favored different forms. Frequently instead of the ellipse itself was recourse had to the stilted semicircle, which the Byzantines employed entirely for their stilted cross vaults.

The Romanesque cross vault with stilting and swelling.

Thereby was created the swelled cross vault, stilted in both directions (Fig. 22), that is to be regarded as the final result of the Romanesque treatment of vaulting. It has the same advantages as for the square bay as for the rectangular, and therefore is employed for both. Two factors constantly worked together in its development, the first being the difficulty in covering the rectangle, and the second was the need for lessening the thrust, the last being an important end for which

the first aided in finding the way. How far the precedent of the far earlier but substantially different stilted vaults of the Byzantines may have aided may remain an open question.

The origin of the stilted and swelled vault is frequently derived directly from the square bay without reference to the rectangle, and therefore the reduction of the thrust is regarded as the sole impulse for the transition from the elliptical to the round arched groins. The way is direct but too unreconciled, it contains a gap, that was first bridged when men took into consideration the necessary procedure by vaulting a rectngle, as such was attempted in the preceding.time. But it should not be denied, that besides the contemporary succession must be assumed a constant alternation of advances, with each that are taken into consideration. Until for the history of c construction and of art equally important questions on the mediaeval development of vaults are fully cleared, must it still require very thorough studies. Unfortunately former drawings of buildings show by comparison with the reality very little confidence concerning the vaults. To correctly investigate the latter are required accurate detail drawings, for which the cooperation of many professionals would be desirable. Never should the particularly favorable restorations be allowed to pass in important works without accurately measuring the forms of the vaulting arches, their diameters and centres, as well as the forms of the compartments and all structural details from the buttresses, so far as they may be of value to publish.

Form of compartmentr of stilted vaults. Groin or channel in the groin lines.

To the stilted and swelled vault represented in Fig. 22, that is quite typical of the later Romanesque works in Germany, is always to be conceived that cross arches are added. Its groin arches as well as the four enclosing arches are semicircles, and they consequently are covered by an ordinary spherical surface. Between these six arches extend the four compartments, for whose form are three possibilities.

1. The compartments are so stongly swelled , that they lie outside the spherical surface, and there results a true cross vault with groin angles projecting beneath. In the sectionof the diagonal view represented in Fig. 23 a is given the curve X–X

the point P appears two acute angle of the cross.

2. The centerpoints lie exactly in the spherical surface. When results no cross vault but a peaconchive zone. But the diagonal zone does not project from the surface, as shown in the section II in Fig. 25.

3. The centerpoints lie inside the spherical surface. In this case are strictly taken are found no cross vaults, but a kind of cloister vault, and the diagonal angles appear when seen from beneath, not as projecting acute angles, but as reentrant angles. See section III.

Vaults of the last kind are indeed not rare in the period of the Renaissance and transition style, and particularly often are found those in which the diagonal arches appear as groins in the great church of S. Maria at Lippstadt, and in the interesting strongly stilted vaults of the district church at Paderborn. Fig. 26. In both examples exist no projecting ribs. The last are very suitable to make hollow intersections of the surfaces of compartments unseen by the eye, wherefore even the occurrence of hollows in Gothic vaults is not usually considered.

If it is desired with semicircular groin and side groins to

-4-

re one driven to a strong swelling. The latter had thus in such vaults the twofold purpose to allow the groin lines to project and to avoid the thumb-like arching at the crown of the compartment. But the swelling was given a heavy accretion, so long

[illegible]

[illegible]

lasting, then must a considerable volume of earth be passed. This was particularly great when two planks were placed as shown in the triangle 1 to 2 of Fig. 22 b. The addition might be reduced somewhat by giving the planks the positions given in the triangle 2 o c, that better equalizes the sinking at the crown. Yet the hooping still remains so considerable, that for many built vaults must be allowed 100 wheelbarrows and more of earth. Such a mass permits it to appear questionable that such work of preparation was so much more elaborate. Where a s

of the compartment I given the larger scale in Fig. 23 c; at the point n appeaas the sharp angle of the groin.

2. The compartments lie exactly in the spherical surface, then results no cross vault but a pendentive dome. But the diagonal arch does not project from the surface, as shown in the section II in Fig. 25.

3. The compartments lie inside the spherical surface. In this case and strictly taken are found no cross vaults, but a kind of cloister vault, and the diagonal angles appear when seen f from beneath, not as projecting groin angles, but as reentrant angles. See section III.

Vaults of the last kind are indeed not rare in the periods of the Romanesque and transition styles, and particulary often are found those in which the diagonal arches appear ar groins in the lower part and is hollows in the upper part, for example in the great church of S. Maria at Lippstadt, and in the interesting strongly stilted vaults of the district church at Paderborn. Fig. 26. In both examples exist no projecting ribs. the last are very suitable to make hollow intersections of the surfaces of compartments unseen by the eye, wherefore even the occurrence of hollows in Gothic vaults is not usually considered.

It it is desired with semicircular groin and side arches to obtain a cross vault with projecting angles of the groins, then is one driven to a strong swelling. The latter had thus in such vaults the twofold purpose to allow the groin lines to project and to avoid the trough-like sinking at the crown of the compartment. But the swelling was always a heavy addition, so long as one constructed the compartments on a complete centering.

When the form of the swelling, as mostly adopted for rubble stone compartments, was prepared for by a form of earth on the lagging, then must a considerable volume of earth be heaped. This was particularly great when the planks were placed as shown in the triangle d o g of Fig. 23 b. The addition might be reduced somewhat by giving the planks the positions given in the triangle g o c, that better equalizes the sinking at the crown. Yet the heaping still remains so considerable, that for many built vaults must be allowed 100 wheelbarrows and more of earth. Such a mass permits it to appear questionable that earth was the sole expedient; but if one must assume a filling of wood, the work of preparation was so much more elaborate. Where a s

suitable material was at hand, even in the Romanesque period freehand vaulting must already have been in use much more than is usually believed.

Gothic vaults with swelled compartments built freehand.

The centering of a swelled vault always remained a great difficulty, one must therefore regard it as a great acquisition, when men generally learned to construct the compartments in freehand. (See compartment masonry later). The independent construction of the projecting ribs on centering arches, and the resulting construction in courses of light swelled compartments without any centeringis to be regarded as the most perfected method of construction of Gothic. It stands higher than all which the art of vaulting had previously undertaken.

Gothic rubble vaults on centering. Avoidance of the swelling, pointed side and cross arches.

The compartments built freehand, in spite of their advantages, were only adopted where bricks and easily wrought limestone ((Isle de France) and also always tufa formed the material of the vault. Vaults of rough and heavy rubble were constructed on centerings until in the 15 th century. In these was again omitted the swelling on account of its difficult execution, but which was only possible by an alteration of the form of the v vault. The swelling resulted from the strong stilting, and the latter must be avoided. For this purpose to lower again the g groins would have been a step backward, and therefore the side arch was raised by stilting the semicircle, or better by the more pleasing pointed arch (Fig. 24). If the groin arches remained semicircles, then must the crown c of the arch be raised to the height of the middle, so that no drop occurred in the ridge o c.

By this vault the second condition is also satisfied, that the groin lines did not become hollows in spite of the omitted swelling. To have an approximately correct indication, whether groin or hollow is to be expected, it is best to draw the diagonal view of the vault. If in this ar in Fig. 24 the projections of the half arches b c and a d remain outside the diagonal arch a o b, then are to be expected projecting groins; commonly if they lie within the diagonal arch as in Fig. 23 a, then may one count on hollows.

In Fig. 24 the groin arches were drawn as semicircles. But

In Fig. 85 the left half above a semicircular groin, and a common ridge is not to be avoided there. At the right is employed a pointed diagonal arch, which permits a straight ascending ridge o g. The possible amount of the rise is made if a tangent is drawn, to the projection of the pointed arch.

The importance of the pointed arch for the groin lines is often unobserved. Even in important places.(Violet-le-Duc again in the passage [illegible] [illegible] [illegible] tion above, that here is to be sought an important reason for [illegible] Chapelsberg have pointed groins with straight rising concentrations. [illegible] [illegible] straight ascending groin arches very nearly pointed. The ridges of the compartments have a steep and straight inclination with a [illegible] region of the [illegible] in Fig. 87 reproduces the [illegible].

Also it [illegible] [illegible] [illegible]

just for these pointed arches may have great advantages. Only with pointed arched groin lines is it generally possible to erect a raised cross vault without convex ridges (Fig. 25,right).

In Fig. 25 the left half shows a semicircular groin, and a convex ridge is not to be avoided there. At the right is employed a pointed diagonal arch, which permits a straight ascendridge o g. The possible amount of the rise is made if a tangent at o is drawn to the projection of the pointed arch.

The importance of the pointed arch for the groin lines is often undervalued. Even in important places.(Viollet-le-Duc andin the earlier editions of this work, the semicircle is regarded as the customary form of the groin). But more exact consideration shows, that here is to be sought an important reason for the introduction of the pointed arch! The Romanesque vaults at Lippoldsberg have pointed groins with straight rising compartments of sandstone. The vaults of the district cumrch of Paderborn (end of 12 th century) with round side arches show high standing groin arches very nearly pointed. The ridges of the compartmentshave a steep and straight inclination with a deflection at the top. Sketch in Fig. 27 represents the vault.

Also at Paderborn are also found other vaults with pointed groins and ridges rising in straight lines, thus in the cloister and the tower of the cathedral. Also on account of its arrangement of buttresses, the latter vault is found far above in the tower, and always to reduce the thrust, is carried so high that the angle at the crown is only about 110°.(Fig. 27).

It appears in the following of the development of the vaults of the 12 th and 13 th centuries, as again and again occurred discussions, that imperatively referred to the introduction of the pointed arch. Here only the most important of the reasons are mentioned, which are derived from the change in forms; to these are added the even more important one of strength. It is particularly the lesser thrust of the pointed arch against the abutment and its own tasteful form, which for most cases of l loading considered (even without a vertex load) comes astonishingly close to the theoretical line of support with very important advantages.

The adoption of the pointed arch forms one of the most important steps in the history of mediaeval construction. The view of laymen must be termed erroneous, that holds Gothic and the

pointed arch as inseparable, yet it is to be desired that the introduction of this form of arch indicates those stages of t the development of mediaeval art, which shows complete freedom from the old fetters and the mediaeval principle is brought to complete acceptance, and to develop every construction from c case to case out of the internal nature of the matter. To take up the old contention over the origin of the pointed arch must be scarcely advisable here, since this question has entirely vanished in comparison with the fact, that it is properly used.

In the preceding is followed briefly the step-like transformation of the cross vault from the Romans to the beginning of Gothic, and its further development in the Gothic period will receive a more thorough discussions in later sections. In the process of development indicated, the middle aisle played a striking part. It occupied itself with the execution of vaults of greater heights with the admission of light at the sides a and with less thrust, which further must be suited to a rectangular division of bays. But there occurred other places in the plan of the church in which were treated forms far more irregular than the rectangle, namely to master trapezoidal and polygonal forms of plan. It is advisable to make clear by at least some examples the inadequacy if the Roman traditions.

Vaulting the trapezoidal bays of the choir aisle.

As stated above, the middle aisle presented the first, but the choir aisle offered the second and greater problem in the vaulting of the basilica. The choir aisle is to be understood as the extended and curved side aisle, and it therefore required the same vaulting that the side aisle received. But from its curved plan resulted difficulties of all sorts.

If the side aisle was covered by a longitudinal tunnel vault, it was easy to continue this in annular form around the apse, but as soon as side compartments appeared, there arose the difficulty, that these were larger on the outer than on the inner circle. Fig. 28.

But if the cross vault (or even the transverse tunnel vault) was employed, then would the dilemman be still greater, for there resulted a trapezoidal bay (Fig. 29), whose outer side a b is longer than the inner one c d. Now if semicircles are erected on both sides and are joined by a connecting surface, this is no longer a half cylinder as before, but has the form

1. The diagonal ridge r s in position 1 --- and then p of the groin
ceases to project when r of the side arch, so that the ceiling
of the arcade and dresses down to rest on the outer wall (fig. 31).

2. The result is as in the last case, yet the arcade arches
are shifted, so that the arcade capitals are at the same height
as the wall capitals (fig. 32).

The last arrangement was the most satisfactory, but it
would increase in the oblique line o r. Another the vertical
[illegible] c j decreased, then the masonry block only
a part have a hexagonal plan instead of a square plan x.
fig 31). This expedient was scarcely employed in the
[illegible] of the 12 th century, but it let us to early beginnings
tion of the central nave from other ideas. It is evident
that the pressure. That the views of writers [illegible]
must have introduced more and more as little as the middle
ought to a satisfactory result especially needle. Likewise w
there were added transverse ribs, well and groove ribs.
ble one [illegible] to rest be concealed but not removed.
rel 4, this could also be applied over a symmetrical [illegible]
ond (as easily over every square ground plan inscribed in a circle,
fig. 34. Here the groins and the four side arches were
circles. For leaning structural faults, the term sag --- see
tion apart but forms or the elevations of the arches of the
groins --- the segmental ogives. Such are intersection of

of a half cone widest outside. The intersections of this conical surface with the annular tunnel vault surrounding the choir form a cross vault of different shape.

There are three peculiarities for this cross vault, if there are excluded the experiments with elliptical and other forms of arch differing from the semicircle.

1. The centres r and p of the side and arcade arches in Fig. 30 lie at the same heights --- so that the ridge rises from n to m. The point of intersection is no longer at the middle of the annular vault (Fig. 30).

2. The ridge m n is horizontal --- and then p of the arcade arch is higher than r of the side arch, so that the capitals of the arcade are higher than those on the outer wall (Fig. 31).

3. The vault is as in the last case, yet the arcade arches are stilted, so that the arcade capitals are at the same height as the wall capitals (Fig. 32).

The last arrangement was the most satisfactory, but it had the defect, that the upper conical surface intersected the vertical intrados in the oblique line o p. Thereby the vertical surface received an ugly appearance. If it is desired to have this intersection o p horizontal, then the masonry block below o p must have a trapezoidal plan instead of a square plan $\frac{x}{x}$. See Fig. 30. This expedient was actually employed in the churches of the 12 th century, but it let to an ugly trapezoidal plan of the capital aside from other faults. It is evident from the preceding, that the forms of vaults based on the ancient Roman traditions here led just as little as for the middle aisle to a satisfactory result generally usable. Likewise when There were added projecting cross, wall and arcade arches, would the weakness at most be concealed but not removed. When m men passed to the Romanesque vaults represented in Figs. 22 and 23, this could also be employed over a symmetrical trapezoid (as generally over every ground plan inscribed in a circle). Fig. 34. Both the two groins and the four side arches were semicircles. But besides structural faults, the form has --- see below under the forms of the elevations of the arches of the cross vault --- the esthetic defect, that the intersection of the groins lies sidewise from the highest point of the vault.

Freedom of the form of the Gothic vault in plan and elevation.

A solution resulting from all requirements for the choir as

aisle and the middle aisle first occurred, when the development of the vault had found its highest aim in the Gothic cross vault. With the Gothic vaults came entirely to light a principle, that step by step tirelessly included in itself nothing less than complete freedom of form. For this form of vault is there no further restraint in plan and the development of the elevation. (Fig. 25).

The vaulted bay may have any form of plan. The enclosing arches can be determined independently of each other as round, pointed or stilted arches, indeed may also have the form of a mathematically formed line of support.

Exactly the same is true for the diagonal arches. The same freedom further exists for the mutual heights of the keystone and crown of the arch. finally the ribs may be multiplied in an unconstrained manner, and combined in the most wonderful s star and net figures. Generally limits were fixed for the Gothic forms only by the laws of the equilibrium of forces and by the requirements of beauty.

These results in the treatment of forms are astonishing, and entirely equal in rank are they established beside those relating to the mode of execution, and which terminate in the final aim to limit the mass of dead material as well as the amount of the rough labor.

Contrast of Roman and Gothic methods of construction.

To correctly understand the aim of the mediaeval method of construction, it is well to contrast it with the Roman.

Their mode of construction (formation of a massive connected body) allowed to the Romans a tolerably great freedom in the form. But they did not utilize this liberty, yet placed the f form in fetters for architectural reasons. For arches and vaults rose the semicircle to an invariable typical structural form, that established itself in the same line with the columnar orders transmitted to them by the Greeks.

The middle ages struck out the contrary path; the form was loosed from this external restraint, its shape is freed from artistic creations, but for this it conversely surrenders just the strong domination of the construction.

The Barocco style rejected both restraints, it loosed itself from strict form and construction, in whose place it set in t the foreground artistic caprice.

For Roman architects was first fixed the architectural form of its construction must be suited to it. The Gothic master m must develop the form from the construction and then give it the artistic stamp. The latter perhaps had the most laborious way, but only thus could he solve his problems in a way impossible to the Roman.

The difference particularly occurs in the limitation of the masses. While Roman vaults seldom exhibit less than 1.2 m and frequently 2 to 3 m in thickness at the crown, and even Romanesque vaults of medium span have a thickness of 40 to 50 c m and more, Gothic vaults are built with a moderate execution of the ribs and even over wide rooms, that require a thickness f for the compartments of only 10 cm or less. The economy in the mass of abutments has an equal advance. Masses generally only occur where forces act; accordingly the perfected Gothic architecture shows a clear separation between the supporting skeleton and the filling surfaces.

That is followed from above downward. In the vaults the compartments form light filling surfaces, while the ribs have to support, and they transmit their vertical forces to the piers, whose thickness only needed to be small; but on the contrary the horizontal thrust was transferred to strongly resting buttresses and flying buttresses. The outer wall only has to enclose here, and according to its purpose it consists of stone or of a freely extended glass surface.

The effect of the forces in both structural arrangements must be regarded as thoroughly different. A true Roman structure f forms a single lifeless and quiet massive body, that is to be compared to a hollowed rock, or if we will, to a terra cotta vessel. The whole is held by internal forces, that depend on the strength of the material.

The Gothic structural system is more like a living and elastic system of numerous separate bodies, that are held in a fixed condition of equilibrium by forces acting on each other. If an external change of form occurs, for example if the foundation wall sinks slightly at one side, the Roman work breaks into fragments in the same way as a vessel, on the contrary the Gothic structure will rather show a displacement of compression in different parts, that in a somewhat changed condition again seeks to assume a quiet position.

The contrast of the two great structural divisions of Roman antiquity and of the middle ages is accordingly decided, but it easily finds its foundation in history. The Romans were a people dominating the world, and they had at command inexhaustible wealth and numberless enslaved laborers. For them it resulted from the nature of the affair without considering the nature and means of building, that a thoughtful master was himself able to erect mighty structures in distant provinces by thousands of mostly unskilled laborers.

The middle ages created under different conditions, the means were moderate and the workmen were paid. Yet if still great w works were to be undertaken, then must the amounts of the rough mass and of work be restricted, no structural member must have anything superfluous. But this was only attainable by a perfectly developed system of construction carefully worked out by the master, and executed by known sympathetic workmen. What the Romans could do with abundance of power, was here accomplished by the work of the intellect.

2. Construction of Vaults. General.

Surfaces of Roman vaults. Surfaces of Gothic vaults.

The most important difference of the Roman from the Gothic vault lies in the relation of its formative surfaces to the e enclosing lines. In the former are these surfaces, being the enclosing surfaces of the body forming the entire vault, the half cylinder or hemisphere, are the determining parts in the manner, that in cross vaults as well as in the dome placed over the square (the so-called Bohemian or segmental vault), the limiting lines are formed by sections of this body. According to the Gothic principle the area to be vaulted is first subdivided by lines resulting from the system adopted, and over these lines are turned the separate arches, that as ribs of the whole receive between them the masonry of the compartments acting as the covering, and support it.

Let there be in Fig. 36 the ground plan of a Roman cross vault, which then in elevation cosists of four similar sections a b c, b c e, etc., of the half cylinder. Over the sider a b, b c of the room are thus turned semicircular arches and a section f g or h ithrough this surface of the vault parallel to these sides forms a segment of the same semicircle, by whose highest point is given the height of the diagonal arch, so

that the same is fixed by these segments. The erection occurs on a centering, that in a way represents the whole of the vault, and whose external surface therefore corresponds to the enternal surface of the vault. On this outer surface is then laid the stones or bricks forming the vault, either set radially (Fig. a), or on it according to the anmient Roman method is laid concrete composed of mortar and broken bricks.

Masonry vaults. Concrete vaults.

In the first case the separate stones either have a wedge shaped form, i.e., their surfaces diverge in the direction of the radii, and the mortar bed between assumes the same form (Fig. 37), or the whole is made of cut stones or shaped bricks; on the other hand if only ordinary bricks are at hand, then as a rule these retained their rectangular shape, and only the mortar beds acquired a greater divergence outward.(Fig. 37 a). Then the mortar beds hold the bricks in place, and the beds between them cannot slip downward.

In concrete vaults the shapes of the inserted stones are just as irregular as the binding mass of mortar, and just in this irregularity lies the durability of the whole, when the mortar penetrates the crevices and between the stones and broken bricks, binding the whole into one mass.

According to the first mode of construction is therefore ensured the position of each stone, and the centering can be removed as soon as the whole is closed, and in the second when the entire mass has hardened. Therefore there results from this properly only a covering composed of inorganic bodies, that only has the arched form so that the load not only opposes the relative but also the reacting strength, while over a smaller area could also be made of them a plane covering, and actually in the 12 th century even window lintels were so constructed.

Erection of the dome.

The construction of a dome may likewise occur in both ways on a hemispherical centering for placing concrete or by actual masonry set on it.

In the manner of the last construction the special peculiarities of the hemisphere introduce and make possible an important simplification. The tunnel vault and the Roman cross vault developed from it a separate straight horizontal courses, that extend in the half cylinder produced by the semicircle or seg-

Construction of parabolic curves.

segmental, and only when this is closed it then obtained an a assured position. A stone of such a course is shown by Fig. 38. On the contrary in the dome each of the likewise horigontal courses forms a circle. Each stone of such a course receives about the shape shown in Fig. 38 a, and extends as a part of the semicircle forming a section of the dome and then of the circle formed by each horizontal course. It also already acquires an assured position as soon as this course is closed. It thus can be done without a centering, and then that is generally superfluous. Conceive such a dome built of bricks, and in the lower third the courses so nearly approach the horizontal, that the separate bricks will continue to lie on each other without binding mortar. Further up the mortar acts, so much the more readily as it is made thicker and the work goes more slowly, so that it can set. But at the same time with each upper course the radius of the circle lying in the horizontal plane becomes smaller, so that if the binding strength of the still wet mortar no longer suffices, the separate bricks can be held in place in other ways. Yet always the closing of the dome remains a difficult work, from the endeavor to keep the bricks in their places, as well as by the need of fitting them to the uppermost circles struck with smaller radii. For the entire construction is needed a means that shall hold each separate brick in its place. In Romberg's Journal for practical Building, the deceased Lassaulx gave such, which consisted in fixing a rod at the centre of the dome that swung in horizontal and vertical planes about that point, thereby determining by its contact the place of each stone.

Construction of pendentive domes.

The before mentioned conditions of stability suffer a change, if the dome is turned over a square, in its lower courses. Such a dome is shown in Fig. 39 in plan, in section through e g in Fig. 39 a, and in perspective in Fig. 40. There the square a b c d is the area to be covered by the dome, the radius of the dome being a C, and the section on the line a b through it is a semicircle. While in the dome turned over the circle a horizontal course forms a complete circle in itsilf, whose radius diminishes in each successive course, this is then the case here if the courses correspond to the circle inscribed with in the square, i.e., above the semicircles turned over the sides

Combination of qualities of long and cross vault.

of the square, thus to the course indicated by f g in Fig. 39a. All courses below f g form only circular arcs, which extend between the sides of the ground form and exert their thrust directly on them. (Note). They will do this in just the same way as if their radii, instead of from the centre C were struck w with entirely assumed radii, i.e. for example, the course lying at the height n o formed the segment h l k or h m k instead of h i k. Therefore retaining the centre C for the courses lying in a horizontal plane is only required in regard to the c commencing courses at e m, which must find their supports on those beneath them.

(Note). It is to be considered, that as might appear from t the preceding, that the little horizontal ring courses do not extend their ends between the walls. In the longitudinal direction of the courses may indeed also occur a ring stress, that exceeds the particular counter pressure of the dome, but it p passes down in a meridional direction from one course to another, until it enters the enclosing walls. See page 56 below.

Combination of qualities of dome and cross vault.

If one now conceives that the two previously developed systems of the cross vault and dome are combined together, then gradually result all the properties of the Gothic vault. It is the object of this combination to give the cross vault the peculiarly assumed location of each course in the dome, or conversely to give the dome the division of the great surface into four smaller ones by the diagonal arches, and thus to adapt both to easier execution. Fig. 41 shows first the horizontal section through the cross vault in Fig. 36. In Fig. 39 is stated that the lower parts or the feet of the dome extend between the arches turned over the sides of the room, as shown in perspective in Fig. 40. Likewise could these extend between t the arches turned over the sides and the diagonals of the cross vault; hence the surfaces of the compartments in plan in Fig. 41 are enclosed by segmental arches instead of straight lines, as shown in Fig. 41 a. At once when the location of the point b lying in the diagonal arch is ensured, also the location of the two courses a b and b c are so, and thus the difference in execution in comparison with the dome only consists in this, that while in the latter one only required centerings for the arches turned over the sides, here such are necessary for the

diagonal arches also. While here all arches remain unchanged both over the sides and diagonals of the room, there enters into the conditions of stability of the entire vault a substantial alteration. In Fig. 41 the separate stones, as already stated, form only a part of the semicircle or segment generating a part of the cylinder, but it also first rests on the diagonal arch, where it meets the arch to which it belongs. Thus the stone first loads the diagonal arch at t. On the contrary in Fig. 41 it extends in the segment a b and thus rests on the s stone of the diagonal arch placed at the same height, and transfers its load downward in the latter. (Note).

(Note. The last assumption is but partially correct. See on this below Figs. 116 to 120).

Semicircular groins.

While then according to Fig. 41 the lowest point of the diagonal arch has the full load of the semicircle, each more projecting point of the smaller segment and finally the crown have nothing more to bear, the converse condition appears here, when the segment of the course directly rests on the vertex, but n not at all on the lowest point of the masonry of the compartment, but is only loaded by the pressure of the arch itself transmitted downward.

To resist this very considerable loading of the vertex is little adapted the diagonal arch line of the Roman cross vault produced by the intersection of two half cylinders, in so far as shown by Fig. 36 b n d, for a considerable length at the ridge n approximates the horizontal, and this is weakest just where its load is greatest. Therefore it must be replaced by a true arch line and is by a semicircle. Now always retaining t the square plan, if we assume the semicircle as the diagonal arch and the entire vault is shaped in the Roman manner, that gives it as actual section line, then will the rectangular section through a quarter of the vault be a vertileal half ellipse, developed in the opposite way from the semicircle. In Fig. 36 the elliptical diagonal arch is developed from the semicircle shown over the side and generating the vault. Such vaults are also found in some English Gothic works, in the transepts of the foundation church at Wetter (Note), and if we do not err, the collegiate church at Mantes. They always exhibit a still incomplete development of the Gothic system, whose natural

sequence led to making all arches true circular arcs, both over diagonals, sides and the ribs of the entire vault.

Note. According to Schäfer (Zent. d. Baum. 1885) not at Metter but in Godehard's chapel at Mayence.

If now the latter arches are again conceived as semicircles, we return to the dome, and only the horizontal section of the compartment surfaces can make the distinction. But the crown of the diagonal arch as in the dome rises high above the arches struck over the sides of the room, and there occurs a loss of height, even in square and still more in rectangular plans, a loss of height above the latter arches, corresponding to the difference between the side and diagonal. Vaults of this kind are found in the cathedral at Trient. (Note). To avoid the loss in height, equally objectionable in practical and esthetic respects, it is necessary to make the heights of the arches independent of their spans, and first to give the arches turned over the sides a greater height. (Note).

Note 1. p.2[illegible]. Mediaeval monuments of art in Austrian Empire.

Note 2. p. 22. The loss of height is relative, according to whether one starts from the vertex of the side arch or abutment of the vault.

Pointed arches over the sides.

But the before mentioned loading of the crown had already led to giving a form strengthening this point of the arches turned o over the sides of the room, thus substatuting the pointed for the round arch. On the origin and sourch of this form of arch, so many opinions have been already expressed, that we avoid r risking a new hypothesis, when this by far not have the importance assigned to it, and a completely Gothic vault can indeed be conceived without a single pointed arch.

Position of courses in the compartments.

But in the choice of the radii of the pointed arch lay a means sufficient for attaining any desired height, and regulating the proportions of the height of the crown at pleasure, so as to avoid any loss of height.

In the Romanesque tunnel and cross vaults the beds of the s separate courses lie parallel to the rigge joining the opposite vertices of the arches. The same direction of the beds was also at first retained in the Gothic cross vault, only being replaced by another in the late brick structures. This is assumed

in Figs 41 and 41a. Fig. 42 then shows a view of a vault built in this way to about half its height, where the voussoirs of the diagonal arch were either cut with an angle as in Fig. 42a, or received a hook shape as in Fig. 42b. In the first case both the through joint and the acute form of the stones formed a defect, and the last was increased in the courses of the compartment were segmental arches as in Fig. 41a. In the latter case the preparation of the hooked stones was tedious and afforded no corresponding utility, since the bond of the surfaces of t the compartment thus formed over the diagonal arch were not at all necessary. Accordingly it was best to leave the diagonal arch to itself, and to shape and to use stones set radially as in Fig. 43, so that the separate courses of the compartment c could rest against it.

Projecting ribs.

Thereby the system of ribbed vaults already invented, and it required only a strengthening of the diagonal arch and the adoption of an independent form for it, to pass from that of Fig. 43 to the shape of Fig. 43a. In accordance with the distinction of the Gothic vault from the Roman given above, there are established as characteristic peculiarities of the former.

1. The forming of the courses of the compartment in segment-arches, i.e., swelling them.
2. The regulation of the heights of the separate arches.
3. Their independent execution and treatment.

But further will appear there what is said, that the Gothic cross vault combines in itself the earlier principles of vaulting of the Roman cross vault and of the dome, and represents itself in a certain measure as the necessary consequence of them.

Names of the parts of the Gothic vault.

Let Fig. 45 be the plan of the room to be vaulted, in which is given the arrangement of the vault. Then the form of the p plan, here the two rectangles a b c d and b d e f, are termed bays of the vault; the arches a b, b e, a e, e f etc., formed over the sides of the bays are generally called side or border arches, or side arches so far as they are closed by walls. Arches separating adjacent bays like b d, that occur instead of the wall a c, are named cross or transverse arches, or if they separate two aisles of churches are dividing or longitudinal arches. Over the diagonals of the bay lie the diagonal or groin

arches, that in the simplest case are lines formed by the intersecting surfaces of the compartments and are well named the groin lines. If they project from the surfaces of the compartments as more or less richly moulded members, they are termed ribs, and thu according to their location on the plan are cross ribs, groin ribs and side ribs. The highest point of the arch is the crown. The length of the ground line over which is turned the arch is the span, and the height of the crown above t the springing is the rise. For example, men say that the arch a b has 5 m span and 3 m rise. The vault surfaces extend between the arches mention are the compartment surfaces, or taken bodily are the compartments, the segmental arch in which the curve of the compartment is vaulted is called the swelling.

In complicated plans of star and net vaults the separate arches as a rule are not named, all ribs or groins being designated as the framework or skeleton of the vault. Yet here are also dossible distinctions between main, intermediate and ridge ribs, etc. The cut stone in which two or more ribs either intersect or meet is termed the keystone or boss.

3. Simple Cross Vault.

Forms of arches.

The Gothic cross vault allows the greatest freedom to the d different arches in the development of their elevations. When the single fundamental condition --- the equilibrium of the forces --- is satisfied, each separate arch may assume its own independent form. Accordingly the vaults of the Gothic exhibit the most varied diversity. Besides the semicircle (Fig. 41 I) the most common forms of arches are the more or less slender pointed arch (II, III), and the stilted or raised pointed arch (IV). The side of the pointed arch is chiefly struck from one centre, but sometimes from several (Figs. 48, 49). For lesser rise prevails the segmental arch (V) and the depressed or broken arch (VI), the latter especially in the English and also in secular German Gothic. Finally occasionally occur the ellipse, oval, horseshoe and others. Predominant is the pointed arch on account of its many qualities.

Equality of radii. Height of crown.

But with all the freedom in the form of the arch, definite considerations lead to bringing the arches into legitimate relations to each other. These appear first in regard to the

radii used in striking the arches, then concerning the corresponding height of the crown. In the first case the nearest construction is that, according to which all side arches are struck with the same radius as the groin arches. See Fig. 45.

The diagonal arches as here assumed are semicircles and are revolved down beside the diagonal. For the end b are laid off the sides a b and a c of the bay on the diagonal at a′b′ and c′b′, and above are erected pointed arches with the radius of the semicircle. The left centre of the pointed arch falls at the centre of the semicircle C, the right hand points lie on the ground line at C′ and C″.

In this construction all arches have exactly the same form in their lower parts, so that as shown by the Fig., they coincide in one half. This shape offers great advantages for the beginning of the vault, giving it a regular appearance and makes its construction easier, especially if numerous members separate in fan shape.

But to the general use of this arrangement were usually opposed difficulties. It is first embarrassing that the height of the crown of the arch is fixed by it; the crowns of the side arches are lower than the middle of the vault, and they also themselves differ in a rectangular vault, indeed so much the more, the more the plan to be vaulted varies from the square. But at the same time, for example, if the sides of the bay are in the ratio of 1 to 3, in b c d l, the arches struck over the short sides assume a very pointed form (lancet). (See b S″ e in Fig. 45.).

Such a form for side arches has not exactly structural objections and affects only the possible insertion of a window, but those defects appear more decidedly if one conceives that the wall b c is replaced by a cross arch, which is concentric with the side arch and receives a still pointed form of inarados. But here will be enhanced in an overpowering way the characteristic peculiarity of the pointed arch, and ~~x~~ a load or force on the haunch will be transformed into a force acting on the keystone upward, that again requires to be neutralized by a loading on the crown. Even more does such a fault make itself felt, if the side thrust of this arch p q acting at e opposes one i i of much wider span. Not only will the ~~play~~ the part of a force acting on the haunch of b e and receive the outward

force, but it will find no resistance in the side thrust b e reduced to a minimum, and therefore makes necessary a substantial increase in the dimensions of the pier e.

The dependence of the height of the crown on the choice of the radius can thus easily become inconvenient, and it may even be entirely necessary to fix first the height of the crowns of the separate arches according to other definite aids. Those d different reasons may require that either all crowns must be at the same height, or that the crowns of the grcin arches and thus the middle of the vault be higher, or even that the vertex of one or of several side arches rise higher than the crown of the diagonal arches. To justify the requirements on both sides, men have attempted to fix the height of the crown according to the requirements, and yet to strike all arches with the same radius.

In the Gothic A B C book of Fr. Hoffstadt (Frankfort-a-M,1840) is a method applied for this purpose, which is explained in Fig.46.

For the side arch with span b e, the crown is fixed at i, for example here at the same height as the crown of the diagonal arches. Then both branches of the pointed arch are struck with the radius of the geometrical arch from the centres k and l.

The arches cut the verticals at the sider and thus take a f form corresponding to the Moorish horseshoe arch. This form of arch is little to be recommended, which but occasionally was constructed in the Gothic period as in Canterbury, for example, and may be avoided by changing the arch below m into a straight line. But then results a stilted broken arch, just as unsatisfactory and is much better replaced by a regular pointed arch.

Since by the construction above the desirable uniformity of the springings of the arches is not attained, but otherwise few advantages result from uniformity of radii, this experiment must be regarded as useless.

Another method of obtaining equal radii, with close heights of construction, places the centres of the larger arches below the base line, whereby result broken arches (Fig. 47). The appearance of the latter for a slight drop of the centres is not very disturbing, and statically it is even favorable. The desirable uniform springing of the arches is indeed also not attained in this way.

Compound pointed arches.

Worthy of consideration is a construction, that already since the end of the 13 th century found great use in English Gothic- it is based on this, that each arch is composed of two parts with different radii, indeed so that all lower parts have the same radius. Fig. 48.

The lower parts a b_1, a b_2 or a b_3, are all struck with the same radius from the centre o. On the contrary the upper parts have their centres at c_1, c_2 of c_3. The crowns are placed at the same height, as a rule in the rich English vaults.

Thus it is possible to obtain uniform springings for the arches and also to be able to fix the heights according to pleasure for the crown of each arch. The frequently recognized forg of arch, which naturally carried from the vault the concentric line of the window, thus does not lack a certain practical justification, though statically this form is less favorable than the ordinary pointed arch.

It would not be objectionable where permissible also to take the form of the pointed arch sketched in Fig. 49, where conversely the upper part of the side of the arch is struck with a smaller radius, for it is statically favorable and is also enentirely staisfactory to the eye with a small change in the r radius. Viollet-le-Duc asserts in his Dictionary, Vol. VI, p. 29, that pointed arches frequently occurred in the 12 th century, whose sides at their middle part are struck with a greater radius. Likewise could a statically favorable form be obtained thereby.

Stilting.

In general it is not advisable to assign too great importance to fixed theoretical rules of construction, but one should rather first be guider by a regard to beauty and the actual practical requirements. Most reach the aim by the ordinary pointed arch. If one neglects the uniformity of the radii, it will be easy to give the arches any desired form, and if necessary to afford any desired height by stilting.

For example, if it be desired to open the side arch by a window or a free opening with a fixed shape of arch, then the side arch is properly drawn concentric as much as the desired height of the crown requires. In this Fig. it is assumed that the side arch has the same height of crown as the groin arch represented with it should have. The difficulty that may be caused by stil-

waiting for the springing of the vault, etc., will be add-
ed at the proper place.

The development of the various forms of the arches for several
different vaulting cells, whether each other, each be
and essential. There can either occur a central
of the radials, or at the entrance of an outer floor
similar manner compels this. The curves are made the same
ent. The choice of different ones and the use of existing
them leads to the aim. Secondly must static requirements and
the difficulty of forming the springers (see that) will
be left out of view.

Having over the

Case of trapezoidal form.

Irregular cases require special consideration. If the
of the area to be vaulted is a trapezoid as in Fig. 52, the
the whole system cannot be turned exactly over the diagonal,
since these will consist of two very different portions.
smaller one either being obtuse, or as in Fig. 52 have
ently results, then not longer will appear over the section.
ore the possibility can be considered. Therefore the crown
[illegible]

[illegible]

[illegible]

stilting for the springing of the vault, etc., will be discussed at the proper place.

The development of the elevations of the arches for several different vaulted bays adjoining each other, such as occur in churches with several aisles, particularly makes careful judgent essential. There can either occur a beneficial graduation of the heights, or if the existence of an upper floor or any similar reason compels this, the crowns are made the same height. The choice of different radii and the use of stilting always leads to the aim. Naturally must static requirements and the difficulty of forming the springings (see that) will not be left out of view.

Vaults over irregular bays.

Bays of trapezoidal form.

Irregular bays require special consideration. If the plan of the room to be vaulted is a trapezoid as in Fig. 51, then the groin arches cannot be turned exactly over the diagonals, since these will consist of two very different branches, the smaller one either being stilted, or as in Fig. 51a must be struck with a far larger radius. But the structural difficulty results, that the larger half presses over the smaller, so that the stability may be endangered. Therefore the crown must be transferred from the intersection C of the diagonals toward the longer side, about the middle of the middle line e f at C′, or better a little farther to the geometric centre or even the c centre of gravity C″ of the trapezoid. Then the four halves of the groin arches will have approximately equal stresses. If one desires to go farther and to place the crown about over the p point g, which is equidistant from the four angles, then would be obtained four entirely similar groin arches, but equilibrium would now be disturbed in the opposite direction. Now would the total thrusts of the arches d k and c g exceed those of the t two others, and thereby the crown would be pressed toward the longer side. Various experimental arrangements are shown by the middle ages at the time, when it was necessary to give the trapezoidal bays of the choir aisle appropriate groin vaults. (See preceding page 16). Thus in the choir of the cathedral at Langres (Viollet-le-Duc, IV, p. 70) the groin arches are still turned over the diagonals, and they had the form of the semicircle. Hence their intersection lay far beneath the crown,

so that the pendant extending downward had an inclined position.

It frequently occurs, that the groin arches in projection on the ground had not four straight but broken lines. Aside from this, that already by the irregular intersection of cylindrical or conical surfaces, this form often resulted of itself, (see Figs. CC and 20 I), it is sometimes employed intentionally in Gothic ribbed vaults, indeed for two reasons. Either because men wish to have two similar springings for the adjacent compartments, and therefore the ribs are allowed to commence in the line bisecting the angle, or because they are wish to avoid a sidewise projection at the intersection of two unequally thrusting compartments. The frequently occurring so-called spiral ribs in the net vaults of the late period will be mentioned later.

Entirely irregular plans.

For entirely irregular plans (Figs. 52, 53)., whether they h have four or more sides, as a rule we proceed best by placing the keystone over the geometrical lentre or even the centre of gravity of the area.(The latter can be found with sufficient accuracy by cutting the area out in thick paper and balancing it on the compass point). But all groins having different lengths, it is best to determine the form of the elevation for t the longer, afterward those of the others.

Similarly but in a different form is theated the vaulting of a triangular bay.

Triangular bay.

This first develops from the simple needs. Let a b c d in Fig. 54 be the area to be vaulted, which is divided by the cross arches e f and g d into rectangular bays so that the triangle g b d is left. Or there may be a rectangular area a b c d in Fig. 55 to be vaulted, against whose longer side are placed partition walls e e, f f and g g. If the outer walls of the room are not sufficiently thick to receive the thrust of the vaults, the partition walls must serve as abutments, thereby compelling the separate bays to take the triangular form of plan e a g, eg f, etc. The sacristy of the church of Ss. Peter and Paul at Stettin exhibits an approximately square plan divided in triangular bays (Fig. 56). Here the addition to the church by a side entrance m seems to be the chief reason for inserting the fifth support e. Another occasion for this form of plan may result from the larger plans of choirs with aisles, that will be ment-

mentioned later.

The erection of the vault over such a triangular bay may occur in various ways. Either the components directly extend to the three side arches and intersect above like hip roofs (Fig. 57)., and as for example such as occur on the choir aisles of the cathedral of Paris and of Notre Dame at Chalons, or the triangular plin is further divided by the three lines a h, h g and h e in Fig. 55, which are the projections of the proper diagonal ribs, while the arches turned over the triangular sides appear as cross arches. A very beautiful vault of the last kind is found in the lower hall of Römers at Frankfort-a-M. Figs. 57 and 58 show the contrast between both forms of vaults in perspective. In the latter may occur the describing of the arches in the same manner as for the cross vault with four sides, so that one begins with the construction of the diagonal arches, which if the triangle is equilateral may again be quadrants. Both forms of vaults may be combined in a vault with t the plan given in Fig. 55, in the way that the longer triangles a e g, etc., are like Fig. 58, and the the smaller ones a e c left at the sides be vaulted like Fig. 57 on account of their smaller size. The hipped vaulting on the corners employed there can also be transferred to bays with four sides, and then originates the interesting but seldom used form of the diagonally placed cloister vault.

Flat arches of vaults.

Where height is lacking, as particularly the case in utility buildings, there can the arches of vaults be made segmental instead of semicircular or pointed arches, or also by the broken arch much preferred in secular English Gothic, i.e., pointed arch with broken haunch. In construction such cross vaults agree with those already described. For example, if in Fig. 59 the arch a g e is the groin line, then with the use of the same radius can be constructed the side arch over a f′ as a broken arch, or to reach the same height at crown as drawn at the right, over the side a f may be turned a segmental arch with a smaller radius.

4. Vaults with a combined System of Ribs.

Hexapartite and octapartite vaults.

The hexapartite vault.

In the 12 th and 13 th centuries is found the system of the

hexapartite vault. In the Norman and northern French works as well as in Germany in the cathedral of Limburg and other places. Its origin is connected with the entire arrangement of the ground plan (see Fig. 15, III), and it was again generally abandoned with the progressive development of Gothic art. Fig. 60 shows the plan and Fig. 61 is its perspective view. It is formed over the middle and transverse aisles of a basilica, and it chiefly has a square plan, that corresponds to the breadth of two bays of the side aisle. It differs from the ordinary cross vault in that to two diagonal arches a b and c d is also added a third groin arch e f, that finds its supports on the intermediate piers a and f. Hence the latter not only serve as supports for the side aisle but also for a part of the middle aisle. The longitudinal walls are each divided into two side arches, whereby the entire vault contains six triangular compartments. The compartments e C b and a C d are ordinary compartments of cross vaults, while the four at the side vaults d f C, b f C, etc., receive an unsymmetrical form that is best realized by a horizontal section made at a certain height that shows the parts hatched in the plan.

The development of the arches is indicated in the ground plan. The groin arches a b and c d are semicircular according to rule, the partial groin c f must naturally be an arch of the same h height. For the six round arches it is on the contrary free, either to bring them to the same height (Fig. 62) or to leave their crowns lower (Fig. 61). In the oldest examples sometimes occurs the form of the stilted semicircular arch, which was v very soon replaced by the pointed arch, much more suitable for this form of vault. The surfaces of the compartments may be straight or swelled, according to the requirements of the form or of the execution.

Figs. 62 &c 62c illustrate the development of the elevation of a hexapartite vault with stilted side arches. Fig. 62 represents the longitudinal section, 62 a and 62b show plans at different heights, while 62c makes clear the form of the lower p pieces of the partial groin. In consequence of stilting the side arches a vertical wall is placed beneath and behind the partial groin, that first farther up passes into the vaulting of the compartment. The thrust that acts on the intermediate piers of the hexapartite vault is manifestly much less than

that acting on the main piers.(Usually but 1/3 as much). Therefore the former require lesser dimensions, as in the cathedral at Limburg, the cathedral of Laon, the church at Mantes, or if the dimensions of the pier are not determined by the thrust of the vaults of the side aisles, it may omit the artificial ensurance by flying buttresses. Hence where these assumptions occur, where either the pier is weaker or the arrangement of flying buttresses is difficult, where otherwise the arrangement of this hexapartite vault would be nearest the execution elsewhere of the system of the oblong cross vault.

For example, we meet such conditions in the arrangement of the transverse aisle. Let Fig. 63 be the plan of such low side aisles and oblong cross vaults over the middle aisle. The thrust of the vault of the middle square against the transept pier b acts against the clearstory walls of the transept and middle aisles, and against the vault of the middle aisle on t the aisle pier a, the flying buttress thrown from a to c. On the contrary there opposes the thrust acting on the point d in the direction d c no wall of sufficient height, and just as l little can a flying buttress be arranged at d, unless the window c d is omitted, and finally from d to c is possible a flying buttress to strike the buttress at c, thus perhaps overthrown. Therefore it is next to reduce the thrust of the vault at d, and this occurs just by the arrangement of a hexapartite groin vault over e i b b of the given plan, as found in the c cathedrals of Paris and of Beauvais. It is possible that in this plan is to be sought the first impulse for the development of the hexapartite vault.

Also for certain kinds of choir plans, whose investigation we must therefore anticapate here, are affonded allied arrangements. If the choir plan adjoins with a half polygon, for example with five sides of the decagon (In Fig. 54), then the natural junction of the ribs lies at the middle of the base of the polygon, and thus at c. But to the side thrust acting here is only opposed the width a b of this rib, an insufficient resistance, if the adjacent rectangular bay of the vault is formed on the system of the oblong cross vault. Therefore this system must be avoided, and this occurs if the ribs from d and e extend to c instead of diagonally, so that the bay a b c d takes the form of a hexapartite cross vault, while the next one is

covered by the usual cross vault. Such arrangements are often found, among others in the church of S. Elisabeth at Marburg and the collegiate church in Wetter.

Just as the arrangement of the hexapartite cross vault is based on the insertion of a pier in each of the two sides of the vault, there results there the insertion of one pier in each side of a square the octagonal cross vault, in which likewise the compartments c C b, a c d (Fig. 60) are also halved. Such an arrangement of piers occurs where the western towers are connected with the plan in five aisles, so that the width of the tower is equal to that of both side aisles. Then indeed if as in Cologne the lower room is divided and includes a chapel besides the entrance, there results a ninth pier at the middle of the square, and the plan of the vault in four equal bays. But where the interior of the tower is entirely united with the interior of the church, as in the cathedral of Paris, the middle pier is superfluous without any special plan of the tower, and there results an octagonal cross vault. A peculiar example of this kind is found in the central tower of S. Maclou in Rouen, whose vault is raised two stories above the vaults of the middle aisle, so that from the church is had a view of the interior of the tower. The side arches are not stilted, but their crowns remain far below the diagonal ribs. From the eight crowns nf the side arches here extend inclined ridges to the keystone, so that the entire vault exhibits the plan given in the right half of Fig. 65.

Star and Net Vaults.

Diminution of the compartments.

The design of the octapartite vault carries with it a reduction of the areas of the compartments, since instead of four great compartments are eight smaller ones. But a diminution of the compartments is there attained only by increasing the number of supports; it can be done without this by the insertion of new ribs, and this then leads to various richer forms, among which are particularly prominent the star and the net vaults.

The endeavor to reduce the size of the compartments must be regarded as a natural result of the Gothic principle once introduced. When men opposed the ribs as supporting members to the compartments as the light supported filling surfaces, it

was a next step to multiply the bearing ribs, so as to more c conveniently vault the filling surfaces and to make the mass lighter. Enjoyment of the pleasing arrangement of the lines aided in ever making the pattern richer, until at the close of the middle ages it fell into capricious degenerations.

Therefore the increase in the ribs is to be regarded ar a previously indicated course of development, and thus different causes have contributed to stimulate it.

In the plan given in Fig. 66 with rectangular bays in the m middle aisle came into use the choir ending mentioned in Fig. 64 with a hexapartite vault. This shows that the compartment d c e is larger than any adjacent one, so that there results a not entirely satisfactory appearance, the vaulting courses of this compartment extend awkwardly, and if they are swelled, a greater rise of the segmental arch forming the swelling is necessary. This extra height may become inconvenient for the beams that extend over the vault. Moreover too large a compartment makes necessary an increase of the thickness of the wall. And this leads to a farther division, as by the two half groin ribs d c′ and e c′, with the ridge rib c′c resisting the side thrust at the crown.

The same condition is repeated in the middle square. If all four compartments are divided here in the same manner, these will pass to the plan of the star vault, as found over the middle square of the cathedral of Beauvais and many others.

The subdivision of the compartments by an increased number of ribs may however be demanded by other considerations. For example, let Fig. 67 be the plan of a tower vault at the centre of which must remain a circular opening, to be able to hoist the bells or some building materials for repairs. This opening is enclosed by a horizontal ring supported by the ribs. If the ring consists of but four pieces a b, b c, etc., the groin ribs suffice to support it, but if the opening is so large that the requires more pieces, say eight, then it is advisable to increase correspondingly the number of points of support. This can be done by the rising vault with ridge ribs k a, etc., or otherwise by the insertion of the ribs h b, h e, will attain the purpose.

Star vaults.

The preceding and many other conditions lead to the division

by ribs in the most diverse ways. Star vaults occupy the most prominent place of all. The clearest and simplest form of these is shown by the crossing bay in Fig. 66. It originated by inserting in each triangular compartment of an ordinary cross vault three ribs bisecting its angles, which unite at a middle point. When this division is applied to the richer kinds of cross vaults, there are produced correspondingly richer forms. The octagonal vault thus gives the form represented in Fig. 68, which is placed for comparisson beside the star form resulting from the octagonal vault. They only differ in that the first has eight points of support placed in a square, but in the latter they are arranged in an octagon. Rooms with a regular polygonal plan lead to particularly beautiful star forms, which a are transferred in corresponding form to the polygonal choir ending. Over oblong bays of vaults may likewise be placed star vaults, as over squares they acquire only a displaced form, that indeed in reality by the curvature of the arches has a far less pleasing effect, than in the plan.

Naturally over entirely irregular plans may also the cross vault be transformed into a star vault.

A somewhat different shape is shown by the star vault that results from the cross vault with projecting ridge ribs (Fig. 70). The crown or ridge ribs, whose origin is to be referred to the technics of vaulting, is found very early in Norman and English vaults, but already occurred in the Romanesque period in Germany, as proved by the churches at Osnabruck and Münster. They connect the crowns of the side arches with the keystone, and as a rule rise toward the latter and are formed as segmental arches. If the ridge rib already divides the surfaces of the vault, this also favors in a high degree a further extension of the division. Fig. 71 shows an addition of ribs that support the ridge ribs at the middle, very desirable with its generally flat form. There results in this way a very widely distributed star form, for example found in employment for the crossing of the cathedral of Amiens. Likewise frequently occurs the arrangement in Fig. 72, that among others is shown by the cathedral at Schwerin and the churches at Köslin (Fig. 74, but more rare is the form taken by the abbey church at Westminster in London (Fig. 73).

The adjunction of the choir polygon to the rectangular bay

of the vault still leads to other varying divisions, for example as shown by Fig. 75. Here the choir ribs a C and d C meet e C and f C going in the same direction. If then for the ribs b C and c C are to be provided a corresponding resistance, there result the partial ribs C g and C h, that extend from the crown of the choir vault to the side of the cross rib e f, and therefore not to bend this sidewise, thus appear to require the arrangement of the ribs g i and h i, thereby continuing t the same movement through the entire width of the bay. We say appear intentionally, for a very compulsory structural need does not exist, the thrust of the ribs e C and f C and of the adjacent compartments bordered by them can suffice to resist that of the polygonal vault; thus there generally results no further advantage from the entire arrangement than the single reduction of the compartments, and we gradually pass to the point, where the ornamental construction passes into structural decoration. We shall not express blame by this, but it is undesirable, that the richer forms like those represented in Figs. 76, 77 and 78, are rather due to an endeavor for greater magnificence, than owe their origin to a structural motive.

Two of these examples already exhibit an interruption of the diagonal ribs, thereby forming a transition to the class of v vaults now to be described. Generally the richness of these forms in plan is capable of the greatest enhancement and diversity.

Net vaults.

Yet more varied than the star forms are the shapes comprised under the name of net vaults. As star vaults are designated all plans of vaults previously considered, so far as they are based on the simple cross vault. In them the square or rectangular bay is divided by groin ribs, and each compartment thus formed was subdivided into a larger or smaller number of subordinate parts. For example, the plan of a simple star vault thus resulted, in that each triangular vault included by the diagonal r ribs was subordinated like a triangular vault first in the richer forms, thus developed like Figs. 77 and 78, the diagonal r ribs appear as interrupted, or rather separated into two ribs meeting at the same angles, i.e., the direction of the principal force is resolved into two components. The further execution of this system, thus the substitution of the resulting by the generating, of the diagonal by the components, and conver-

conversely, now forms the theme, which is to be varied in the compound piers, and that with the principle of an increase in the intersections makes possible these richer forms. Thus in the plan of the simple star vault may the groin ribs be conceived to be replaced by other ribs and therefore be omitted, thus passing to the plan given in Fig. 79, that again represents a simplification of the star vault. It exhibits the net vault in the simplest form.

It would be incorrect to desire to refer the origin of the net vault to only a capricious enrichment of the former. W What justifiable ground for this can be deduced is shown by the consideration of an elongated bay of a vault. In this there result angles of very different sizes at the beginning of the ribs as in Fig. 80. But from these besides the very ugly appearance results the fault that the members intersect each other very irregularly, and that on account of the different forms of the various compartments, the ribs easily receive a greater thrust from one side than from the other. Equal sizes of angles in the spandrel are for these reasons advantages for every ribbed vault. But if on a rectangular bay the ribs are drawn in the direction of the line bisecting the angle and not in the diagonal, there results the net vault represented in Fig. 81 instead of the simple cross vault. This arrangement of the ribs makes possible at the same time a skilful adjunction to the choir vault, that is frequently executed in the manner sketched --- for example in the church at Notteln in Westphalia.

What free forms can be assumed by the net vaults are shown by the examples represented in Figs. 82 and 83 from the church S. Maria at Danzig and the cathedral at Kaschau. Here are found engaging general patterns, in the place of which frequently appears a confused web of lines in the late period.

All these shapes exhibit only an interruption of the diagonal arches, and the cross arches being retained. If the latter are omitted, then the character of the net vault appears more decidedly.

We shall now examine the condition of the cross arches, effected by the division in the various bays. In the works of the earlier period their importance was really structural, among others by the strong stilting or required by the arrangement of the flying buttresses. (As will later be shown and treated

in that section). Accordingly they received a greater thickness and bolder profile than the groin ribs. But for the erection of the vault itself, this enlargement was generally unnecessary, and therefore it was omitted already in the 13 th century in many cases. The cross ribs then received forms similar in shape and size to the groin ribs, and thus there resulted a condition not essentially differing from the cross ribs but a more perfected expression. But thereby was it only accidental, that they retained their direction, and according to the principle already applied to the groin arches, it could be replaced by the s sides of a lozeng.. Thereby men advanced to the plan of the elongated net vault (Figs. 84, 85, 86).

Net vaults of tunnel form.

The most characteristic indication of the latter is therefore to be sought in this, that the groin ribs vanish, that thus the subdivision in bays disappears, and the ribs now possessing the same function for the vault rise from piers or from s separate springings at the wall, but without emphasizing these anywhere in the entire scheme, weave the most varied lines over the area to be vaulted.

In a very decided way this character is expressed in the manner reproduced in Fig. 86 at many places, for example in the choir at Freiburg-i-B, in the Catholic church in Marburg, etc., with certain variations concerning the number of divisions, a and the plan of the springings of the ribs repeated in the plan of the vault, from which with the rise of the before mentioned principles of formation can be developed the most varied forms.

In elevation these vaults mostly have the shape like tunnel vaults, only that along the wall are triangular compartments cut in the form of segmental compartments (hatched in Figs. 84 and 85).

The entire surface of the vault is covered with lozenge panels, that are termed meshes. According to the number of panels placed in the width of the vault are distinguished net vaults of one, two, three meshes, etc. If the distance between the p points of support corresponds to only one mesh (left half of Fig. 84), the ribs for a regularly coursed network, intersected by the side triangles a b m, etc. as segmental compartments. On the other hand if the distance between points of support are

greater, for example equal to two such lengths, as in the right half of the same Figure, then also is consequently increased the size of the segmental compartment c d h. The parts o v and p v of the ribs can be continued over the segmental compartments, and then form a a break at o and p, to rise to the crown V. T they are generally omitted as on the left half of Fig. 85. As a rule men even went further, when they also omitted the parts i s and t k, but added therefor the transverse rib pieces t k and c l, etc. (Right half of Fig). Thus resulted an arrangement, which occurs with special frequency, and which is represented in plan and elevation in Fig. 86.

In this vault nearly all points of the ribs, that in the same longitudinal section fall on a horizontal, lie exactly the same as in the tunnel vault. Yet there remains the principal difference between this and the tunnel vault consists in, first the compartments are vaulted on ribs and thus are supported by them, second that compartments may have a special swelling varying from the surface of the tunnel vault, and third that the form of elevation is not determined for the cross rib but for the rib extending obliquely.

For edtermining the form of the elevation is selected a rib as a long continuous rib, giving it the desired form as a round, pointed or depressed arch. This arch is termed the principal arch, and all other parts of ribs have the same form. On this further see the following chapter (p. 82 et seq.).

If the principal arch is a semicircle, the cross section of the vault will be a vertical half ellipse. Accordingly that generally emphasized agreement with the tunnel vault is only based on an external and accidental similarity, that entirely disappears when the plan of the direction of the ribs experien- the least change according to the principles mentioned above.

For example in the plan of Fig. 87 the oblique ribs are broken at places, and are rpplaced by transversely directed parts of ribs. Since no through ribs exist, care is taken in such cases to assume the principal rib for a broken course of the rib, i.e., as shown in Fig. 87a, the plan lengths of the rib course c e a″d are successively laid off, and the principal arch is struck over their added lengths, this arch being a quadrant, for example verticals erected at the points e and a″ determine at E and A the heights of the intersections, while

the rib pieces c E, E A and A″D exhibit the actual form and l length of the corresponding rib pieces. Eny other piece of the rib, for example d is found in the same way by laying off the plan length at the corresponding place on the principal arch. Verticals erected at the ends here again lout anpiece Oi D from the principal arch, which gives the actual form of the rib piece. If in this manner the vault is determined by a principal arch on the broken rib course, its form differs in both longitudinal and cross section as shown by Fig. 87b.

On the advantages and defects of such structures according to a principal arch, see the following Chapter, p. 66.

From the plans of Figs. 86 and 87 can be developed all possible forms by simplifying as well as by richer combinations, by changing the relations and locating piers, directions of ribs, by continued application of the system of revolving diagontals into sides explained above, conversely sides with the diagonals.

Thus in Fig. 86 all meshes along the sides may be reduced or increased at pleasure. From Fig. 87 may be derived forms like Figs. 88 and 89. The derivation of evey nev forms of network can in general be carried to infinity.

Staggered springing points.

In the two plans last given in Figs. 88 and 89 the springing points of the vault do not lie opposite each other, but are staggered. Such forms are already found in many Romanesque works and they increase in the Gothic period.

An arrangement based on the system of the cross vault is found on the side aisles of the church of barefoot friars in Erurt belonging to the 13 th century, on a plan corresponding to Fig. 90. The aisle piers stand at a and b, and are connected by the side arches. Above the crown of the last lie the two diagonal arches c d and d e as quadrants, thus determining t the heights of the points f, also the form of the diagonal arches a g and b g, whose halves correspond to the parts e f and c f, which are pointed arches. From g then extends the cross arch g d as half a stilted pointed arch toward the side arch. A perspective view of this with the entire arrangement of the section connected with the arrangement is shown by Fig. 90a.

Similar conditions of the location of the piers are found

in the cross church at Breslau from the 14 th century and in S. Stephen in Vienna from the 15 th century. In the former the solution is effected by the division of the vault of the side a aisle into three triangular bays as in Fig. 91, and in the latter by the peculiar treatment of the net vault. With particular ease is the latter vault adapted to the solution is such cases, as in general to all irregularities. If we return to Fig. 87, then a location of the piers corresponding to Fig. 90 would be easily combined with the scheme of the first Fig., if g g be assumed ar the distance between piers at the outer wall and c e as half the distance between the detached piers.

As in all these cases to each inner pier corresponds an outer one, and only the number of the latter is doubled, but also with the same number the opposite placing may be omitted, or the row of the points of support in the different rows may be no longer 1 to 2 but even 2 to 3 or as otherwise desired.

Examples of the placing of points of support in equal numbers have been developed in Figs. 88 and 89. In the last case the support always lies on one side of the middle of the arch opposite the other row, in the first case is found a less regular condition.

The ratio of the supports of 2 to 3 occurs in the east wing of the cloister of the cathedral of Meissen, whose ground plan is shown in Fig. 92.

The crowns here are a b c d e f, the generating arches are g b, m c, m a, as well as k l extending from the point n. The point k lies just as far from the crown b as the point l from the crown a. At the southeast angle as seen by our Fig., is r represented a transition to a simplified plan.

Forms of the late period.

In the late period were added to the ribs various enrichments, they were set doubled and opposite, furnished with perforated tracery. The wall spandrel rising vertically above the ribs g gave the first impulse to these forms.

Already in the Romanesque period the irregular intersection of vault surfaces led to such solutions. For example if the rectangular bay a b c d in Fig. 93 is spanned by a semicircle at the long side, on the short side by a stilted semicircle, there will rise in elevation over the haunch of the cross arch the triangular wall m n o. Raised or stilted arches usually

lead easily to this form, that is especially often found on p polygonal choir endings, also generally with hexapartite vaults. (Fig. 62). The later rich net vaults afford still more opportunity for such forms. Thus it occurs that in the net vauld represented in Fig. 86a, only the ribs extend down to the low support a; on their backs are carried up a high vertical wall, a against which the surfaces of the compartments are first placed far above at the height b c. This then forms over each rib a vertical piece a b c of wall, as shown in section and plan in Fig. 86b. It is next to perforate this bit of wall, resolving it into tracery.

Perforated spandrels over ribs.

A very rich example of such an arrangement of a vault is shown by the cloister of S. Stephen in Mayence, a sketch of this southeast angle being given in Fig. 94. Since the angle projecs into the room, it combines a great number of ribs. The intersection of lower and upper ribs must naturally be wrought in one block, that Figa 95 represents in perspective. Also the function of the upper rib with the vertical descending shaft consists of one piece, which requires tolerably large dimensions, as shown by the plan in Fig. 95 e. The cross section of a rib assumed in Fig. 95 a shows that the mass a b c must be continued, that is favored by the development of a of a cusp at this place. If the cusp is repeated on the cut stone lying beneath this, then results a treatment of the perforation like tracery. In larger proportions the tracery will naturally be enriched, and can be utilized in the opposed stresses in the upper and lower ribs.

The system on which the entire form is based, the perforation of the wall erected on the rib, meanwhile occurs on many earlier works, even if in varied execution. We mean the ceilings of stone slabs, for example that in the tower of freiberg cathedral occur over the lower hall in the octagonal tower, and similarly over the chapel in the northern wing of the cloister of Magdeburg cathedral, and which thereby alone differs from an actual vault, that the courses turned in flat arches in the latter between the ribs are here replaced by stone slabs, which then join over the back of the rib. Here in order to offer this support, the latter must be made horizontal, and this leveling is effected either by a system of mullions and arches as

in Freiburg, and shown in perspective in Fig. 96, or by a direct insertion of circles and other forms of tracery in the spandrels between the back of the arch and the horizontal above as in Magdeburg. The sole difference between the construction in Figs. 95 and 96 lies in this, that the rib c in the first Fig. forms an arch, but a horizontal in this. Also while the first only requires support by an abutment, the latter needs a continuous support, which again must be borne by the rib c″. Therefore if as in Fig. 96 this support is made by posts or little columns, then must the seats for these be wrought on t the cut stone forming the rib e″, and at the same time must t these pieces be long enough for each to receive at least one mullion, since with unequal loading the different pieces that are less or not at all loadedmust be forced upward. More exact conclusions concerning the loading of the ribs easily give the easily constbucted line of support for this case. Therefore e each single piece of the rib must have about the form shown in Fig. 96a. Indeed in Freiburg the execution is less careful, for the seat is only cut for about half the width of the mullion, the other half being sunk in the back of the rib, whereby stone material is economized for the rib.

The loading of the rib forms an essential advantage in the construction of Fig. 96, when thereby the rib is ensured against slipping sidewise. The lack of this safety in Fig. 94 compels the use of artificial means such as iron cramps or dowels, shown by the before mentioned cloister of Mayence, and forms a weakness in this otherwise entirely structural arrangement. But the structural character is entirely denied in certain forms of ribs in late Gothis, whose origin can be sought only in the pleasure in the varied effect of such perforations. In many places and among others in S. Leonard in Frankfort are found two ribs with different radii under each other. This arrangement can be justified structurally, if the load of the compartment is taken on the upper rib, and the lower two cross points are stressed. But the lower rib is often an entirely useless accessory, that sometimes can retain its position only by an iron dowel. Fig. 97 shows this indeed rich but entirely capricious design.

Doubled system of ribs.

Striving for rich forms was expressed more and more in the

designs of double systems of ribs likewise belonging to the end of the 15 th century. Only the upper one of these is connected to the compartment, while the lower one starting from the same shaft is entirely separate from the upper and forms its connections after an entirely different scheme, so that the upper edge of the lower rib extends beneath the bottom edge of the upper one. An example of this arrangement is found in the church of the village of Langenstein near Harburg (Fig. 180. The effect of the two systems, of the two forms intersecting each other, is still enhanced by the shadows cast by the free ribs below on the surface of the compartment, and is truly surprising. But not less astonishing is the thoughtful arrangement of the plan, the extremely skilful connection of the development of the square or octagon with that from the triangle or hexagon. In this respect it is not unfruitful to compare the past with the present. There are few mechanics or even architects able to devise an arrangement at this time, to which a plain village mason in Langenstein was equal. And it is not the certainty of execution won by repeated practice, it is in a far higher degree the direct invention, in short the real purpose that makes the fame of the old works.

The vault mentioned first belongs to the late period, it suffers in an enhanced degree from the before mentioned weaknesses, particularly in the need of iron cramps for the lower system, and is merely a study than intended for imitation. A still richer example of the same kind is found in S. Willibrod in Oberwesel.

Suspended arches and cusps.

In the late period the lower edges of the arches and vaults frequently are a continuous series of suspended arches. This is the same mode of ornamentation already found far earlier on arches of portals. They were then transferred to the cross arches in the interiors of churches as in the Romanesque church of S. Isidoro at Leon in Spain and later in the church of S. Jacob at Liege, where they are even arranged in two parallel rows. They finally appear on the cross arches and ribs. For the latter they are found either only on the parts next the keystone and are omitted downward, as in the choir of S. Sebald's church in Nuremberg, or are continued on the entire rib from the capital to the keystone, as in the northern side aisle

of the cathedral of Mayence, and in the peculiarly graceful s stairway tower of the house in Rue de Forge in Dijon (Fig. 90; Fig. 99 shows a section of the rib).

Likewise there belongs to the more ornamental forms of ribs what often occurs in connection with the richer plans of star and net vaults, the placing of the cusps on the sides of the ribs, which then extend freely below the surfaces of the compartments, but at the same time require a greater width of the stone for cutting the rib. Such an example is found in one of the chapels of S. Maria im Capitol in Cologne.(Fig. 100). As a rule the profile of the rib is contracted at the top to allow the cusp to rest free (Fig. 100a).

Bent series.

Directly from a continuation of the ribs with cusps results the form of the ribs that make a sort of tracery in plan, that in the beginning occurred in connection with straight ribs, as in the vestibule of S. Maria at Mühlhausen (Fig. 101), while later the entire plan of the vault is so formed. Slightly curved ribs can be justified, as already stated on pa 28, but there is no mention of it in the so-called sinuous ribs of the late Gothic, of which a chapel on the southern side of Strasburg minster gives most decided evidence. The effect of the ribs winding in space like snakes is truly distressing and extremely poor. Fig. 102 shows such an example in plan.

The construction of the elevation of the arch of this curved rib is first done for its chord, thus for the part a b of the arch in Fig. 102 over the chord a b etc. From the elevation arch constructed for the chord a b is therefore then determined the arch of the rib proper in this manner, for example the height of the point d″ equals that of the point e, etc. If in Fig. 102a the arch a d e b is the isometric projection of the arch turned over the chord, then is the arch a″d″e″b″ the actual arch of the piece of the rib, that is formed in the same way from b e etc.

In the same manner then also occurs the practical execution, for the side surfaces of the centering set over the chord a b are made of a piece of wood of corresponding curvature.

The direction of the joints in the compartments can be assumed in various ways. The plan of the vault generally decides the choice. This decision may either be a separate one for each c

compartment, so that the joints for the different compartments meet at oblique angles on the backs of the ribs as in Fig. 103, or the joints pass in straight lines over the backs of the ribs and intersect at an oblique angle on the next rib as given in Fig. 100. Finally they may pass in a straight line over all r ribs and intersect on the middle line of the compartment just as on the ridge line of the cross vault. The last arrangement is reproduced in Fig. 326 for a vault without ribs.(More on the construction of the compartments follows in a special Chapter.

5. Form of elevation of the vault according to static and practical respects.

In the preceding Chapter the richer forms of vaults are chiefly treated according to the form of the plan of their ribs, and now will be collected the most important requirements for form and elevation.

The diversity of forms of the elevation is no less than that of the formsof plan. The principal of Gothic to develop every part of the structure strictly from the actual requirements becomes particularly prominent in vaulting. For two different basal needs Gothic also creates two different vaults. The variable plan of the bay, actual height of the construction, nature of the materials, absolute dimensions of the vaulting, height of openings for light, kind of distribution of the supports of the vaults,--- all these perpetually produce changed c conditions, which afford a welcome tendency to ever new solutions. In this permanent change rest in great part the magical charm of mediaeval works.

It was natural that under similar conditions also similar f forms resulted, which received a commonly recurring stamp, but at first never led to any rigidity. The masons' lodges must first have lent themselves to training in the proper nature of the matter, not to a dead cramming of all formulas; this could not even in the later time be too rigid, as proved by the free exchange of the animated transformation of forms recognizable from decade to dec.de. At the last end of the middle ages the art of the compasses and ruler certainly came once more into honor, and the living art of construction became a dead mechanical form, that passed into the Renaissance period and there ossified without any relation to practical creation, until it only lost itself in its last remains in our days.

Very instructive in this respect is the extensive work of Hoffstadt --- Gothic A B C --- Frankfort in 1840. Based on traditions of the late middle ages and still more on those from the 16 th and 18 th centuries (drawings, masters and writings, drawings, etc.), the author has developed an entore system of geometric rules of construction, that extend to all parts of Gothic structures, as thickness of walls and piers, tracery, arches and vaults, and even to the late Gothic foliage. The geometric relations are mostly developed from the square (ratio of side to diagonal), the equilateral triangle and the circle. However instructive the work, one can assign to it no important value for the understanding of early and m middle Gothic. Some rules for the construction of star and net vaults introduced by Hoffstadt will be described at the close of this Chapter, but it seems first required to consider these vaults from other different points of view.

Treatment of the form of the vault with reference to the equilibrium of forces.

a. Opposed position of intersections.

The vault as a system of beams.

To render easier the idea of the location of alternate dependence of the intersections of a rich ribbed vault, first conceive the components to be omitted and the ribs replaced by straight beams or struts, so arranged that they can transmit only compression and not tension. Meanwhile neglect the weight or loading of those beams, all existing loads being conceived at the joints or intersections, that are to be regarded as movable hinges.

An ordinary cross vault then takes the form of a rectagular pyramid (Fig. 104). For a simple star vault there is added on each side of the preceding pyramid a triangular pyramid (see left side of the Fig.). If the four supporting points A, B, C, D, are fixed and immovable, then the apex S is also a fixed point. Then are all three base points A, B, S, of the little pyramid and its vertex E also fixed. This apex could also on its own part again furnish the fixed support for a ridge rib E F. The entire star vault is thus represented as a fixed and immovable framework of struts.

The little pyramid can be much flatter than it is drawn in the Fig., the apex can be moved back quite close to the basal

area (M E in Fig. 105). As long as an upward movement of the crown S is prevented (see on this pa 43), so long will the permanence of the pyramid be ever assured thereby, only the c compressile stresses in the stone struts will be greater, the flatter this inclination. But now if the apex is moved back y yet farther, so that it falls in the basal area A, B, C, or e even under it, then will the limit of pressure be reached or exceeded, the three struts will fall (since safety from tension was not assumed).

Safe height of crown.

From this follows the first fundamental condition for the durability of the vault; every crown must lie above the plane passing through the basal points of its supporting ribs.

Let the wooden struts now be replaced by the actual stone ribs. Very short ribs could be formed as straight stone beams, just as the English late Gothic actually employed straight ribs of small lengths, that must naturally consist of a single piece. When longer ribs are composed of a greater number of s stones, this forms the natural transition here as in other cases from the stone beam to the arch. The form of the arch will be already decided by the weight of the rib, and still more by the side load of the compartments. By the curved shape of the rib must a second basal condition for the durability of a terminal point be added. In Fig. 106 the terminal point E may indeed be supported by the straight beams A E and B E, but not by the arched ribs over them. These show a deflection between B and S, that would cause the middle stones of the rib to fall. If the crown cannot be hung on an upper separate bearing arch or a similar expedient, then may be given to the back as much size as required by the safe transmission of the compression, or a longer piece v w is made of a single stone, that will not break under the forces acting in it. Under such conditions are actually found examples of somewhat sunken crowns. (For example in the rich net vaults of the cloister at Aix-la-Chapelle.

Avoidance of sunken intersections.

From the preceding follows the second basal condition:- the supporting ribs must not sink greatly at the crown, or special security must be provided.

Supporting the crown by ribs.

Further results the third condition, that each crown must be supported by at least three ribs, so directed that every vertical plane passed through the crown on both sides has at least one rib.

As a limiting case will be the supporting of the terminal point be only two ribs lying in a plane, and the latter would stand upright like two rafters opposing each other, so long as no transverse side force acts on them. But this would always require sidewise bracing, that could be required from other ribs or from the set surfaces of the compartments.

Very frequently were then three ribs meeting at the crown, indeed four, six, eight and even more. Among them must be at least three that satisfy the previous conditions; Strictly taken in general only with the existence of three supporting ribs can the distribution of the compression to each be accurately determined, for with a greater number the consthuction is no longer statically determinate, and there could occur here by accidents in execution a distribution of the compression calculated with great difficulty (just as for a table with three legs, each always bears its part, while for four or more legs this cannot be assumed). In the execution of the vault the ductility of the mortar takes a great part, to first produce an independent suitable distribution of the forces. F For a rectangular vault for each may one assume with great safety that each of the four ribs properly transmits its part of the force, and then there must occur a substantial movement in the position of the abutment or in the loading.

When more than three ribs unite in a crown, they cannot all support it, or besides the required supporting ribs may also occur loading ribs. (See D S in Fig. 107 and F E in Fig. 104). A rib must act as a load if its lower end lies above a plane m n o (Fig. 107), which passes through the terminal point S parallel to the ground plane A B C of the supporting ribs.

Still more common than such ribs, that support with one end and load by the other, are such that have to support a crown with one end, or more correctly have to keep two points apart. Such bracing ribs especially occur on net vaults, which often exhibit entire series of them. One must regard them as a supporting rib for each of the two crowns, for example, o s, p s, etc., in the net vault of Fig. 109 a. In the star vault of Fig.

108, on the contrary n s must be considered as loading the point s and as a rib supporting the point n.

The beams of a framework corresponding to the star or net vault generally receive compression for a vertical loading of the intersections. But in special cases may also occur tension in one beam (or rib), as shown by the following consideration.

In Fig. 105 (scheme of the ordinary star vault) the point E may lie very near above its ground area A E S and be strongly loaded by G. This produces compression in all three supporting ribs, of which the rib E S seeks to raise the crown S, in case it does not have in its load P a sufficient counterpoise. A r rise of the crown would produce tension in the ribs A S etc.

4. Inadmissability of tension in the ribs.

Since an ordinary masonry rib can bear no tension, such a vault would not be durable. Therefore must be established the following fourth basal condition:- The form and loading of a vault must be of such a kind, that no rib needs to receive tension in its length.

Location of intersections in star and net vaults.

The preceding four conditions must always be satisfied, if the mutual positions of the intersections shall be ensured. For one part of the vault they alone suffice, not at all for others. For this may at once be seen that a decisive indication may be derived for the simple cross vault on one hand and for the net vault on the other.

For a star vault with intersections all are supported by immovably fixed points (Figs. 108 a, b), that makes possible every arrangement of the intersections, which corresponds to the established four conditions, as a structural vault that can be erected. However these conditions still allow great freedom, and thus in Fig. 108 the intersection m may be dropped within the permitted limits, and on the other hand may follow a raising of the point n. Such a shape forms an unchangeable equilibrium of the intersections with each other, that also continues when the load changes.(For a changed form or a different loading there naturally change correspondingly the magnitude of the forces in the different ribs, whose dimensions and forms must be suited to the loads). The advaytages of these star forms thus consist in this, that they permit a tolerably free arrangement of the intersections, and that the entire

system itself cannot be displaced or is stiff for a changed l loading of the intersections.

It is different for the net vault of Figs. 109a,b(likewise for Figs. 82, 83 and many others). Since the diagonal arches are broken, the crown S is not firmly supported by them, and consequently it cannot be regarded ar an immovable support for the other ribs s o, s m, etc. The entire system of ribs finds itself in a condition of labile equilibrium, if it is conceived as a system of beams with hinged joints. The intersections are not fixed but allow themselves to be moved against each o other. A condition of rest for the system is only possible for an entirely determinate mode of loading belonging to this position. If the distribution of the load changes in the least, all beams would lose their position at rest and fall together. Each new load requires a different position of the beam for its safe reception. For example, if the crown S is more heavily loaded, then must it first be raised higher; generally a heavier load requires an elevation, or the removal of a load the lowering of the intersection concerned, so that equilibrium may occur. It is evident, that for such labile net forms the heights of the intersections are not entirely free; if the supporting points and also the rise of the arch are given, the further heights of the different intersections will be fixed by the loading. As shown, to this is opposed the great freedom in the form of the fixed star vault, which particularly in the earlier time of its use was frequently utilized; but the more in the later Gothic men passed to the variable net forms with meshes, the more must they choose uniformly curved forms of vaults, surfaces like spheres or cylinders.

Distinction between star and net vaults.

We believe that the distinction between star and net vaults is best made known, as that by a star form is to be understood a fixed, and on the contrary by the net form is a labile connected system of ribs. (In this sense will both expressions be employed in the following).

With an unchangeable or stiff system of struts (star vault) it occurs, that with m intersections and n supporting points, there are required to be at least 3m + n - 3 arches (the enclosing side arches are not here included as arches).

In reality net vaults with a fixed mass are naturally long

because not so labile as an assumed theoretic and unreal system of struts. The bodily extent of the ribs, the stiffness of the intersections and the bracing by the compartments make the vaults immovable to a certain degree. For not too great changes in the loading the corresponding lines of support will not greatly differ from each other, so that they all find a safe place in the interior of the material ribs. If consequently the net vault is correctly constructed in form and thickness, it is not so much inferior to the dorability of the star vault, but it always in contrast to that shows the restraint of less freedom in the location of the heights of the separate end points.

If all is once briefly included, it appears that the first established four conditions suffice to prove a safe location of the intersections for the firmly supported star forms, but that they do not suffice for the net system of meshes. For this must be applied the generally valid requirement (also including those conditions), that the arrangement of every ribbed vault must correspond in position to equilibrium resulting from the loading.

Equilibrium polygon and equilibrium network.

The theoretical obtaining of the equilibrium location for the intersection of such a vault will mostly be not very simple. Its form will be best be represented, if beneath the vault is conceived a suspended network, whose intersections are loaded exactly like those of the vault above. This net will assume a form that gives a faithful reflection of a system of struts corresponding to the vault above it. The difference between a net of struts and the equilibrium network is this, that in the former all struts are in compression, in the latter all strings are in equal tension, and further the network of struts are in labile equilibrium, but the equilibrium net is in hanging equilibrium, i.e., the former strives to leave its position of equilibrium, and the latter always to return to it. If the loading of the equilibrium network is changed, it passes of itself into another position corresponding to this loading. The last must be given to the network of struts, so that it may stand under the new load. When in the network of struts for a loading an unexpected tension arises anywhere, this is also immediately to be recognized in the equilibrium network. There the intersections will move toward each other and the string lying bet-

between them will slacken.

Such an equilibrium network is for the vault the same as an equilibrium polygon for an arch, which is loaded exactly like the arch itself, thus the theoretically correct arch line will appear in the reflection. If the loads are continually added, the curve will appear; but since the loads are suspended at certain points, there results a broken line as the equilibrium polygon. The theoretically correct form of arch corresponding to it will be termed the line of support or line of pressures in the arch. Graphic statics constructs this line simply by m manifold applications of the parallellogram of forces (Müller-Breslau, Elements of Graphic Statics, and others). By this is found not only the directions but also the magnitudes of the forces. An extension of graphic statics to network forms in space is not so very simple for most cases, since its constructions can only be executed in the plane. For example, if one has to do with three forces in space acting at a point, two m must be combined in a resultant, which lies in a plane with t the third.

For special cases in practice it can be actually recommended to make network models with cords or strings (perhaps at 1/10 full size), which is not so extremely difficult, as experiments of the author have shown. If only the determination of the intersections is concerned, then may one be able to properly combine the loads found in the surroundings in the proper manner. The string extended between the points will then appear as a straight line.

If it is desired to obtain in this way also the theoretically most nearly correct curvature of the ribs (which will be treated below), then is the equilibrium polygon corresponding to e each rib is to be divided into a suitable number of separate parts, and to each part is given a force existing in it, computed from the weight of the rib, the thrust and weight of the part of the compartment added, as well as any extra load. In this way may be obtained the theoretically correct form of the rib. If a variable load is to be assumed, they would also the loading of the model be correspondingly changed and its thrusts be observed. All positions of the strings resulting thus must naturally find place within the later body of the rib with sufficient safety.

Net vault with close meshes.

A sound structural feeling in the middle ages, at times made more acute by bad experiences, permitted the correct form, itself always to be found for the richest ribbed vaults. When the network of the ribs was tolerably close, and the loading was not too irregularly distributed, then the skeleton of a net vault assumed a quite regularly curved general form, that was the more uniform, the smaller its meshes. For central bays mostly result a spherical form, for long rooms usually tunnel shapes. Very close networks according to their structural form may be boldly treated as simple vaulted surfaces, but it is not fully stated, that in them one must return absolutely to the accurate sphere or semicircular cylinder of the Romans. Other heights of crowns produce other forms, and further those Roman forms require a diminution in the thickness of the vault, since it agrees little with the theoretic form of equilibrium. On t the latter see further below (p. 53 et seq.).

Between the original vault ribs and the later system of meshes is to be recognized a notable difference. Those were strongly loaded supporting arches, that as a firm framework divided the entire surface of the vault into separate compartments' t the latter finally extend as a network beneath a uniformly curved surface, the compartments often les bearing than stiffening, sometimes, whereby is always obtained also a light vault, a l light framework and an esthetic subdivision.

b. Distribution and compression in the compartments.

The preceding treated the general form of the vault, particularly of the reciprocal position of the terminal points. The ribs are but incidentally mentioned and the compartments extending between them are not even mentioned. In a great part of the system of vaulting --- the only cross and star forms --- could one arrange freely within the limits established by the conditions and location of the terminal points. But the more this freedom is used, the closer attention must be paid to a correct form of the ribs and compartments.

To obtain in a strictly scientific way the forces, that may occur in the compartments of a thousand forms, would be extremely difficult, and yet would lead only to conditionally solvable problems in statics. This would be of little use to the practical architect, for whom it is important to obtain a gen-

general view of the effects of the forces, and to take into account in the simplest yet sufficient way his works. To this first belongs a clear idea of the actual conditions; after this is obtained he can easily unravel and traverse such a domain, pathless at sight, as the rich Gothic forms of vaults, as it may be attempted in the following.

Influence and position of the courses.

Very common is the false view that the direction of the pressure in the vault depends on the position of the course of masonry in the compartment. This is generally not the case, for the pressure of a tunnel vault against the abutment and for o other like forms, it has little importance whether the courses are horizontal or diagonal. The position of the courses has its great importance in the erection of the vault and later considerably lessens (although it may always play a certain part in possible failures, at least if the angle between the direction of the pressure anf the course is too acute. See further under compartment masonry). In Early Christian, Romanesque and even Gothic times is accordingly observed a varied change in the location of the courses.(On the position of the courses with the Byzantines, see the preceding Figs. 12, 13, and on those of Gothic vaults, see the later Figs. 298, 319).

Irregular distribution of the pressures.

The essential factor in the distribution of pressure is not the position of the courses, but the shape of the vault, yet even this is not alone decisive, for various other conditions and even accidents may exert a very important influence.

Few constructions are so generally dependend on accidents as vaults. Some may be enumerated here. The stresses in vaults are affected by:- 1, incorrectly arranged strength of the abutments, that cause a certain yielding of the weak and fixing of the strong points; 2, Varied stresses in the abutmentcaused by external forces, for example the thrusts of adjacent vaults, that exceed those of the vault in question; 3, different settlements of the abutments; 4, mortar of varied mixtures or unequally stiff, employed in a vault; 5, interruptions and varied rapidity in executing the masonry; 6, strong stresses in certain courses, while others are loose in the mortar; 7, more or less great variation of the centering, mode and time of its removal. All such cirmumstances may produce little displacements

or movements, which with the small elasticity of the material used may at once cause a notable alteration in the pressures. If in heavy pressures a separation of some parts of the vault be a crack, then willthe transfer of the force be the more influenced.

Influence of the mortar on distribution of pressures.

In all these accidental conditions the mortar plays a certain part, and it may equalize or even increase it, and it has a p particular effect whether the mortar is still soft or has already hardened. In general soft mortar favors a uniform distribution of the pressure following the form of the vault, while after hardening of the mortar, the pressure is chiefly transmitted where it finds greatest resistance.

These phenomena are connected with the plastic and elastic qualities of the materials in question. To understand them, c conceive a tough flowing mass near hardening (mortar, asphalt) slipping down and they supported by a jagged body (Fig. 110). The principal load will rest on the projections a, and by compressions and slides of the mass will be distributed also a s smaller load in the depressions b, but the gaps c will be entirely empty. After this change of form the mass comes to rest and hardens. The more stiff was the body, the more will its e entire pressure be brought on a few projections, and the more flexible it was, the more will its load also settle in hollows.
or fluid

Similar procedures occur in a vault. If a rectangle be conceived as covered by a convex tunnel or Bohemian vault, this will exert pressure on the longitudinal sides and the transverse ends. But how this pressure is distributed in both directions after the setting of the mortar cannot be definitely stated, as it partly depends on accidental facts. If the end walls are omitted at D C and E F (Fig. 111),(they perhaps also sink by a settlement of the foundations), then the portion of the vault over them will drop and perhaps the lower part entirely falls, over it being formed oblique cracks, when the vault gradually comes to rest and is only supported by the longitudinal walls. (Fig. 111a). Conversely if for the sage vault the end walls are extremely firm (Fig. 111b), while the side walls exist but are too weak, (since entire failure would no longer be possible in the case sketched), then the thin side walls w would settle or yield under the pressure of the vault. Consequently the vault will move until its principal mass is suppor-

supported on the firm end walls. Thus here the ends support, which were previously free. On the side wall falls but a small part of the pressure, just as much as they can receive. If more was given to them than they can bear, then would continue the movement for further lessening the load. Thus the compression and movement would occur until a new condition of rest with a different distribution of the load is found. Naturally the abutments must not be too imperfect, since no condition of rest could occur, but the displacements would continue until destruction.

The more the mortar has set and the better it adheres, the more it allows a strong abutment to appear for a weak one; but the softer it is this is less possible. The change of pressure in consequence of displacements of the abutments in hardened vaults is frequently astonishing in old buildings, and one can see by the cracks that the load of the vault has been transferred entirely to other points. In some circumstances may such a change be useful, but in general it is serviceable to the structureto permanently receive its stresses in the same manner as attributed to it.

But for a favorable universal distribution of pressure first to occur, this is to be secured just by the soft mortar. If a part of the abutment yields somewhat, then it will not be relieved of the load by soft mortar, but it rather moves outward a part of the vault under certain pressures until a state of rest exists. If an important part of the abutment is built so weak, that it generally no longer comes to rest, then a soft vault moves till it falls. It is a peculiarity of soft mortar, that bad constructions may become full of mystery, but which is welcome for correct designs, for which will be attained a pressure acting on each part of the vault, which appertains to it by its form.

What degree of softness is favorable for the mortar can only be decided in each case, for it will be better to avoid too flexible mortar, since it may produce unexpected and strong pressures. It usually suffices if only a very slight degree of ductility exists at the removal of the centering. In free-hand masonrw there usually suffices the usual stiff mortar, so that the compartments have a movement corresponding to the ever varying load, so that if a powerful wedging together of

some courses does not occur, the distribution of the pressure finally follows the form of the vault.

Dependence of the pressure on the form of the vault.

It follows from these considerations, that an architect with a clear conscience will not remove the centering ofhis vault too late in order to utilize the favorable activity if the mortar. In some circumstances a very safe construction may be favored by a longer hardening, since then perhaps the work of t the yielding of bad structural parts may be relieved by others built with abundant strength. It will be assumed in the following, that there is a correct distribution of the pressure according to the shape of the vault, favored by the plastic qualities of the mortar. Then can generally be established the hypothesis, that in a vaulted comparment each part of it chiefly transmits this pressure to the abutment in the direction, which a rolling ball would take, or in other words, that the pressure always seeks to pass in the steepest direction.

The forms of compartments employed by the middle ages are innumerable, for most of them have no mathematical names. Partly horizontal, ascending and swelled surfaces like cylinders, conic sections and all possible spherical or swelled shapes occur, but with all diversity they may be chiefly classed in two divisions, in those curved in one direction with surfaces like tunnels, and those curved in all directions or swelled surfaces.

Division of compartments into strips.

If according to the preceding hypothesis is assumed the transmission of pressure in the vault, then for the true surfaces is a parallel division in strips (Fig. 112), and for a dome r results a radial division of the surface (Figs. 113, 114). F For swelled surfaces not differing much from the dome of rotation, may be used without great error the same radial division extending from the highest point (vertex). For any portions of such surfaces like the curved triangles m n o and p q r as well as the rectangle t u v w in Fig. 114, the division into strips naturally remains the same. Necessarily such parts of surfaces are first to be completed of their vertices, so that these may proceed the division. If surfaces occur that differ greatly f from both tunnel vaults and domes, for example the ascending forms between the two, then according to the rule of the ball rolling downward results a varied division in strips, as inser-

inserted in the Fig. In this case. But generally sufficient accuracy will be obtained, if as in the examples of Figs. 112., 113 they are made parallel or radial, or both are combined.

According to these examples it is easy to determine for a c cross vault in what manner the surfaces of the compartments are to be properly divided in strips, thereby finding the mode of distribution of pressure in the different directions. This is shown in several illustrations, that will be understood without further explanation.

1. Cross vault with straight ridge. The surfaces of the wault have a tunnel form and are therefore divided in parallel strips perpendicular to the ridge. As an example is taken a half polygonal choir vault (Figs. 116, 116a), that is pointed, and it is immaterial whether the side arches and compartment surfaces are curved according to the semicircle, pointed arch or any other curve. Each strip of the compartment transfers its part of the load and thrust of the vault to its corresponding rib, so that the piece a b of the rib receives the two hatched strips abutting there. In the same manner the load on the pieces c d and e f of the ribs are indicated by hatching. So that the rib may not bend outward, the thrusts exerted by the strips on both ribs must be perpendicular to the rib, as still mentioned. For vaults with an ascending ridge will the strips take the d direction indicated b: dotted lines on the right side of Fig.116.

2. Cross vaults with swelled compartments (Figs. 117, 118, 119). For each compartment is sought its highest point s and from this are drawn rays on the plan, that divide the surface into triangular strips. On a piece v w of a rib (Fig. 117) comes the weight and thrust of the hatched triangle. In Fig. 117 the vertices lie at about the middle of the compartment surfaces, and the pressure is therefore distributed in about equal parts on the ribs and side arches. If the vertex of the compartment is placed close to the keystone (Fig. 118, left), the side arch receives the greatest pressure, and conversely the ribs receive the greatest portion if the vertex falls near the side arch (Fig. 118, right). The compartment may even rise so much at one side that the vertex no longer lies in the triangular compartment but must be placed outside it. In the left triangle of Fig. 119 the side arch receives only a vertical load from the strip, the ribs will not be loaded except

by the never failing horizontal thrust and they bear rather a force directed upward. Conversely in the triangular compartment lying at the right the ribs are strongly loaded, while t the side arch receives an upward pressure.

In this way it is possible by the choice of the form of compartment to place the thrust of the vault on one or the other arch, which has great importance for the erection of such vaults. It is always important to see that the ribs are pressed as equally as possible on both sides, which is most perfectly attained, when the vertices of the adjoining compartments lie symmetrically with the rib. The point s is to be sought in the plan about sidewise from the middle point of the ridge arch.

3. Cross vault with sharp swelled ridge of compartment (Fig. 120). The vault is produced when from the pointed side arch extends the curved ridges f m and g m to the middle, against which the surfaces of the compartments intersect like a roof. Each of the compartment halves is to be regarded as a triangle cut from a surface like a sphere (p q in Fig. 114). The probable vertex s must be extended and the rays drawn from it are shown in the Figure.

4. Star vaults. Here result quite corresponding changes as before, some of which are represented in Fig. 121. According to the location of the vertex may one also here load one or t the other rib or finally also the side arch. An equal thrust from both sides of the ribs is also naturally to be striven for here.

5. Net vaults. For the preceding kinds of vaults each separate part of a compartment can be swelled in the most diverse ways (see left sides of Figs. 122, 123); or as mentioned in other places, there can be assumed for the entire vault a common compartment surface. In the last case the pressure is directly transferred from the surface of one compartment to that of another without the ribs having any special importance. The division of the surface of long net vaults would be into parallel strips as for the tunnel vault (Fig. 123, right); central net vaults conversely would have a radial division (Fig. 122, right); from the middle point s, which extends over the compartments with the single exception of the outer triangles, that ordinarily rise toward the side arches (vertices s_2, s_3 Fig. 122).

c. Correct form of the compartments.

The division of the compartments into separate strips, which then are arranged according to the shape of the compartments and not in the direction of the courses of stone was very simple to execute, but it now concerns the fixing of a proper curvature of such a compartment strip,and the pressure exerted by it on the rib separating it. Men seek to make the thickness of the vault and compartment as small as possible, and this generally is 10 to 15 cm, but can be reduced still further. According to the usual assumption that the middle line of pressure lies in the middle third of the thickness of the vault, there remains no great play space, i.e., in other words, if we desire to turn this compartment, we must adhere accurately to the theoretically found line of pressure in the curvature of the vault; this is very particullarly true for the compartments in tunnel form. A parallel strip cut from the tunnel compartment is to be treated exactly like a quite ordinary masonry arch, and the line of support is found for it exactly as for the latter.

Obtaining the line of support.

The arch or the strip of the compartment is to be divided into any number of equal parts --- for example 11---$ Its weight is calculated for each part, which acts at its centre of gravity vertically according to the scale of lines 1 to 11 in Fig. 124. In a separate force polygon of Fig. 124a are laid off the computed weights on a vertical line at an assumed scale, for example here 20 kil. = 1 mm. This produces the line A B representing the total weight of the arch. If in this case the arch has a symmetrical form and loading, then is drawn a horizontal through the middle point C of A B, on which is to be laid off the horizontal force H existing in the arch. Since H is not to be accurately found beforehand (?), to it is first given an assumed length C P. From the pole P at the end are drawn rays connecting it with the divisions of the vertical A B, that are indicated by Roman numerals I to XII. Assuming H to be correctly assumed, these lines represent in the arch the pressures continuing from one part to another in magnitude and direction, from which is very easily constructed the line of pressure, as done below the arch in Fig. 124b. Here a parallel is drawn to each ray I to XII to produce a polygonal series of

lines, whose angles lie on the verticals 1 to 11. A force IV lies in the force polygon between the verticals 3 and 4also lies in the line of pressure between those verticals.

The line of pressure thus found for a provisionally assumed H can be inserted in the arch above (see S R), which shows that in this case the line of pressure is much too flat; if it remains within the arch it must be steeper or more curved. This is obtained by assuming a smaller horizontal thrust H, for there corresponds to a higher arch a smaller, and to a flatter arch a greater thrust. Therefore is assumed in the force polygon a smaller H by moving the pole P to P¶. For this pole is made the same construction of the line of support, and if necessary the process is continued, until the most favorable line of support is obtained, i.e., that line of pressure or support which differs the least possible from the middle line of the arch (U S' V in the Fig.). It is assumed that in a vault in good condition that pressure is striven for, according to which the most favorable line of support is transmitted, and m generally an arch or vault is held to be durable, so long as a satisfactory line of support is possible in it. Further on the construction of lines of support, see the proper manuals, among others in Müller-Breslau, Elements of Graphic Statics.

The line of pressure U S'V in this case does not conincide with the middle line of the arch, but lies in general within the middle third, at the crown approaching at c the lower limit of this third, but on the contrary at the base reaches the outlimit, also nearing the limits at the sections d e and f g, the outer one in the first and the inner in the last. These places are to be considered, for the more that the line of pressure varies from the middle line the more unequally is the pressure distributed over the section concerned. Where the line of pressure passes exactly through the middle as at i k, the arch there receives uniform pressure over its entire cross section. (For example, if the total compression obtained by the force polygon at a certain place were 1000 kil., but the area of the cross section is 800 q. cm, there is to be taken a pressure everywhere of 1.25 kil per q cm). It is otherwise at the section e d, for the more the line of pressure approaches the outer edge d, the greater becomes the pressure at that edge, while it is comparatively reduced at the inner edge. If the line of pres-

pressure passes at 1/3 of the width, then is a pressure at the outer edge d twice as great as that on the cross section (thus 2.5 kil instead of 1.25 kil as by the assumption of a total pressure of 1000 kil and 800 q cm of surface), and at the inner edge the pressure is 0. If the line of pressure be moved still farther outward, the pressure at the outer edge is rapidly increased, while tension occurs at the inner edge. If the latter cannot be resisted by the mortar, the joint opens (in fact this can be seen in numerous arches and vaults). If the line of pressure passes entirely outside the arch, then in case the mortar cannot resist the tension, destruction certainly follows. The movement thus occurring is illustrated in the sketch of Fig. 124c, and it is seen as at the dangerous places the so-called joints of breaking alternately open outside and inside. These joints of rupture always open at the side farthest from the line of pressure. It is further recognized how important it is for thin compartments to fit them as closely as possible to the line of pressure, since slight variations are already conceivable.

Line of support for tunnel vaults of uniform thickness.

If the consideration of the line of pressure is so important, then the question is what is its proper form. The vault compartments in question were almost always of uniform thickness, for example built equal to a half brick. But it results for tunnel vaulted arches and vaults of uniform thickness the same fixed line of support, which may be found in the way just described, but it can also be determined analytically (see Hagen, Form and thickness of vaulted arches and domes). In Fig. 125 is reproduced the form of the line of support, somewhat like a parabola, but the lower branches would finally become vertical when prolonged to infinity. An infinite portion of this or of any other line of support can never be vertical below at the support, but will always show a certain inclination outward. If a flat arch is to be erected, then will be employed only the upper portion of the line of support, and for high arches there will be cut a piece from the line of support, which extends down till the necessary ratio of rise to span is reached. (Naturally the form of the line of support can be enlarged or reduced as a similar figure, according as demanded by the absolute size of the vault or by the scale of the drawing made for the

vault to be erected.

Pointed tunnel vaults and compartments.

If the line of support be now compared with the forms of arches usual in practice, it appears that a very flat arch shows no important difference from the corresponding portion of that line, and this is also the case for high arches. For comparison the forms of arches are inserted in the Fig. as dotted lines, at the left being the semicircle and the stilted arch, at the right being the pointed arch. It is evident that the two first are very ugly, they vary very much from the line of support, that even by great heaping of the material at the abutments can the line of pressure be brought a little nearer to it. If the line of pressure safely finds its place in the vaults, t then these two forms of arch always give a thick and heavy masonry to the compartment. Incomparably more graceful is a not too slender pointed arch whose preference over the stilted semicircle is strikingly visible, it varies from the theoretically curve a little outward at the bottom and inward above, but everywhere shows only a slight distance from it. When the uniform loading of the compartment is but slightly changed by backing masonry at the lowest spandrel and the addition of a certain load at the vertex, the line of support is changed at once, so that it almost exactly coincides with the pointed arch. No simple arch line struck with the compasses is so near the curve of support as the pointed arch.

We find here the explanation why in the 12 th and 13 th centuries men so readily accepted the pointed arch for the compartments in reducing the thickness of the vault, and also for the side and cross arches. Even the early time of the Normans introduced loaded ridge or crown ribs, which find their practical and also structural reason. What we can obtain theoretically, the middle ages won empirically by its struggles conscious of their aim.

The proportions of the pointed arch differ much in their r rise, and very high and slender arches require special consideration. Of the joints of rupture I, II and III in Fig. 124c, for low pointed arches the lower III comes more in question, and for high arches are rather the upper I and I. In falling a low vault acts as in the sketch 127A; a high one moves as in sketch 127B. The low one is statically favorable if the load

on the crown is small or entirely lacking, but there is behind it a good backing of masonry (Fig. 127C) at least 1/3 of the height, or better is required about 1/2 the height (for the s semicircle is even best more than half). The more slender the arch, so much the more important is the load on the crown, and for an equilateral or 60° pointed arch (circumscribed about an equilateral triangle), this is properly assumed at 1/7 to 1/4 the weight of the arch, and thus is admissable already a heavy keystone or for tunnel vaults a strengthened ridge line.

If the pointed arch is still more slender (Fig. 127D) it can only be erected generally with an uniform reduced thickness of the vault, when the keystone or crown is strongly loaded, by extra masonry if necessary, that is even increased to the weight of the other part of the arch or more. The proper weight of the load on the crown is easily found by experimental construction of the line of support. If executed in proper dimensions, then is in place the strong backing of the spandrel to 1/3 or 1/2 the height, but if the load on the crown is too small, the lower backing must not be too heavy in any case, since otherwise would occur a lifting of the apex of the arch; 1/3 of the weight would certainly be too high in this case, it being assumed to be well bonded.

If it be desired to omit the load on the crown, then for arches of excessive rise are only the following expedients. a, the thickness of the vault is increased so that the line of pressure will certainly find its place, a mode that indeed stands lowest; b, the acute internal angle at the crown is filled (Fig. 127E). Just at this place will the line of pressure fall below the arch most easily without a load on the crown, and if this place be filled by a corresponding closing stone E (or a ridge rib not too narrow), then is much won. Also the addition of a cut stone with long sides will also help (F), it being assumed to be sufficiently strong not to break with the excentric position of the line of pressure. c, a more favorable arch line is chosen, and as such may come in question a pointed arch, whose spandrel below is with a larger, and above with a smaller radius (Figs. 127G, K; also Fig. 49). d, the pointed arch is stilted (Fig. 127H, right). A curvature is thereby produced at top, which agrees better with the line of pressure, and certainly the thrust of the abutment will be greater and applied higher.

The stilted arch was a good and much used expedient in ancient and modern times. The already mentioned compound arch certainly attains the end better, especially if struck with more than two radii; it thereby produces with a great rise a form far more pleasing to the eye than the usual lancet arch with its crown always having a hard effect (Figs. 127J, K).

In every case the easily drawn line of pressure will show which expedient is best.

Correct form of domes and swelled compartments.

Tunnel vaulted compartments require a difficult adherence to a definite form of support; which is much less the case for swelled compartments, as will be stated hereafter. In the tunnel vault the side force acting between two parallel strips can be regarded as = O, but in spherical surfaces curved in all directions the mutual forces acting between two adjacent meridian strips cannot be neglected, they extend in the direction of a horizontal ring from strip to strip and may either be compression or tension. Whether ring compression or ring tension occurs in the spherical surface depends entirely on its form, and naturally a definite spherical form is possible, where neither compression nor tension occurs in the direction of a ring, and to obtain this form is of great interest. If from such a dome is cut a narrow meridian strip or a slit, no transmission of force can occur between the side surfaces, i.e., such a curved triangle will stand independent, so long as it is held above at the crown and below at the abutment. The form to be given to this strip (and thus to the dome)is very easily found by t the graphic method, when it is considered as an entirely ordinary masonry arch, and its line of pressure is sought for its loading in the usual way. We are here troubled again by the case which occurs with a uniform thickness of the dome or swelled compartment. The form of support then occurring is drawn in Fig. 126, and it is likewise treated analytically by Hagen at the place mentioned. Domes or swelled compartments that have this cross section exhibit neither ring compression nor tension.

It is to be noted, that for the vicinity of the crown the c curve is not entirely correct, for ring compression must always occur there, since the meridian strips end too sharply at the middle to be able to transmit any horizontal thrust.

Hagen recommends in his work; On the form and thickness of

thickness of vaulted arches and domes, Berlin, 1875, p. 59), to form the cross section of the dome directly according to the line established by him and represented in Fig. 126 --- we might not agree to this. The advantage of swelled surfaces consists just in the possibility of a stress in all directions; rejecting the ring compression takes away a great advantage.

On the contrary we might regard as favorable just such spherical surfaces or swelled compartments, which at all heights show a certain ring compression. For such surfaces can never occur ring tendsions even with important variations of loading, for ring compression occurs for the stresses that cannot be received in a meridian direction. Thereby will it be possible to make a swelled compartment very thin even for variations of loading, while a tunnel vault in such cases requires increased thickness to make a safe line of pressure.

From these points of view are to be designated as statically allowable numberless forms of domes, so long as at no point of the meridian or ring compression has exceeded the limit fixed by the strength of the material, and just so long as the dome is permanent. To illustrate this are given the forms of cross section I to V in Fig. 126.

I. Section I is to be termed particularly favorable, if the ring compression from the top to bottom is everywhere equally great, and in this example it is as great as the meridian compression increasing from top to bottom, at about a point A.

II. A cone with an angle of 90° at the apex is represented here as a dome, which with uniform thickness has equal ring and meridian compressions at any height. Such a cone indeed exerts at base a greater thrust than the preceding cross section, but otherwise is a good form of dome. Likewise slender cones or pyramids (roofs of towers) are to be regarded as statically favorable domes.

III. A concave curve within certain limits gives a statically possible dome. But the ring compression below and the thrust against the abutment will be found very great, and indeed the more the flatter is the curve here.

IV. A recurved line (oriental dome) can very well be erected without ring tensions or clamps, in case at no point is it too much rounded externally or anywhere approaches too closely a vertical or horizontal. On the contrary domes curved in bulbous

form below are impossible without special provision against tension, and therefore are absurd as vaults.

V. Most strikingly appear the effects of ring compression in the diagonal section of the pendentive dome (curve V). In spite of the bend inward the dome can be erected as in numberless examples. It is an advantage that the directions a b at the b bend remain rather far from the vertical, which the Byzantines skilfully attained by slight changes in form.

Similar effects result when the dome bears at top a heavy lantern, and a great ring compression exists there, which is less, the more steeply the line of the dome meets the lantern.

For comparison are also given in the Fig. the current vault lines, at the right being the pointed arch, at the left the s simple and the stilted semicircle. It may be perhaps assumed that compression occurs where they lie within Hagen's line, a and that on the contrary in the parts cut out above occurs ring tension. Accordingly the stilted semicircle is very unfavorable, and also disadvantageous is the simple semicircle or hemisphere, where tension is found for fully 3/5 the height, that can be overcome only by a correspondingly high backing or an increased thickness of the lower part of the dome, unless recourse is had to iron rings. A slender pointed arch is much preferable and requires backing only in the lower part.

Generally lines passing into the vertical below are unfavorable, and if they are chosen, there must be assumed the necessary thickening externally to receive compression, the internal lower masonry being a filling mass. The preceding will have sufficiently explained what a great freedom surfaces curved in all directions exhibit in opposition to the tunnel vault; in the preference of the swelled compartment the middle ages consequently shows again in a wonderful way its refined and also practical and statical feeling. In a practical sense the swelled compartment favors freehand vaulting, in a statical sense it permits vaulting of extremely thin compartments in quite capricious forms, that even in strong transfers of the loading or pressures always continue stable.

d. Form of the ribs.

Pressure of compartments on ribs.

The last considerations are true for the form of the compartments of the vault, but almost more important than them is the

pressure against the abutment, that each strip of the compartment exerts on the supporting rib or side arch. If the line of pressure has been found for the strip of the compartment, then at the same time are found its end forces, but one can obtain the latter approximately correct, if the rather tedious construction of the line of pressure does not seem required.

If a strip of the compartment is regarded as a separate whole, there usually come in question only three forces, the weight and the two end forces. The weight (G in Fig. 128) is naturally drawn vertically through the centre of gravity and can be calculated, being composed of the weight of the arch and the extra load resting thereon. The direction of the end forces W_1 and W_2 must coincide quite correctly with the directions of t the ends of the arch (or their tangents), since the mostly very thin compartments allow no great play to the compressile f forces lying within them, and further the end forces must pass through an intersection O lying on the line G. Hence the probable location of the forces can be inserted in the drawing with approximate accuracy. If such a position cannot be obtained, this is to be regarded as proof that the compartments have a statically unfavorable form, and therefore must be changed. (Fig. 128a). If the directions of the end forces are assumed, their magnitudes are found by resolving the force G simply by the parallelogram of forces.

Magnitudes and directions of end forces directly depend on the direction of the compartment as hown in Fig. 129. Here a compartment strip extends from the rib A to the rib B, which may give the positions I, II and III, and the eud forces belonging to these positions are indicated by the corresponding numbers. It is evident that their directions differ widely, making clear at the same time their differences in magnitude, as assumed in Figs. 129a, b, c, in the resolved forces for the three cases. First the weight G is assumed equal for the three cases and is resolved into the end forces W_1 and W_2, and at t the supporting points these are resolved into horizontal forces (H_1 and H_2) and vertical forces (V_1 and V_2). The first is the side thrust received by the rib and the latter is the vertical load on the rib. In a horizontal direction occur only the forces H_1 and H_2 and therefore these must be in equilibrium and be equal. In a vertical direction the algebraic sum of V1 and

V_2 must equal the load G on the compartment. The great diversity of thrusts and loads on the support appears still strikingly from the figures.

Thus in the case that in a very strongly curved compartment the thrust is small, and the vertical load is divided between the two ribs A and B.

In the second case, i.e., for a moderately curved compartment the thrust is already greater and the vertical force here falls only on the support A (thus $V_1 = G$ and $V_2 = 0$), since the upper end of the rib is horizontal at the rib B.

In the third case for a very flat compartment, the horizontal thrust is very great. Concerning the pressure on the support there occurs here a peculiarity to be well considered, for the pressure V_2 is directed upward, while V_1 on the lower rib is greater than the load G by V_2 (for $V_1 - V_2 = G$). This case must always occur when one end of an arch or vault is inclined obliquely at top, so that this end cannot exert a downward pressure on the support, as it rather seeks to force the support upward. In the present case the rib B is not loaded by the compartment, but will be supported by it or even be forced upward if not sufficiently heavy to equal by its weight the upward force.

The horizontal force is naturally unaffected at the upper e

end of the arch as in general the thrust is less from the direction of the end of the arch then depends on the curvature or the ratio of the rise of the arch. The flatter and heavier the arch the greater will be its thrust, which is always true, the points of support being as we please in relation to each other.

According to Fig. 129, there are wide limits in the choice of the curvature of compartments, for regulating the magnitude of the thrust is required, a fact that has the highest importance to the vault ribs. The latter are so narrow that with a strong sidewise thrust they bend sidewise at once, and therefore the thrusts of both compartments must fall in a direction transverse to the direction of the rib. For example, if a rib A (Fig. 130) is very much pressed sidewise from the left by a great and heavy compartment, it would be very faulty to place at the right a light and strongly curved compartment; that would be forced upward by the excess of the thrust from the other compartment. Rather must the right compartment be very flat and

necessarily loaded extra so as to supply an equal thrust, that naturally also appears at another support C (Fig. 130a). For example such considerations may be necessary for the side compartments of net or star vaults.

If the smaller compartment on the right is too high in the direction of the thrust, and on the other hand is flat enough in any oblique direction, then the excess of thrust will take that direction and thereby be received.

Bending of the ribs.

Thus far the form of the compartments of vaults have been treated, then obtaining the forces exerted by the parts of compartments on the ribs, and it only remains now to deduce from these forces the correct form of the ribs. This would be most simply possible if the ribs were loaded by compartment strips, that on the plan are seen (Fig. 131) to abut on both sides perpendicular to the ribs. Each strip transfers in Fig. 129a, etc. to the rib a horizontal thrust H and a vertical load V. For c correct construction the horizontal thrusts neutralize each o other on both sides, and thus there remain only the vertical forces of both strips that are added to that of the weight of the piece of rib to make its total weight. All pieces of the rib receive in this way their corresponding vertical loads, w with which is constructed the line of support for the rib exactly as shown in Fig. 124, and at the same time is found the correct curvature of the rib, to which the form hre rib is adapted as far as possible.

That the load of the compartment strip strikes the rib exactly at right angles is seldom to be assumed, as shown in Figs. 116 to 121, but the strips mostly join obliquely like Fig. 132. Each of the two ends of the strips again exert a vertical load on the support and a thrust, the two vertical forces again forming a vertical load V with the weight of the piece of the rib.

But the horizontal thrust H_1 of the end of a rib in Fig. 132 meets the rib obliquely, and it is to be resolved into a force N, perpendicular to the plane of the rib and into a thrust S_1, that lies in the plane of the rib. The force H_1 must be neutralized by the corresponding component from the other end of t the strip, but on the other hand the force S_1 combines with t the corresponding force from the other compartment strip in a common thrust S1, that seeks to move the piece of the rib. Thus

on each piece of the rib acts two forces, the vertical force V and the thrust S. If the force polygon be drawn for one branch of the rib, it appears stepped but otherwise the construction of the line of pressure shows no difference (Fig.133).

Making end of rib secure by backing masonry.

To this line of support must be adapted the form of the rib as closely as possible, which is sometimes scarcely to be done, since Fig. 134 shows that the line of pressure does not approach the vertical below but remains quite flat. The ellipse of the Roman vault is quite ugly for the groin, the semicircle a and the pointed arch are preferable for their greater height, and still better is a broken or a segmental arch for low vaults, that like the line of support cuts the abutment obliquely. It is important in any case to connect the lower end of the rib very firmly with the abutment, so that the line of support may be unrestricted and be left above the abutting point of the p profile of the rib and can be made secure by backing masonry. For cut stone are in place great through springing stones, but for bricks the spandrel must be also strongly connected with masonry carried high and also laid with the addition of cement. A light and subsequent filling of the lowest spandrel is very doubtful.

The very important question, how far shall be carried up the backing masonrw in the spandrel is difficult to answer in general, but for important cases it is recommended to carry up t the line of pressure with reference to the weight of the spandrel. As an approximate value may be assumed, that for the elliptical groins of a Roman vault it may be freely built to 2/3 the height, while for round and pointed arches 1/3 to 1/2 is sufficient. For slender pointed arches the backing must not be carried too high, since otherwise the crown of the arch would be forced upward. (Also see Fig. 127 with explanatory text).

Compression on the cross section.

If the cross arch or the rib becomes an actual support of t the pressures combining together, then must the line of pressure find a safe place in it, and further in no cross section of the rib should the compression be too great. As a permissible compression per q cm area may be taken on good (not porous) bricks in lime mortar 7 kil, for very hard bricks in cement about 11 kil, for cut stone in good mortar or set with lead 20

kil and more. But since the mortar in vaulting is but little hard at first, it is better to assume smaller stresses. Uniform distribution over the entire area of the cross section must o only be assumed, as already stated, when the line of pressure passes exactly through the middle point of the cross section, if it drops below to the limit of the middle third (more correctly to the bottom of the kern of the cross section), then the compression at the bottmo edge equals twice the average c compression, but if the compression becomes more excentric, then the pressure at the edge becomes still greater. (On the distribution of the compression over the cross section see Abutments below, Figs. 375 to 386). Naturally the compression on the most stressed edge must not exceed the given values per q cm. If the line of pressure approaches or even falls above the upper limit for the section, it is less important, for t then the adjacent part of the compartment takes part in the transfer of the compression. With very small or lacking ribs the parts of the compartments adjoining the edges of the groins take the transmission of the compression entirely on themselves; In such cases for larger vaults is required a strengthening m masonry on the backs of the groins (Fig. 324 below).

Example; compression in a rectangular cross section.

As a close to these considerations, let the way be briefly sketched, which would be followed in obtaining the forces for a rectangular cross section with straight edges. The compartments of such a vault have a cylindrical form, and therefore are to be divided in strips parallel to their directions (Fig. 135). A piece of rib m n is loaded by two half strips m n c d and m n b a. If it is desired to test the form of elevation of these strips, the line of pressure can be constructed for them, otherwise one can be satisfied by obtaining their pressure on the rib in a simplified way according to the proportion in Figs. 128, 129. The ground areasof the two strips are equal in rectngular vaults, in consequence of which with equal thickness of the compartments their weights will be tolerably equal also, and hence they have about equal vertical pressures on the piece of the rib. On the contrary the horizontal forces differ and that of the longer strip is mucrh larger. For equal height of ridge, equal thickness, and a form of compartment not too unlike the line of support, the magnitude of the thrust will be

in proportion as the magnitudes of the sides of the rectangle, which is the result, that the resultant of both thrusts falls in the direction of the rib. Hence a bending sidewise of the rib is not to be feared for such rectangular and also naturally for square bays. (If the thrust against the rib were greater from one side, then for the ordinary cross vault one need not be too anxious, since almost always a flatter direction occurs in the compartment, that can act as stiffening). With the vertical loads and thrusts of the separate pieces of the rib is considered as in Fig. 133 the line of pressure for the rib, t thus obtaining also the thrust of the entire vault against the supporting wall mass.

If in the preceding is left a certain freedom to static requirements in tthe form of the vault, then is the architect less compelled to make an exact determination of the forces for each vault to be erected; much rather will he be able to form a correct idea of the actual forces to be expected in his vault, t that will warn him of faulty steps.

General form of rich ribbed vaults and construction of t the rib arches.

General forms.

The form of the elevation of the ribs of an ordinary cross vault has already been explained in a special Chapter, and here will be treated the richer star and net forms. In what manner the static requirements are to be calculated have just been d developed, and then appears particularly the difference between firmly connected and freer forms of elevations afforded by star forms and the variable system of meshes based on a single general form. Earlier Gothic utilized the greater independence of its star forms, it placed the side terminal points sometimes higher and sometimes lower than the middle one, and allowed t the compartments to rise sometimes at the middle and sometimes toward the outside, entirely according to the actual need, so far as permitted by the demands of equilibrium. The later net forms must for static reasons be adapted to more uniformly curved general forms, to which men adhered the more, the more they saw themselves unable to clearly survey the ever more complex directions of the forces. Star vaults were even if unnecessarily, drawn into this course of development, so that most later vaults exhibit an expressed general form. For this the following

general types predominate.

1. Ribbed vaults of tunnel form extending over long rooms.(Fig. 136).

2. Vaults formed of intersecting surfaces, for example to form of the usual intersecting tunnel vaults (Fig. 137).

3. Domical curved ribbed vaults (Fig. 138).

4. Fan vaults (Fig. 139).

Fan vaults.

The last two deserve greater consideration, of which the fan form of vault will be placed first. In it all ribs lie in a s surface of rotation about the vertical axis of the pier. This is especially favored by English Gothic, but also elsewhere vault forms widely found have no further basis than the justified endeavor to shape the beginning of the vault as regular as possible. It is thereby obtained, that first the difference between cross arches and ribs vanishes, so that all ribs are curved to unite in their lower parts, and finally the angles in plan between each pair of ribs are as equal as possible. If these rules are regarded as required by reason of easy construction, then the fan vault originates of itself entirely, whether the cross, star or net vault is its basis. The more ribs appear, the more prominent is its regular fan or palm-like growth together, but the most perfect expression is always the entire development of the ribs over an isolated column.

Each horizontal section produces a circular ring in which lie the ribs (Fig. 140). The compartments between each pair of ribs are generally turned with horizontal courses (with ends lying at equal heights). A course may as well be either curved or straight (Figs. 140a, b).

If in the fan vault of Fig. 141 a circle is struck with a m, then must the points n o p etc. lie at one height, but the ribs rise still farther above this point to e d c. The point c will naturally lie highest, and therefore the ridge line m c rises in a wavy arch from m to c as shown by the section 141a. If t the diagonal arch a c b is pointed (Fig. 141b), then the parts a m, a e, etc., of the ribs are pieces of this pointed arch and accordingly are very simply extended in their correct form, but if instead of this the longest rib is a semicircle (Fig.141c), then will the differences of the heights of the parts be very small, and consequently the ridge will have but an unimportant

weve. This may be a reason to avoid entirely the wavy ridge o and to meke it entirely horizontal. The English Gothic is aided in such cases as already shown in Fig. 48, by arches struch w with two radii, and then the ribs are only alike in their lower parts.

A particular form belonging to the latest time is yet to be mentioned, in which the ribs of similar shape terminate above a tangent horizontal circle. The square spaces left between t the circles are either closed by a stone slab or by any other suitable way.

In the fan vault the chief point is the beauty of the springing of the vault and on the contrary the form of the ridge is inferior. Conversely a graceful treatment of the middle of the vault may be placed first and this subordinated to the form of abutments, so that one is then referred to the three forms of Figs. 136, 137 and 138.

Net vaults of tunnel form.

The tunnel-like net vaults are usually employed over long rooms. Close networks of ribs lie in these vaults according to similar static requirements, like simple smooth surfaced v vaults, and therefore the most favorable cross section of such a net vault about coincides with the line of pressure for an ordinary tunnel vault of like thickness, and this curve is illustrated in Fig. 125. The form of the vault is to be determined in the manner, that the oblique ribs are pointed, semicular, or even a depressed arch. The cross section of the tunnel will therefore be the narrower projection of such an arch, the vertical ellipse produced by the semicircle, or from the pointed parts of two elliptical arcs. These projections approximate the correct line of support far more nearly than their generating curves themselves, and thus the elongated net vault of the late Gothic has a statically more favorable form than the tunnel vault of the Roman and Renaissance periods, a fact than certainly was necessary for the constructions on p. 53, when one thinks of executing vaults like tunnels with the least use of materials.

The cross compartments resolved into a network of ribs (Fig. 137) has to satisfy similar requirements of equilibrium as the tunnel, the diagonal ribs require special consideration, to them falling much greater stress than to others, therefore their st.

structural and architectural importance can be a rib section of stronger form.

Domical net vaults.

Swelled or spherical net vaults in static respects have similar favorable qualities like swelled compartments or plain d domes. If a ring stress is possible, whether from cross ribs or steep surfaces of compartments, then can the position of equilibrium move within much wider limits than in vaults of tunnel form. All cross section curves, whose curvature nowhere exceeds the corresponding curvature of Hagen's lines shown in Fig. 126, are usable for such vaults, so long as the ring compression can be safely received with sufficient safety. But there are very many such cross sections. The pointed arch is not favorable with a little lower piece, nor the round arch in a greater one, but both can be safely employed, if they receive a secure backing to the proper height.

If it is desired to form net vaults over a square bay according to an accurate surface of rotation, whose vertical axis passes through the keystone, then will result the form of a pendentive dome (Fig. 142). The springing of the vault over a detached pier will assume in plan the outline of a square that has concave sides (Fig. 142a). For rectangular bays this square passes into an elongated form (Fig. 143). The border or side arches a b, c d, etc., enclosing the bay are semicircles for the hemispherical dome, and produced by the pointed dome are curves similar to the ellipse. They are loaded particularly heavy, and therefore in the latest time are sometimes formed as stronger cross arches. Certainly men chiefly sought also to give this arch the same profile as the rib, but then an unloading of it is desirable, most easily attained when to them is given the form of a higher pointed arch with added side compartment, also favorable on other grounds. In Fig. 144 these side compartments are hatched, the middle portion left white retains the spherical form, the section through the crown is drawn in Fig. 144b, while Fig. 144a shows the already somewhat more centrally shaped impost of the vault in plan. If the impost is yet more rounded, then must the ribs resting on the concave sides at v t w would be preferred, but thus the regular surface of rotation would be given up. Finally if the springings of the ribs are brought into a regular circular plan, Fig. 145, then

is created the change of the fan vault. Thus follows a stepped transition from the surface of rotation about the middle axis of the bay of the vault to the surface of rotation about the middle of the pier. On executed works may these steps be observed in various ways.

Ascertaining the arches of the vaults.

There will be required as a rule to be chosen neither an accurate surface of rotation about the middle of the vault nor such a one about the axis of the pier, but to arrange one between the two in a proper manner.

The procedure of determining the vault will be about as follows. After the general form is drawn to correspond to the governing conditions, the plan of the ribs is determined, and one then passes to the cross section of the vault in the direction of the diagonals, the cross arches and the ridge of the vault are approximately assumed, always with the view that a graceful general shape is produced, for the last will always be sketched thus. In this main form are now the intersections to be arranged in height, where it is to be considered, that none shall seem sunken and each is sufficiently stiffened by its ribs (see on this p. 43). If then the terminal points come to lie on a surface curved on all sides, then is used with the static advantages of the latter, even not to take too much care in net forms for the assumed mutual condition of equilibrium of the intersections of the ribs. It is now to assign their forms to the rib arches, when a graceful growth from the imposts of the vault is to be especially kept in mind, and if not obtained, it is necessary to change somewhat the location of the intersections. If one can freely mostly strike the reb arches with the same radius, then may this be done with stone ribs, but with construction in brick no great practical use is seen in it. A good metwork of arched ribs must be so created that it can transmit its forces safely without requiring stresses through t the compartments. The latter is naturallw desired for greater safety. In inserting the compartments attention is to be chiefly devoted, that for no rib occurs the danger of bending sidewise.

In such wise it will be easy for smaller vaults to obtain s satisfactory forms with only an oversight of the conditions coming in question. For special cases the constructions given later will give means to undertake an examination of the vaulh

for its static peculiarities.

To the foreman on the work is to be given besides an accurate plan of the vault and of the springings, particularly the ordinates of the heights of each terminal point, and for swelled compartments also the location on the plan and the height of the ridge of the compartment. Never should it be omitted to take a view of the established framework of the centerings of the arches, since a fault can better be seen there than on the best drawing.

Usual rules for laying out.

The best vault will always be that in the actual case is developed from the determining requirements. For convenience some general rules of construction have been adopted, partly ascribed to the middle ages, whether rightly or not is hard to say. These rules concerning the laying out of the ribs are now to be given at the close of the Chapter, and are to be explained critically as far as necessary.

a.Ribs in a spherical surface.

a. Laying out a ribbed vault with ribs lying in a spherical surface. Although for reasons already mentioned vaults seldom form an accurately spherical surface, yet this case is first treated as the simplest (Fig. 146).

All ribs in their entire course lie on a hemisphere, whose plan is given in the Fig. If any piece m n of an arch is to be drawn in its position, length and shape, then its plan is to be produced to intersect the ground circle thereby producing the chord r s. A vertical plane erected on r s cuts the hemisphere in a semicircle, for every vertical section of a hemisphere gives such. On this semicircle conceived vertically over r s must lie the rib m n, it may then be revolved into the ground plana, which is simply done by striking on r s as a base sidewise a semicircle and erecting perpendiculars at m and n, which cut the semicircle is the points M and N. Then the arc M N is the rib in actual length and curvature, and the lines Mm and Nn are the heights of both ends above the ground plane. But that is all which is to be obtained thus. We proceed exactly in the same manner for each other piece of rib, and in the Fig. the arcs E O and a B are laid off as further examples. All ribs passing through the centre o lie on the so-called great circle of the sphere, while the others like a b and m n lie on

smaller circles of the sphere. Therefore these have smaller diameters and also a greater curvature.

b. Principal arches over the diagonals.

Laying off a vault according to the principal arch struck over the diagonal (Figs. 147, 147a). By this procedure all arches are struck with the same radius, which was not the case before. The diagonal rib is first assumed as a pointed, segmental or round arch, from one half of which principal arch are derived all other arches.

For a better comparison with the preceding construction the diagonal arch is assumed to be a semicircle, so that the principal arch is also a quadrant. The rib over a o is then given directly as a quadrant laid off as e O in the subordinate Fig. 147a, and it is now to determine the rib over b c and e o. For this purpose the last two lines are laid off on the ground line from the point e, as o e, and e, b. At e, is erected a perpendicular to cut the quadrant, whose height e, E gives the height of the end over e, while the arc E O is the rib over e o shown in position and size. The rib b e in the plan must lie over b, e,, the upper end E is already found, and thus it is only to connect b, with E by an arc struck with the given radius r of the principal arch from the point x. The centre x lies below the ground line, so that the rib starts as a broken arch from the impost. The side arches c d and h g can be made as pointed arches with the radius r to consistently carry out the principle of the principal arch; their crowns then lying higher than the adjacent ends i and f.

The vault laid out in this way agrees pretty accurately with the sphere, merely a portion of the ribs projecting onward in their curvatwre, but all intersections as well as ribs passing through the middle of the vault likewise lie in the spherical surface by this method.

Like all corresponding ribs, b c leaves the impost obliquely, but statically most are not unfavorable. But the springings of the vault by alternating vertiteal and oblique feet of the ribs may have such an irregular form, that in some cases a departure from this construction is preferable.

This construction differs from the preceding only, that the broken rib line b e o (Fig. 147) on the ground line of the subordinate Fig. 148 is not laid off left from the point o, but

from a, to the right, the rib b e is found as a,E, and coincides with the lower part of the principal arch, the ridge rib e o must have one end at E and the other at a point O, at the same height as O. The arc is again struck from the centre x with the radius r. The side arches can be as before pointed arches with the same radius, but now the intersections and f will be higher than their crowns.

Thus originates a vault with regular springings, the side intersections e, f, etc. in Fig. 147 no longer lie on the spherical surface but are higher, so that they attain approximately the height of the crown of the vault. If it is desired to obtain a greater difference in the heights of the side and middle ends, then for the static reasons it is better to choose the preferable pointed arch as principal or diagonal.(Fig. 149).

Principal arch over a broken rib line.

c. Laying off according to the principal arch above a rib l line broken in plan (Fig. 150). There may be again taken the smae simple star vault of Fig. 147. The principal arch is now not struck over the half diagonal, but above a ground line obtained by adding the lengths b e and e o, and it is again a quadrant.

In Fig. 150 the plan lengths of the ribs coming in question are laid off as the line b,e,O, in course and the principal arch b,O is struck above them. The two parts b,E and E O of t this give directly the form of these arches. Also to obtain the diagonal arch, we lay off its length on plan as a,o, and then join the points a, and O by an arch. If the radius of the principal arch is to be employed for this arch, then the centre is lowered to x somewhat above the ground line, which would l lead to a horseshoe shape. But it would be better to depart f from unity of radius and make the diagonal ribs pointed.

Since the last principal arch has a longer line it leads to a greater height of the crown. If height is lacking for the c construction, a flatter line must be taken as basis instead of the quadrant, but whilh will not form a too nearly horizontal ellipse unfavorable for static reasons.

Considerations against the last rule.

The principal arch above a broken rib line appears particularly in place for net vaults, where the continuous diagonal rib is wanting, its use is also recommended for them, and still

it must be adopted here only with the greatest foresight, as will be illustrated in plan in Fig. 151.

As the ground line of the principal arch that may here again be a pointed arch, one would naturally take the rib line a e f g o, in the side Fig. 151a is the construction that presents nothing novel, most ribs being obtained directly as parts of the principal arch; the arch over a f is not contained in the rib line and is to be laid off separately as p P. If in such wise the locations of the ends and the form of the ribs are fixed, then is easily drawn the diagonal section of the v vault in Fig. 151b. But this is opposed to the simplest requirements of durability (see p. 43 et seq.), the intersection E is sunk inward in such a striking way, that the fall of the vault should be feared --- Thus we should be very badly advised to employ this procedure.

It is justly asked where this doubtful construction originated. It was chiefly circulated by Hoffstadt (Gothic A B C), and it is substantially based on a manuscript of the Danzig master mason Barthel Ranisch belonging to the year 1695, whom we can scarcely recognize as an authority. We know that remains of Gothic rules of construction have been constantly preserved nearly to our days, yet it is hardly to be assumed, that every master who had finally transformed the forms of the antique a and the Renaissance into the most silly scroll work must have preserved the traditions of the middle ages in a pure form.

That the latest period of Gothic employed in its vaults as everywhere certain mechancila rules, indeed is quite conceivable, and an explanation of them will perhaps afford numerous accurate means, that could easily be executed in the ceilings of cloisters. There is often an appearance that if such relations could be followed, whether arrangements of intersections on a spherical surface, or a course of ribs relating to a principal arch, usually are found near those arches that fit no s system. It is not quite improbable, that the use of a uniform radius, expressed in the principal arch was striven for within rational limits. From the form of the springing and the execution in entstone this has a certain value, but the mode of c centering may have led to it. Men always set the centres first under the cross and diagonal arches, the latter even when the ribs were interrupeted in places, when the end was supported

by a vertical wooden prop. Then were also inserted the centres corresponding to the other ribs, as they could best be fixed, first the larger and then the samller, when the intersections were supported so far as necessary. It was everywhere a convenience to set beforehand the centering for rich vaults, to draw them with the same radius, then fitting and cutting them just as required. Thus many reasons may have combined to cause the endeavor for a consistent execution with the same radius. But where this led to injurious results, there a thoughtful architect in even those days would not have carried the love of a hard worked principle so far as to have sacrificed to it the beauty or even the safety of his work.

6. Form of sections of ribs.

Projecting ribs of vaults generally occur, since after the 12 th century, not the vault surfaces but the vault lines were the determinants for the treatment of the vault. (See p. 11). Then was rapidly completed that important change, the new forms of arches were introduced that changed the shape and the execution of the surfaces of the compartments and received its crown when the supporting arch was separated from the supported surface.

The most effective impulse toward the introduction of the ribs was given by the difficulties in the construction of the irregular grcin angles and the great stress in them by the forces transferred just by these intersecting lines.

The cross section of the ribs must first afford the required bearing for the courses of the compartment, but then be suited in dimensions and shape to properly receive the corresponding loads with sufficient safety, thus dividen in two parts, the upper abutment and the moulding projecting beneath.

Consideration of the compartments.

The support required by the courses of the compartments either consisted of a flat surface (Fig. 152), of two roof shaped surfaces inclined to each other (Fig. 153), or of a back extending through both compartments, with its sides best so inclined as to receive the compartment at right angles (Fig. 154). This projecting back occurs both for cut stone and brick in the earliest period, as proved by the earliest sections belonging to Gothic (Figs. 195, 196). From the ruins of Walkenried in the Harz mountains and the remains of the abbey church of Chorin,

likewise from the 13 th century. Brick architecture especially adopted this form of support, that also again became the favorite for new structures. It had the advantage among others, t that the rib could not move beneath the compartment, which is occasionally observed in old works (Market church at Hanover). An interesting section belonging to the Renaissance and perhaps imitated from earlier works is found in the ruins of the church of the order at Doblen in Courland (Fig. 155), that sows a dovetail back that must have been advisable when the compartment partly sprung from the ribs. Instead of the reduced backs the brick ribs usually show a back of the rib piece in its entire width, as at the entrance of the German Catholic church at Wilna, at the cloister of Riga cathedral (Figs. 156, 157).

Moulding.

As for what concerns the moulding of the part projecting from the surface of the compartment, this is extremely varied, yet is alwayu first dominated by the law of resistance. For the strength of a certain arch first depends on its depth. If a movement sidewise does not come in question, and the arch has receive predominating vertical loads, then the breadth has far less importance for its resistance . This predominating importance of the depth must be expressed in the profile of the rib, both in its dimensions and the character of the mouldings. Therefore the height is to be made at least equal to the breadth and better more, so that it is to the breadth as the diagonal of a square to its side. The ancients very soon recognized the importance of depth, and this is often expressively emphasized already in Romanesque cross sections.

In form the cross section of the rib naturally tends in its basal form to the far older arches, as in these were the rectangle or the round. The rectangle sometimes appears in the first occurrence in the simplest way indeed in the simplest form in the ribs of the crypt at Gloucester belonging still to the 11 th century (Fig. 158). But mostly as for cross arches the angles are more richly treated, for example in Notre Dame at Paris (Fig. 159), or are merely chamfered (Fig. 160). Instead of the early chamfering at 45° there often occured later a steeper and more expressive direction of about 60° (Fig. 161). The round appears up to the transition time as a simple semicircle with or without stilting (Fig. 162), but it is most fre-

frequently attached to a rectangle (Figs. 163, 164).

The predominating importance of the height is most clearly expressed in the last form with two parts placed below each other, and this therefore also became quite particularly the starting point for further changes. An example of this kind is shown in Fig. 165, taken from a side chapel of the cathedral in Fritzlar and in a measure may be regarded as the root of t the richer forms represented in Figs. 166 to 172. In the choir of the churhc at Wetter the lower member is reduced and the c chamfer is replaced by a cove in Fig. 166. In the nave of the same church a few years later the connection between the cove and the vertical slab is made by a chamfer (right half of the same Fig.). The effect is still heavy here but is better if t the cove is cut deeper (Fig. 167). Yet more strongly is the c cove separated from the vertical side surface by a round inserted between (like the profile of the rib of the collegiate church in Treysa belonging to the 13 th century, Fig. 168) and to the cathedral at Magdeburg (Fig. 169). This round is sometimes repeated in smaller form before the beginning of the cavetto on the rib as in the chapter hall of the monastery of Haina in the 13 th century (Fig. 171), of Cologne cathedral (Fig. 1 170), and in the Marburg castle chapel founded in 1288 (Fig.172).

Simple forms directly result from the chamfered rectangle ((Figs.160, 161), if the oblique surfaces are replaced by flat cavettos (Fig. 173), from which then by doubling or deepening the cavetto are developed the form of Figs. 174 and 175 belonging to the late period.

The purpose to more clearly express the intersection of the ribs led in the late period to a general use of the cross section divided below and represented in Fig. 176, but which also occurred already in the earliest time on ribs and cross ribs.

Lower round of the rib.

As shown by a glance at the given cross section shown, most end below in a round. As a rule this is shaped by the compasses, (Fig. 171), also being sometimes drawn from two centres, either as a pointed rib (Fig. 178) as it occurs already in the earliest time, or as a wide depressed round (Fig. 179), such as late Gothic works indeed show it.

To the round was already added in the 13 th century an addition henceforth peculiar to it in the form of an edge or fillet

extending along its lower surface. This edge composed of two intersecting surfaces first occurred, but was soon followed by the projecting fillet in Fig. 181, which then became more commonly employed than the former. The use of this member might have been prepared by the pointed profile in Fig. 178, and even might favor setting on the centering, yet the main reason for its introduction must be sought in the artistic effect. With the great height and the confused lighting a simple round easily appears rather uncertain, the shadow limits often extending as long wavy lines on it. On the contrary the eye clearly sees at a glance the shape, if it can pass along a sharp projecting angle. Since only bold members are effective in this place, t the fillet soon projected quite strongly until the pyriform s section of Fig. 182 was reached.

In Fig. 182 is indicated how this form of round was composed of circular arcs. Such a sketch in circular lines was not always the rule in the middle ages, but much rather measurings of cross sections of ribs from Cologne, Aix-la-Chapelle etc., show that these members like many others were frequently drawn freehand in pleasing curves.

Sometimes fillets were added on both sides of rounds, which project so far, that they nearly or quite unite with the bottom fillet, allowing the round to vanish as in Fig. 183. The other little rounds of the rib were later also furnished with an edge or fillet, which then also passed on the members extending down the pieces as well as on the moulding of the window jambs etc.

General form of cross section.

If one places on the cross section of the rib a series of l lines touching the high points, he recognizes that generally in the course of time the rectangular ground form disappeared more and more, and on the other hand the shape of a triangle pointed downward ever became more expressed.

Such a series of lines enclosing a cross section or its principal points (for example the centres of rounds and cavettos) frequently and undeniably exhibits a regular geometrical figure, for example a square, isosceles right angled triangle, an equilateral triangle, etc. 1 : 1, 1 : 2, also 3 : 5 (about the golden mean), as well as occasionally the ratio of the side to the diagonal of the square.

It is not to be denied, that a basis of such simple relations greatly facilitated the existence of an expressive form, and further the convenience of starting points for sketching and executing a block of stone. This advantage was also justly utilized by the middle ages, but in the best period at least the true limit was never exceeded. The cross sections of the ribs show that geometrical constructions first come in question only after the requirements of resistance and of artistic expression had made their rights felt. The vault arches appeared confused in the broken light and foreshortened at a great height, and even exhibited requirements in their mouldings, that scarcely were expressed in the geometrical ratios of the cross section, but much rather appeared in the entire peculiar forms of the members, for example in the origin of the pyriform sound.

Relation between rib and cross rib.

When the ribs first appeared, they were sometimes made as large as the cross rib, thus on many French works from the end of the 12 th century, as well as in Germany at Walkenried, in the choir at Magdeburg, etc. But men soon recognized that the ribs only required a smaller cross section, and therefore they were made generally weaker than the cross rib, as already usual in Romanesque works in Germany. That was rightly done, for then the stilted vaults usual at the time like domes, transferred a considerable load to the cross rib. A strong cross rib further gives a desirable and firm resistance of the opposite pier to variations of loading, wind pressure, etc. But especially necessary is a broad cross rib where adjacent unequal vaults exert different side thrusts on the cross rib. Where extra loads of masonry or the framework of the roof is entrusted to the cross rib, a corresponding strength is also naturally required for it.

Such reasons can demand very great strength for the cross rib, where however they do not appear too compulsory, one is satisfied by assuming the difference in width between cross r rib and rib at about 5 : 3.

Where in many forms of vaults of the advanced Gothic period there is referred to the cross rib only the work of an ordinary rib, it was quite logical again to make the size and form of cross section exactly like that of the other ribs. For the elongated net vaults the cross rib was often externally omitted.

But as soon as a reason occurred for the existence of the cross rib, it appeared in the latest time in a moderate size.

The cross section of the cross rib is mostly a rectangle with more or less richly treated angles (Fig. 184).

Cross section of the cross rib.

Very frequently is placed a semicircular or polygonal projection under the rectangle (Figs. 185, 186). Of these forms the Gothic takes particularly the simple rectangle, which is membered iy the most varied ways. The chamfer, cove and still more the round remain favorites in the entire Gothic period. When the breadth was relatively small, already after the transition period the lower surfaces quite back off and on, so that the double forms in Figs. 188 to 191 result. Figs 189 and 190 are taken from the choir vault of Magdeburg cathedral, while Fig. 191 found egployment at Strasburg, Freiburg, etc.

Other more frequently occurring forms are given in Figs. 192 to 194.

The membering of cross ribs and ribs differs as shown in the examples represented, generally from each other and are often quite different in the same vault, even if an extensive harmony is always intended. Even in those of the earliest works, that exhibit cross sections of equal sizes for both ribs, the members often vary as shown in the compared examples from the abbey church of Walkenried. (The same cross sections are found over the upper choir aisle of Magdeburg cathedral and on the nobles' refectory at Maulbronn.

The correct feeling expressed in the different examples of the actually supporting and the occasionally bearing and dividing cross ribs in the membering is followed in late Gothic, but besides it occurs also quite early the endeavor to treat both ribs alike. In many cases the cross section of the cross rib is merely a widening or enriching of the form of the corresponding rib. Thus the forms represented above are in great part transferred to the cross ribs, and an example of this kind is reproduced in Fig. 198 from Cologne cathedral.

In determining the membering of both ribs it must not be overlooked, that beautiful and regular growth of the same at the springing of the vault (see that) must determine their forms.

Like the pier ribs, greater cross sections of cross ribs are composed of several layers of stone above each other. In the

earlier time also in brick in rowlock arches was favored, where now the bricks are set in the usual bonding. A brick cross rib from the 13 th century from the chapter hall of the cathedral at Riga is shown in Fig. 197.

In the later time the upper angles of cross, diagonal and side arches are frequently replaced by slopes (Fig. 199), by which in the execution results a neater junction and a rather desirable bearing for the placing of the centering.

Side arches.

Side arches may either project from the face of the wall or lie in it.

In the first case (Fig. 200) the side arch is bonded, i.e., the ashlars or bricks composing it extend for at least half the arch into the wall, and therefore must be set with the erection of the same and project with at least half the members of the corresponding cross rib, or better have an independent section before the face of the wall and form with its back a curved ledge on which the compartment rests.

Where the side rib does not project from the face of the wall, then must this bearing b be formed by a groove sunk in the wall corresponding to the side arch. This recess in Fig. 201 is sometimes formed in works executed with a certain economy in ashlar masonry by cutting it in the usual bond of horizontal and vertical joints, afterwards striking the true circle. In rubble work the stones are sometimes merely set back in the ordinary bond, whereby very irregular rib lines result, or in brickwork is formed by three concentric arches (Fig. 201), although by the last method compared to that of a bonded side arch, easy execution is scarcely obtained.

In the early time it was especially common to recess the wall above the side rib, whereby a safe bearing was produced in the simplest manner possible above the side rib. An example in cut stone is shown by the ruins of the church of S. Maria at Lippstadt (Fig. 202) and a similar one for brick from the cloister of Riga (Fig. 203).

Size of the cross section of the rib.

On the absolute size of cross sections of the ribs of vaults it is difficult to make an absolute statement. As the statements of the preceding Chapter (p. 59) teach, the magnitude of the compression to be transmitted by the ribs come into consid-

consideration less than to receive it correctly at the middle of the cross section, or in other words the proper position of the line of pressure. It it is desired to calculate the section of the rib only according to the magnitude of the compression, then would usually result sections of such small dimensions as to be practically not executed. With regard to a safe reception of the line of pressure, the size of the section is properly not to be limited too much. It is assumed in practice that subordinate ribs of cut stone 15 cm wide and 22 cm deep should s suffice for vaults with diagonals to about 9 m. Brick ribs of the cross section of a flat brick (12 × 25 cm) including some back projection will often be executed up to nearly the same span. But it must be recommended to increase the section for such spanr by large special bricks or by several bonded bricks. To this can be added in place a thickening of the compartments over the back (compartment masonry above).

As the lowest limit for the breadth and depth of the cross section of the rib may be designated indeed 9 and 15 cm, although for small bricks nothing prevents making,them still smaller. In fact we find also on old works sometimes yet smaller sections, for example in the triforium of the church of S. Mary in Stargardin Pomerania being those whose projecting portion measures only about 8 × 10 cm.

The erection of rib arches will be mentioned in the last Chapter (centerings etc., p. 118 et seq.).

7. On Closing Stones.

Keystones of arches.

Closing stones of masonry arches.

For a simple wall arch it is in structural respects most immaterial, whether it shows a joint or a stone at the crown. Yet it was a rule that the Romans had to give the middle of t the arch a keystone, which became a favored ornament as on the triumphal arch. The middle ages abandoned this rule and according to daily judgment sometimes ordered a joint and sometimes a stone, but in the pointed arch the joint at the crown is even preferred; Figs. 204 to 207. The other joints raidate from the centre of the braoch of the curve. With small stones or bricks were not avoided intersections formed as in Fig. 206. Only occasionally as at the city gates of Pisa (middle of the 12 th century) did men strive for a gradual transition in the direction

of the beds, when either a part of the upper beds were allowed to run toward a different centre d (Fig. 207), or all beds were directed toward a common centre c. In brick arches such a gradual transition is more common.

But a separate keystone is not rare for pointed arches, but was especially employed for heavily moulded arches to make possible a more solid junction of the members (Fig. 208). The hook-shaped reentrant angle at a sometimes afforded opportunity to form there a projection (rosette) or a cylindrical body growing out of the lower members of the arch. Examples of this kind are shown by the side arches of the minster in Freiburg. That this filling in slender arches is statically favorable is shown in Fig. 127 E.

Keystone of the vault.

Closing stone of the vault.

Among the vaults the tunnel like the wall arch may have a c continuous joint or a closing course of stone at the crown. Naturally a single keystone cannot be mentioned for a vault, for this first comes in question for vaults with raised middle and for cross vaults. But all real importance succeeded the keystone for ribbed vaults. Here unite at the crown of ordinary cross vaults four rib branches at a point, six in the hexapartite vault, and frequently more in the choir and star vaults. Such a closing stone has also to satisfy requirements of strength, of solid execution and of beauty, for it must hold fast the branch ribs and unite them immovably, make safe a transfer of the middle, must receive the arch members in an ornamental way, and also finally have a worthy appearance as the climax of the internally visible construction.

Simple intersection of the ribs.

To this importance of the keystone corresponds the fact, that it was strongly accented already in the Romanesque period after the appearance of the first ribs and was richly treated. Only the more modest works show at all times a simple intersection of the rib mouldings. Thise will first be mentioned ar simple coverings of the rib branches forming terminal points.

It is theoretically permissible to allow the mitred ends of the ribs to meet in the goints a b, c d, etc. in Fig. 209, as frequently occurs in brick ribs. But it would be difficult to carry out this in stone, with acute easily broken edges, incon-

inconvenient setting (especially with many ribs), easy displacement and finally an ugly appearance of the joints. Therefore only one arrangement can come in consideration, which makes t of a single common cut stone the middle part with projections for each rib (Fig. 210).

The size of the closing stone is arranged according to the size and number of the ribs; where sufficiently large cut stones are at hand, it is advisable not to make the separate projections for the ribs too short. On the contrary if there is a reduction in size, all projectins will be made so short that the mouldings can develop freely.

Independently developed closing stone.

When the ribs meeting in a closing stone have different forms of arches, thus striking the closing point with different inclinations, then must the projection for each rib be made by itself. In this case the members of the different rib branches do not regularly intersect, but the intersection of the ribs acquires an ugly appearance. Moreover since there the unity of the importance of the closing stone is not expressed, and thus the useless removal of a considerable portion of the stone is required, and the middle ages preferably replaced the naked intersection of the ribs by an independent form of closing stone, that shows the most manifold variations. Men inserted between the ribs a round or angular body, that frequently extended into a ring. Then the junction between two ribs was enriched by foliage or angels' heads, or it was concealed by a great d disk placed beneath, or finally the closing stone was allowed to project far below and it was covered by rich plant and figure ornament.

A particular favorite was to allow the ribs to terminate against a cylindrical nucleus on which the projections for the ribs were cut, so that now the entire closing stone received the form given in Fig. 211. It was next to omit the projections for the ribs and to cut the sides of the nucleus with the radial directions of the separate arches, thus giving it a conical form. But since the junction of the joint surface of the ribs with a round surface demanded a conical form thereby caused a flushing of the courses (Fig. 211a), small projections then always became necessary and made a conical form of nucleus unnecessary, since on it was formed the radial joints. The least

radius of the cylindrical nucleus was the length b a; as a rule it was made greater, so that the surface of the cylinder was everywhere visible between the projections for the ribs. At t the height of the face of the compartment this cylinder is reduced, so that a projecting ledge remains, and the smaller diameter continues through the thickness of the compartment as s shown in Fig. 211. As a rule for cylindrical closing stones t this recessed projection that passes through the thickness of the compartment is round. It then chiefly exists when these remain without starts for ribs, and is necessary for closing stones with an opening in the middle.

On certain works from the beginning of the 16 th century t the portion of the closing stone passing through the thickness of the compartment has a greater diameter than the visible cylinder, so that the projecting ledge is formed in the converse sense, that lies on the backs of the ribs.(Fig. 212). The ribs themselves terminate then at the sides of the cylinder with a vertical joint, but to avoid a hollowed form of the joint surface are let into the surface of the cylinder. Likewise the b backs of the ribs must pass so far under the projecting ledge of the closing stone and be dressed horizontal if the rib is not horizontal next the middle. But both requirements make the construction disadvantageous.

The moulding of the ribs is generally carried around the sides of the cylindrical nucleus of the closing stone (Fig. 213); frequently the member also takes an entirely different form as in Fig. 214. The last arrangement is preferable when the ribs are pointed arches, since on account of the oblique joint against the closing stone, the moulding on the latter will have a different and much higher form. When the ribs join at different vertical angles, carrying the same moulding around the closing stone is usually no longer to be attained, since at each rib a different section of the moulding is formed, and it then remains best to leave the sides of the closing stone entirely plain. The mouldings of the closing stones in Figs. 213 and 214 show a projecting disk beneath, that affords opportunity for rich ornamentation (see below). On many early Gothic works, these disks project far beyond the projections for the ribs, so that the junction of the ribs is concealed, as at the monastery of Walkenried and the collegiate church at Lippstadt. It

is unnecessary for a cylindrical nucleus to exist above this disk, and this in Walkenried (Fig. 215) the ribs simply abut against each other, after having been changed to a rectangular cross section. The disks beneath instead of the round shape, often assume an entirely independent form as a trefoil, quatrefoil, or even a combination of the trefoil and triangle oh the quatrefoil and square.

But sometimes such a form of ground plan is directly carried up as a nucleus, so that all lower projection is omitted, and the members designed for the sides of the members now extend around this basal form. Fig. 216 exhibits such an example, where the ribs enter the projections of the quatrefoil, while with a round nucleus they could also have joined in the direction a b.

Usually the round ground form of the nucleus is merely conventional. It certainly affords the advantage, that the ribs passing in different directions toward the middle of the closing stone intersect the side at right angles, but the originally square shape of the block now appears only in the projections for the ribs. But more clearly expressed is the cut block in square or approximately square form of the closing stone, as shown in Figs. 216, 217. In the rectangular end of the choir of the church of Volkmarsen the ground form of the closing stone is a diagonal square, so that the ribs join its corners. On the underside is found the lamb and the banner of the cross in a panel sunk by a flat moulding, in the angles of which are made four rosettes. Likewise as the closing stone not rarely found in the form of the vesica piscis, then bearing a figure of S. Mary.

But in the cloister of Erfurt cathedral is also found the converse arrangement, for the nucleus of the closing stone is in form of a square or flat four ribs, the ribs joining its sides and their joints are here concealed beneath by an attached and richly ornamented disk (Fig. 218).

Loading the crown. Pendent closing stone.

The size of the closing stone is not capriciously assumed, for round arched ribs could support only a light closing stone, while conversely steep pointed ribs require a greater load at the crown, whose magnitude can be found by constructing the line of support. The load on the crown can be obtained by a

case

In every case must be taken for this projection in establishing the centering. I.e., the upper edge must be made sufficiently low at the crown as to leave space for setting the pendant closing stone. Further see under Centerings.

The principle of loading the crown, which in general led to the cancelment and the decided treatment of the closing stone, is not more clearly expressed by the form of the pendant.

ing a vertical plane, and thus assume the appearance of corbels [illegible]. [illegible] pendant stem on which are placed two rows of leaves each about like cross flowers (fig. 214). In this case with the low centre of the diagonal ribs not exceeding a semicircle, hence the loading of the crown was entirely unnecessary, anything is undesirable, yet the writer sees in moderation. by which was attained reversing it and resting at nothing.

Another is more labored, since an attached structural ornament in the form of the closing stone. It looks to a bag or the ordinary style of the cathedral of Lyons, where it assumes the form of a suspended canopy of rouen fronted iron. the closing stones that represent the springing of the ribs for the more ignorance of completing the bodies in continuation of the ribs. [illegible] [illegible] [illegible]

corresponding breadth and depth of the closing stone, or in s some cases by a great specific weight of the building material.

Usually the closing stone projects below the lowest bottom of a rib as shown by many examples mentioned, thus in Figs. 215, 216, 218. This drop either remains plain on the underside and only has a moulding around its edge, or a more or less rich ornament is given to the mass, and expresses the loading of the crown demanded by the upward pressure of the pointed arch, and at the same time affords opportunity by its richer execution for bringing to a conclusion the effect of the vault in ornamental respects.

In every case must be taken care for this projection in establishing the centering, i.e., the upper edge must be made sufficiently low at the crown as to leave space for setting the pendant closing stone. Further see under Centerings.

The principle of loading the crown, which in general led to the pendent and the decided treatment of the closing stone, is yet more clearly expressed by the form of the pendant.

In the simplest case this takes the form shown in Figs 216, 218, because the separate leaves have a position more approaching a vertical plane, and thus assume the appearance of corbels or capitals. In the church of S. Mary in Mühlhausen is found a pendant stem on which are placed two rows of leaves each almost like cross flowers (Fig. 219). In this case with the low shape of the diagonal ribs not exceeding a semicircle, where the loading of the crown was entirely unnecessary, caprice is undeniable, yet the refined sense is wonderful, by which was attempthe the transformation of the cross flower, instead of merely reversing it and leaving it unchanged.

Another is more labored, since an affected structural significance in the form of the closing stone is found in a bay of the northern side aisle of the cathedral of Mayence, where it assumes the form of a suspended canopy of square ground form, against whose angles terminate the ribs. Likewise belong there the closing stones that represent the springings of the ribs as resting on a rising corbel, so that also the projections f for the ribs instead of penetrating the nucleus in continuation of the rib arches, are bent near the joint and spring in an arch of smaller radius from the corbel forminm the lower ending.

But the entire principle is carried to an extreme and appears

in the exclusively late Gothic design of the suspended vault, that is especially common in England but more rarely found in France and Germany. An application of this construction to the plan of the net vault is shown by the cloister of S. Stephen in Mentz. The closing stone has here become a formal suspended column and like an actual hanging column with struts in wooden construction, thus it here supports the ribs of the true vault above, continues down to the height of the ground line of the vault and ends in a free knob. Above the knob are then found the projections for the rising ribs. Fig. 220 exhibits this c construction in section. There is the closing stone supporting the arches, b is the suspended closing stone, c the ribs and d the compartments of the vault.

The rich and picturesque effect of such vaults expiatees in their reality the exaggeration of the principle. However it m may be regarded as a mere decoration, and thus forming only an accessory development from the construction, in nowise merely something fastened or glued to it, in which production architecture is so rich.

Broad development of the closing stone.

It was shown in the preceding how far the development in height of the closing stone could be carried, so on the other h hand is it also capable of a greater breadth. particularly the meeting of a great number of ribs leads to an enlarged closing stone. In many works of the transition period, especially in the Westphalian churches at Billerbeck, Leyden and also in the great church of S. Mary at Lippstadt as at the cathedral at Minden, the meeting of eight ribs has been so arranged, that only four extend to the closing point, while the other four are received by a concentric circle. See Fig. 211 for the church at Billerbeck (after Lübke). There are to be mentioned here similar more sportive treatments of the middle of the vault, among which is a great ending of ribs like a wheel in a vault of the middle aisle of the cathedral at Paderborn is particularly prominent.

Perforated closing stones.

Large closing stones are frequently perforated, and also smaller ones have often received central openings. The perforations may serve for very different purposes, for lowering ropes for scaffolds, hanging chandeliers, ventilating the interior,

and finally are intended for hoisting larger objects like bells. In arranging the openings for emission of air is necessary a certain economy, since especially in little churches there easily originates an unbearable draught, which at least leads to the temporary closing of the opening.

Around the inner side of the opening somemtimes extends again the moulding of the ribs or one entirely different (Fig. 222). Usually the sides of this opening are also simply vertical as in Fig. 231, it forms in a certain sense the middle of a rosette as in Fig. 231, or it is otherwise connected with the ornament of the closing stone as in Fig. 216, where the perforated opening of the mouth of a head represents it.

Those middle openings further enclosed by a circle of ribs in vaults, required for hoisting larger objects to the upper rooms like attics of towers, were also executed in vaults with compound rib systems. The opening itself is then generally closed by a stone slab laid over it or by a lighter removable cover of planks.

Piece of rib.

With an unequal length of the sides of a bay, there will remain considerable spaces between the beginnings of the ribs on the sides of the closing stones in the direction opposed to l longer sides, while in the directions opposed to the shorter sides the beginnings of ribs may join each other. In such cases these intervals are sometimes decorated by heads projecting f from the sides of the closing stones. Such an example from the former Franciscan church in Fritzlar is shown in perspective by Fig. 223. The same case occurs on the closing stone of the hexapartite cross vault, as for example in the southern transept of the cathedral of Wetzlar, and further in those of the choir vaults (Fig. 224). In both the last cases is found the same, an arrangement of such heads producing a very happy effect. Viollet-le-Duc gives several French examples of this kind.

In a different manner are the inequalities of closing stones of choirs lessened, if the closing stone is moved a bit a b in Fig. 224a from the middle of the polygon, but where the direction of the ribs toward the first point remains unchanged. Thereby is indeed produced a more oblique and irregular position of the starts on the cylinder, and also the heights at which these sometimes succeed are changed.

In Fig. 224a the middle lines of all ribs meet at the same point a in height. Now since the distance c x to the junction of the rib is less than from the same point for the rib d y, thus a c is smaller than a d, d will be lower than c. Hence t the closing stone must have a depth greater than is required by the junction of one rib, and this extra height must equal the difference in the heights of the points c and d. But the lines of the backs of the cross sections of the ribs also strike the closing stone at different heights as well as the two sides of the same rib. Thus the point f lies higher than the point g of the same rib, and both are again higher than the also unequal points e and h. Therefore the compartments also join the closing stone not in a horizontal line, but in one r rising from h toward i, and hence the before mentioned ledge of the nucleus must be wrought to such a line (Fig. 211). The entire construction is expressed clearly in the perspective of Fig. 224b, where for greater clearness a pointed form of the rib arches is assumed, such as usually occurs in practice, so that both the irregularity of the joinings and also the inclined position of the projecting ledge of the nucleus appears in labored distinctness.

Quite similar conditions require the forms of closing stones if the ribs join with different inclinations. Fig. 225 will m make this condition clear, and there is the proper crown of the vault. At the left side of the Figure are drawn the joinings of two ribs, both being directed to c. Hence the steeper one meets the surface of the closing stone lower at b a, but the flatter is higher at d e, so that the entire depth of the closing stone must be b d.

All these irregularities may be avoided when the junctions of the ribs have a horizontal transition as in Fig. 225a, and the arches of the separate ribs are accordingly constructed so that they are not struck from the point c, but from the points d and e, that would be shown by the first closing stone to be constructed.

Esthetic development.

It still remains to cast a glance at the esthetic development of the closing stone. In whatever manner the side surfaces are formed, membered by varied mouldings or enriched by projecting heads, etc., is already shown at the proper place. Yet more was

busied the artistic joy of creation in the mode of treatment of the underside, and there Gothic ornamentation prowed itself as an inexhaustible wealth in the most splendid manner. These closing stones, like the sun in the vault of heaven, frequently in even the dryest works are found most richly adorned, forming the sole decoration of them. This effect was again heightened by the often still visible painting not merely of not merely the closing stone itself, but also of the adjacent parts of t the ribs joining it, unfortunately now in most places made unrecognizable by whitewash or dirt. The destruction of the splendor of color is the more to be regretted,,since even the relief treatment of the foliage was so arranged, that it required color in many and indeed in most places to make visible, how the separate leaves entirely flow into each other, no longer separated by color but blackened by smoke.

We can here merely specify the principal modes of treatment and illustrate by examples taken for the reasons mentioned, in great part from the smaller churches, cloisters etc. The simplest treatment originates when the ornament is sunk into the underside of the cylinder, so as to leave it enclosed by a border lying in this underside.

Forms of this kind are frequently found even in the transition style, as in the vaults of the cathedral of Mayence, but in the later time they chiefly appear in very simple works. An example from the church in Trendelburg near Karlshafen is shown in Fig. 226. It is better when the ornament joins a disk projecting downward, so that its recesses no longer extend into the nucleus further than to the bottom of the ribs. A sunken wrought ornament then either lies on a surface recessed by a member as in Figs. 232, 232a, or on a flat sunken underside as in Figs. 230 and 231.

Usually it also occurs that the foliage without an enclosing border springs strongly and directly from the underside of the closing stone and not rarely projects beyond the border. Examples of this kind are shown by Figs. 227 from the church in Trendel burg with section in Fig. 227a, then by Figs. 228 and 229 from the church in Volksmarsen (Fig. 228a is the section of the latter), Fig. 216 from the cathedral in Freiburg, Fig. 230 from S. Blasien in Mühlhausen. Sometimes the foliage also lies on the disk concealing the underside of the closing stone,

as shown by Fig. 218 from the western side of the cloister in Erfurt.

The arrangement of the foliage is extremely varied. In the simplest case the leaves radiate indeed from the middle of the edge, or more rarely in the reverse direction as on certain closing stones from S. Blasien. In the first case is thereby prudoced a rosette, either single or filled by several rows of leaves. The arrangement of such rosettes is the advantage of easy recognition, and sometimes is found as the conventionalized imitation of a rose, as shown by Fig. 227 in a form belonging to the 14 th century.

Instead of the straight position the separate leaves are sometimes curved sidewise as in Fig. 228 or placed in a cluster as in Fig. 232, or they are mixed with leaves directed otherwise, so that the rosette entirely disappears as in Figs. 229 and 216, or finally the ornament consists of a branch laid on the underside and bent in a circle, its leaves then covering the surface as in Figs. 230 and 218.

In the earlier examples like Fig. 231, the ground between the separate leaves is still visible and the modeling is merely indicated, so that the leaf is chiefl: effective by its outline, while already at the end of the 13 th century the modeling gains so much in projection, and the leaves lie under and over each other, that the ground almost disappears, but the perception of the whole suffers as in Figs. 228, 216. Yet in both c cases is still retained a general effect, iu the first by the symmetrical arrangement and in the latter by the head placed at the centre. Yet attention is called to this, that the effect is essentially richer if the separate leaves alternately show their upper and under sides as iu Fig. 229, whereby at the same time ttheir different colors have a motive. Just as in Fig. 216 the head forms the centre of the closing stone, such often forms the entire ornament so that to obtain a regular form either the hair and beard are utilized or the latter is replaced by leaves.

Usually figure ornament produces the richest and most beautiful decoration of the closing stone. Here belong first the symmetrical representations, such as the symbols of the evangelists, the pelican (Fig. 234) from the cloister of the cathedral in Fritzlar, the lamb with the standard of the cross, etc.,

then the sun and moon (Fig. 226), or even more fanciful animal forms like Fig. 233 from the eastern side of the cloister in Erfurt. Particularly in such forms does the refined feeling for ornament strive, that guided the forms of the ancients. Thus all animals are conventinalized with wonderful intelligence, that is derived from their natural peculiarities so much as necessary for their designation, and thus by the stronger accenting of these peculiarities, by the omission of all indifferent parts is obtained a representation, which expresses the desired object in a more acute way. The most decided opposition to this method of treatment is found in so many modern symbols, that are formed in the most naturalistic manner, and allow the characteristics to appear so indefinitely, that the most amusing alterations are produced. Thus the lion is now represented with metal hair and is scarcely recognizable at the least distance, the hair does not cause a clear outline of the form, and merely forms a covering of the mass. The outline should reproduce the natural sharpness but loses definiteness and energy, the movement is crippled and the entire conception is far removed from expressing in the least the idea of the lion. Likewise is it with all other heraldic figures as well as with most of those animal forms, that afford an integral portion of modern ornament. As in so many cases, here mediaeval modes of representation agree with the antique, but the modern is entirely opposed. Thus for example in the collection of Roman antiques and casts in Mentz are certain animal forms engraved on helmets, shields etc., that could almost be held to be mediaeval drawings.

Figures are found on the cloisters already in the first times, particularly of the holy Virgin, the patron of the church or of the order, angels, etc., but frequently also a representation from sacred history, especially common being the coronation of S. Maria. Beside the figures there sometimes remains a free space in thecloister filled by foliage that encloses the figures. Of the greatest beauty are such examples in Viollet-le-Duc's Dictionary. Also the arms of the founder, even cities, families and states find there place there. Sometimes are even found vacant shields on which probably coats of arms were originally merely painted. See Fig. 217 from the church in Gottsbüren.

Intersections of ribs beneath the crown of the vault.

In star and net vaults occur intersections of the ribs that

the rules the cross of the vault;

[illegible] for [illegible] cases, [illegible]
section or against a closing slope. The difficulties result
because the cross section of the ribs are not perpendicular t
all others on the vault. Fig. 295, but are everywhere ver
[illegible].

This difference shows little or not at all for the diagona
rib, but very strongly for the transverse rib. In Fig. 295
where the cross rib is perpendicular to the direction of the
[illegible]. See example of next [illegible] regularly. See al
the contrary in Fig. 296 is found an irregular intersection.
the ribs are arranged so that at least their lowest edge
predominant, and the reason for it is that the radial applica-
[illegible] [illegible] [illegible].
makes difficult the placing of the centerings as well as
cutting and setting of the ribs, and finally leads to the out-
ward projections of the ribs on the plan, as they were usually
executed in the late time.

The theoretical intersection of the members of the ribs, m
is observed on all rock vaults, if not very strong-ribbed or
even those with the vicinity of the abutments. The requisite
[illegible] [illegible]
conformance to the needs of the case, that the different
members of the intersection.

The angle r of the cross rib rises in Fig. 298 by the diag-
onal r s above the back of the other rib. Accordingly the cr
requires the cross rib to be [illegible] [illegible].
the main rib which is set at the height s. These [illegible]
are only combined by raising the lower back by a wedge-shape
f r to the same height or in any case so that the other
reduces to height.

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible],
[illegible] [illegible] [illegible] [illegible] [illegible] [illegible]
imply here the cross rib must be raised at one side, either b
an upper surface inclined sidewise (Fig. 297), or by [illegible]
tion (Fig. 298), and finally by the existence of an abutting
fillet can be obtained by the additions of different heights
[illegible] [illegible] [illegible]. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].
[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]
298, in perspective in Fig. 84.

lie below the crown of the vault; these always afford opportunity for irregular forms, whether they arise from a mere intersection or against a closing stone . The difficulties result because the cross section of the ribs are not perpendicular to all directions of the vault, Fig. 235, but are everywhere vertical underneath, Fig. 236.

This difference shows little or not at all for the diagonal ribs, but very strongly for the transverse ribs. In Fig. 235 where the cross rib is perpendicular to the direction of the vault, the members of both ribs intersect regularly, but on the contrary in Fig. 236 is found an irregular intersection. the ribs are arranged so that at least their lowest edge was predominant, and the reason for it is that the radial arrangement of the ribs leads to other and greater difficulties, and makes difficult the placing of the centerings as well as the cutting and setting of the ribs, and finally leads to the curved projections of the ribs on the plan, as they were actually executed in the late time.

The irregular intersection of the members of the ribs, which is observed on all net vaults, is not very embarrrassing and even increases the variety of its appearance. The particular difficulty of the arrangement increases by the junction of the compartments to the backs of the ribs, that lie at different heights at the intersection.

The angle r of the cross rib rises in Fig. 236 by the distance r s above the back of the other rib. Accordingly the cross rib requires the cross rib to be set at the height r, but not the main rib which is set at the height s. These requirements are only combined by raising the lower back by a wedge-shaped f r to the same height or in any case so that the other ribs are reduced in height.

At the other side of the cross rib the condition is reversed, and here the back o′ of the latter lies too low by o p. Accordingly here the cross rib must be raised at one side, either by an upper surface inclined sidewise (Fig. 237), or by anprojection (Fig. 238), and finally by the existence of an abutting fillet can be obtained by the additions of different heights at both its sides (Fig. 239). The entire intersection will have its form shown with the assumption of the stepped back of Fig. 238, in perspective in Fig. 240.

Covering the intersections.

Another intersection of the ribs is represented in Figs. 242 to 242c that corresponds to the point A in the net vault of Fig. 241. The intersection is symmetrically formed at both sides of the axis MM. Fig. 242a shows that a wedge on top is necessary as for the two lower additions of the ribs, and Fig. 242c shows the cut block for the case that an abutting fillet exists on the back of the arch. The intersection of more or l less than four ends of ribs is arranged according to the same points of view. As a rule these intersections of star and net vaults consist of a simple crossing, while only in the ridges are arranged actual closing stones. But sometimes there are found in the angles between the additions to the ribs leaves or clusters of leaves. The same treatment however occurs also on the closing stones on the ridge. Likewise the view of the intersections from below is sometimes concealed by a more or less richly ornamented disk lying on it.

On the contrary more rarely is the intersection avoided by the arrangement of a cylindrical nucleus. The position of this cylinder strictly taken must be perpendicular. The case when the projections for the ribs extend obliquely from the nucleus may lead to make these inclined. But it is undesirable that the vertical position of the side surfaces of the rib sections is in essential conflict with the inclined position of the surface of the cylinder, and thereby another arrangement is sought, as an intersection concealed by foliage might represent the proper treatment. An extremely rich and happy treatment of such intersections is shown by the choir vaults of the church of S. Martin in Cassel, where the twelve crossings of the ribs are concealed by the sculptured figures of the twelve apostles cut on the respective ashlars, while the closing stone of the whole bears the crucifix.

Forms of the late period.

In certain works of the late Gothic the intersections of the ribs are so intentionally labored and enhanced, that one of t them not continued beyond the intersection is extended as a short piece purely for ornament and is then cut off at right angles. Fig. 244. For this is sought a certain reason. Yet it might appear more correct to allow the part of the cut stone, that must be omitted between the ends of the ribs, to remain

in the surface of the compartment, and thereby reduce yet the mass of the stone by just that cut off piece of the rib.

But men went still farther, when in certain cases where the plan of the vault required no abutting or intersections of the separate ribs, when these last simply pursued their course, to indicate a stopping point by allowing a rib piece cut in the same profile at both sides to cross at right angles in Fig. 243. We have had no opportunity for a closer investigation, that exception is found in the church of S. Katherine at Eschwege. However it is not improbable that men desired to ensure a certain bonding of the ribs extending flat beneath the compartment with the masonry of the compartment by just these longer crossing pieces perhaps passing through the thickness of the compartment. These corresponded to the rib arches inserted at certain distances and the cylinders passing through the compartments in certain early Gothic churches of Westphalia, which effected a similar strengthening like the bonds on the columns and little shafts of the early Gothic.

8. Springings of Vaults.

The springings are the lowest parts of vaults, so far as they are connected with walls or piers, and therefore must be executed at the same time as those, while the erection of the proper vaults only follows later, after the whole is under roof.

Dangers and securing the springings.

In the pregothic the lower ends of arches and vaults were m mostly built free in the body of the supporting masonry. Fig. 245. This arrangement is safe and good if no rising masonry r rests on the lower end of the arch, thus if the upper high wall is set back to the line n n. But on the contrary if the wall is continued above in the same thickness, it rests partly on the extrados of the arch, and there finds the unsafe bearing. When the joint between tha arch and wall is open, their occurs a danger of a sinking of the wall at one side. This case can be conceived to occur in the middle walls of a basilica above the vaults of the side aisles. Still more insecure is the arrangement where above a strongly loaded pier carried up high, arches at both sides or wall arches cut in, as in Fig. 246. The heavy upper pier thrusts like a wedge driven between the arch springing, forces them apart and crushes at A the lower little stones until by continued movement destruction occurs.

In several new architectural structures have disasters occurred by this procedure. The endangered place may be secured by setting the pier and ends of the arches in good cement mortar, thereby making an entirel block of stone; also the wedge ing effect may be reduced by stepping the voussoirs (Fig. 247); but far more allowable are the logically deduced constructions introduced and tested in the middle ages.

Where it was concerned, Romanesque art sought to create relief there in the most effective way, so that the cutting of t the springings of the vaults into the masonry was avoided, the cross arches and also indeed the groins being supported by strong projections. Fig. 248. This expedient was naturally also most secure, so far as it went and Gothic also retained it.

But the projections restricted injuriously the internal space, and particularly made the aisle piers undesirably large. Men first sought a projection by strong projecting capitals, also indeed by the partial use of corbels, thus making the lower w wall thinner than above at the vaults. But this did not always suffice, and men usually saw themselves compelled to insert at least partly the springings in the wall. But they allowed now --- and that was the result of the new method of construction --- to be one with the wall. Figs. 249, 250. The beds were horizontal for the thickness of the wall, and in the projection they were either continued horizontally (see a and b in Fig. 250), or were made radial there.

The beds only continue horizontally if the face angle was n not too acute, and if sliding of the upper stones on the lower was not to be feared. But a slipping of the upper on the lower stones is possible if the angle a in Fig. 251 between the direction of the pressure in the vault and a perpendicular to the bed does not exceed the angle of repose. (Between stone and soft mortar about 30° to 45°, more for hardened mortar).

Springings of brick ribs.

The cut stones forming the springings of a cross vault are so large as to make it possible to transfer the strong thrust to the vault combined here, uniformly and quickly to a surface as large as possible on the supporting masonry. For brick it is advisable where possible to have a cut stone for the springing of the vault, but otherwise care must be taken to have hard bricksand strong mortar (recently cement or trass) with good

bonding. Particularly is good execution required where rib sections of moulded bricks are at the springing.

Let Fig. 252 be the cross section of the rib, then if Fig. 253 shows the plan of the springing of the rib, its execution is made by the omission of the part a d c of the brick cross rib and the part a b c of the brick diagonal rib and thus the lowest bricks of the under course take the form shown in the right half of the Fig. In the following course of bricks for bonding are cut in a different manner.

Rib springings composed of moulded bricks are necessary for making the joints, mostly radial in the first course, and the separate bricks then maintain their places as in walls by the adhesion of the mortar in spite of the inclined position. But where the vault was to be plastered later, and the springing was cut from ordinary bricks, it is better to corbel out the entire height in horizontal courses, or at least the lower part of the springing of the vault.

Springings of vaults without ribs.

The rules to be followed in the execution of the springings of vaults also naturally are applied to ribless vaults. The s springing is to be reckoned there to the place where the bricks of the compartment become free from the wall. If in ribless b brick vaults there exist the brick strength groins projecting above usual in them, then naturally for them the springing extends to their becoming free from the angle of the wall. The springing will best be built up to this height at the same time.

The height of the beginning of the stiffening groin is determined by laying off its cross section a g on the extension of the diagonal groin a x in the plan in Fig. 254, carrying up t the groin rib a′x′ and g′i′ (Fig. 254b). From the plan and erecting a vertical at a′, which cuts the back of the rib at i, x, then a′i′ is the height to which the springing of the vault is joined to the wall, therefore the corresponding part of the c centering must be set at the required height in constructing the wall, and on it will be built the lower spandrel of the v vault. In practice in brick vaults, especially if segmental, the springing is often not carried up at the same time, so that it must be the more carefully inserted later.

If several bays adjoin each other and are separated by cross arches, in case they bear an upper wall, these must be vaulted

at the same time with the rising masonry. When they serve only to strengthen the vault, it suffices to carry them to the height of the compartment spandrel adjoining them, so that the F Fig. 255 gives a perspective view of such a spandrel.

Springing of ribbed vault.

More complicated and differently formed are the rpringings of ribbed vaults. Already for ordinary cross vaults with projecting side arches, occur at a vault spandrel three arches at the corner of a room, five on the flat wall, and seven for a projecting angle. These arches may be turned free beside each other on an abacus slab of the capital or corbel as in the Romanesque period, or their plan may be carried sp together more or less (Fig. 257).

In the first case there is no mention of executing the springing of the ribs at the same time with the wall. It is only necessary to set the capital or corbel in erecting the wall and constructing the wall ribs together with it, while the other ribs, for whose bearing must exist the necessary space on the corbel, are then placed when the vault is to be completed. All banding with the wall is omitted. But on the one hand this connection is essential to the safe transfer of the thrust of the vault, especially for a lonsiderable stress, and on the other the large surface occupied by the separate ribs demands a great projection from the face of the wall, which requires either the arrangement of wall piers ending beneath it or a massive and high corbelling, by which moderate or limited dimensions are obstructive. For both reasons in contrast to the Romanesque, in the spandrels of vaults in Gothic art, as a rule the plan of the ribs compines together and with the wall, first separating at the height where the ribs have allowed the space necessary horizontally.

There are chiefly two points to consider. First must the springings of the ribs be easily and properly inscribed on the s surface on which they rest. Hence their plan at bottom is first arranged according to whether a separate shaft is to be placed for each rib or a common round or corbel is set for all. In t the first case the polygonal shape of the abacus follows the plan of each separate rib, while in the other the general plan of the ribs must correspond to the form of the abacus of the capital. The second point to be considered concerns the upper

surface op the springing of the ribs and consists in this, that it is advantageous for the execution of the compartments if the outside points of the different ribs if possible are freed from each other at the same height. The fulfilment of this condition depends on the form of the bottom plan. The possibility of it is indeed given in all cases; but with an irregular plan of t the bay it is only difficult and attained by continued experiments, and the bottom plan may compel such an unsuitable shape, that it is often better to deviate from it.

The more regular is the sparation of the members and the freeing of the mouldings from each other, the more beautiful will be the appearance and the easier is the preparation of the stone.

Requirements for the regular separation.

An entirely regular separation of the arches then occurs when the following conditions are indeed satisfied.

1. All middle lines of all arches intersect in one point in the plan.

2. All angles between the directions of the arches are equal to each other.

3. All arches have the same cross sections.

4. All arches are struch with the same radius, at least in their lower parts, when all arches are not stilted at all, or are so to equal heights.

The endeavor to take these requirements into account as much as possible, has sensibly influenced the further development of the Gothic vault, and it is completely fulfilled in the fan vault. The ordinary cross vault can only unite all the points when it is executed over square bays. Fig. 257 shows the springing of the vaults at the junction of three square bays, where all the preceding conditions are satisfied.

Most vaults and especially early Gothic do not combine the above requirements in a regular treatment of the springing, indeed often having not a single one of them. Yet by skilful solutions the existing irregularities are so far reduced, that they are not greatly disturbing. To afford the beginning for overcoming the irregularities in designing, a brief explanation of the four points mentioned may pind place here.

Intersection of the middle lines in plan. The intersection best lies in the face of the wall or its angle (Figs. 257, 260), yet frequently reasons (to be explained below) lead to placing

it farther back (Figs. 261, 265).

If the union of all directions of the arches at one point is not attained, but two or more points of intersection occur, t then the thrusts of the arches may cause an effect of rotation as shown in the plan of Fig. 258. The thrust I turns the point A in one direction and the thrust III turns the point B in the other. But unless the difference of the points A and B is not too great, this effect is not dangerous. A slight eccentricity is also not very notable in the appearance. Therefore a transfer of the point of intersection is often made intentionally to relieve other greater irregularities, as will soon be shown in Fig. 264.

2. Equal angles in plan between directions of the arches. In spite of the great advantages of equal angles in plan, they a are mostly different. This case already occurs in cross vaults over rectangular bays, as in Fig. 259 the angles a are notably larger than B. Fig. 260 shows that the upper surface of Fig. 260a is the prespective view of the corresponding springing of the vault. Ribs and cross arch separate at the same height at the points c and d, and the ribs and side arch have already separated at the much lower points m and n. Consequently the spandrels of the compartments start at different heights at t the places where the two front spandrels begin at c and d, the side ones already have the width a b or e f. These side springings of the compartments must be cut in the stone, as first begins the compartment masonry above the line a b. Structurally the beginning of the compartments at different heights presents no disadvantage, and when for sake of a good appearance a separation at the same height should be attained, relief can be obtained by setting back the intersection O to O in Fig. 261. The compartments begin at the same height at the points b c d e. On the other hand there result slight irregularities in the lower connection of the side arch with the rib in case the former does not have a suitable radius or is stilted. Likewise p pacing the point O' back too far easily has the disadvantage, that below the arches very quickly pass into the surface of the wall. For this reason it is often best to allow the start of compartments at different heights, and the perspective in Fig. 260a shows that the effect is really not so very disturbing.

Not to be avoided is the placing of the spandrels of the

compartments at different heights, where join two bays of unequal widths. Fig. 262 shows such a plan where all four angles are different. The beginnings of the spandrels of the compartments separate at different heeghts (Fig. 263), even if the middle lines of the arches all run to the point of intersection O. By a transfer of the middle lines of the arches so that they intersect in two different points O′ and O″ behind the face of the wall (Fig. 264), the three points of separation c d e are brought to the same height, but a lower beginning of a spandrel at a b is not to be avoided.

2. Similarity of the cross sections of the ribs. As mentioned in other places, like cross sections are used for some of the earliest works, but they again come into use chiefly in the later Gothic. Otherwise a difference between cross arches and ribs forms the rule.

Most regularly developed is the springing when all arches are alike (Fig. 257). Yet the influence of wider cross arches if no other difficulties occur, is very easily equalized by placing the intersection of the middle line of the arch back of o′. (Fig. 265). If the side arch is half the width of the cross arch, then the intersection o′ regularly falls in the plane of the wall.

More difficulties are produced by the compression of the cross arch and rib members on the abacus. A broader cross arch differing little from the rectangle cannot usually be contracted much below, at least the underside a b must remain the same. This fact explains why also for the cross arches, especially in the late time, a form of cross section reduced on the underside was favored, that is often shaped to entirely correspond with the arches. When the cross arch according to its internal nature is moulded differently from the rib, then care must be taken that in the combination of the members arises a beautiful intersection, of which one can judge by drawing horizontal cross sections at different heights of the springing.

4. Agreement of the radius of the arch and the stilting. A beautiful combination is easily obtained if all arches are struck with the same radius, their centres lyong in the same horizontal plane. But since requirements in the heights of the arches frequently oppose fulfolment of this law, then in many c cases it cannot be carried out, particularly for elongated bays,

unless the expedieot iotriduced by English Gothic of arch lines struck with several radii is adopted. (See the preceding elevation of the arch (Fig. 48)).

Where different radii occur, usually the irregularity shown in Fig. 266 cannot be avoided. If the arch I is struck with a greater and the arch II with a smaller radius, then will II at a certain height have receded horizontally already farther than I, i.e., the back of II has already receded to the point b while the back of I is only found at a. Consequently there is formed beneath the line a b a vertical surface of the spandrel, that above a b passes into the warped surface of the compartment, whose shape is shown by the drawn joint.

Frequently a stilting of certain arches cannot be avoided, and this usually also leads to the warped form of the spandrel of the compartment represented in Fig. 266, but in a still higher degree. Particularly commonly occurs the stilting of the s side arches on elongated bays. This stilting extends to a height where the other ribs have already separated far apart, so that there results the plans in Fig. 267a to d). Characteristic of this development of the vault is the weak connection between the springing and the wall, that is limited to the same measure m n in the entire height of the stilting. Since just at this point is to be rtransferred the thrust of the vault to the wall, care must be taken for a very strong construction with large bonding cut stones or hard bricks in excellent mortar. Further at these points the compression of the members should not be carried too far, especially where two bays of unequal width exert different thrusts from both sides. An example of a particularly great stilting of the side arch is afforded by the middle aisle of S. Paul of Liege.

For narrow side aisles frequently occur the converse case, and elongated bays are formed with side arches on the longer sides with a cross arch at the end. The cross arch must then be stilted, which produces a wide form of springing, like that drawn in Fig. 268 from the early Gothic collegiate church of S. Maria at Lippstadt, and shown in an expressive way. Fig. 268a is the plan above the capital, Fig. 268b is that above the stilting, and Fig. 2681 is above the separation of the cross arch.

Particularly striking is the appearance in polygonal chapels

or choir endings. The difference in length between the sides of the polygon and the ribs is so great in the plan, that the former are mostly spanned by very highly stilted side arches. Where the stilted always rise straight, the rims are already very far removed from the wall (Fig. 266a). A particularly high stilting is shown by the side arches in the Gothic choir of the cathedral at Aix-la-Chapelle (about 1400).

Laying out the cut stone.

As for what now concerns the practical execution of this springing of the ribs, this is done in stone, so that it consists of one or more courses of cut stones laid on each other with horizontal beds. On the upper surface of the springing the separate arches may either be cut with either horizontal or radial beds. The radial position is then to be preferred for other reasons, if the horizontal beds would intersect the different arches at acute angles, as allowed by the nature of the stone. It is of the greatest importance, that the laying out of this springing is done with the greatest accuracy, so that the pieces of stone to be set thereon later shall make no bend horizontally or vertically with the members wrought on the springing. (Note). Hence the construction of these cut blocks is illustrated for the different courses in the following.

Note. In the numerous old works a careful eye can see a bend over the springing, that partly is to be referred to inaccurate execution, and also indeed partly to later crushing.

Laying out the cut stones. Example 1.

The springing of the vault consists of a cross and two diagonal arches, the widths of the bays and the radii of the different arches are equal, their centres lying in the same horizontal plane. Figs. 269, 269a.

The middle lines of the three ribs intersect at the point b, the underside of the strongly projecting members resting on t the capital or corbel as drawn. The springing must consist of one block that extends up to the separation of the ribs. It is then to lay off the height and the upper surface of the stone.

On the middle line A B as base is revolved into the plan the side elevation of the cross rib, whose inner arch a x and back is b y. The point where the ribs separate is d in the plan, and at it is erected a vertical to cut the back of the arch in e. This point e is the point of separation of the ribs in elevation.

a horizontal through e gives the height of the block, which is thus already found.

The upper bed may be entirely a horizontal surface g e h or it may be radial for the depth of the ribs e f. The beds are here assumed to be permissible as horizontal. There is now to be projected on the plan the outline of the upper bed. This is simply done by dropping verticals from the profile and determining the points m″n″ etc. on the plan, where by drawing horizontal lines through the points m n the desired points m′n′h′ are obtained. For more accurate drawing of the profile, other points may be found in the same way.

The top bed of the rib thus appears as the elongated profile d h′n′m′. For the diagonal ribs is employed the same procedure, and for example the elevation of the lower diagonal rib is revolved likewise about the line b r, etc. But it is not necessary in this case, since with the present assumption of horizontal beds of both ribs they are exactly the same as that of the cross arch, thus they are found by simply transferring.

Thus the top surface is thereby found for the three arches. Yet it appears that at this height the arches are already far removed from the wall, and therefore it is now to fill the spaces between the points s or t of the wall. That depends in the form of the side arch. In this example is no projecting section of the side arch, but only one line of the side arch is struck with the same radius as the three ribs, then while the angles of the ribs are moved back from p to s, and the side arch would be moved back from p to w. Consequently the line s w is to be drawn, which lies in the surface of the compartment. The masonry of the compartment will later be set on this line. The spandrel of the compartment below s w will also be wrought in the stone block, and is drawn down to the point p.

If the side arch is struck with a greater radius, or is somewhat stilted, then it is not set forward so far at the top of the block, at perhaps first passes from w to v (lower half of the Fig. In this case the line t u limits the cut block. But the side arch may also be stilted for the entire height of the springing. The side arch then rises at the point u, so that t u is the outline of the cut block. The springing is then made v vertical below u t, being warped above u t as shown in Fig. 267. A more accurate conclusion concerning the relation of the

side arch is thereby obtained, as it is laid down on the line n p in the plan.

On the cut block is wrought a projection anchored in the wall and as wide as the stone employed permits.

Laying out cut stones. Example 2.

2. The springing of the vault consists of a cross rib, two diagonal ribs and two side arch ribs, the width of the adjacent bays being so different that the points of separation of the ribs lie at different heights. Since further both the crowns of the vaults lie at the same height, then must the radius of the narrow bay be greater. On the contrary the crown of the s side arch must lie somewhat lower than that of the diagonal arches, so that those of the wider bay can be struck with the radius of the latter through a point in the common base line. The side arches of the smaller bay required the same height, struck with their span as radius, and are stilted by the difference of the resulting rise and that of the greater side arch. Fig. 270 sows in the hatched portion of the plan the springing of the rib resting on a hexagonal capital.

There is drawn above the middle line of the little cross rib its side elevation and revolved about a b as base and struck with the rib a c etc.. In plan the back of the rib separates at the point e from the adjacent rib, therefore at e is drawn a perpendicular to a b that in elevation cuts the back of the arch at f. Hence g f indicates the greatest height of the spring of the rib. According as the upper joint is horizontal or radial, it will be indicated by the line f h or f h′. By projection downwadr there results for the horizontal joint the profile e k i, or for the radial joint e k w as the projection on the plan.

The side elevation of the cross arch is likewise revolved a about the line l m as a base. On the back is assumed a point that has the same height above the base as the point f from the base a b. The radial joint at this height would as before be projected on the plan, showing the profile p r x, and a horizontal upper joint as in the former case would give the longer profile p r g. Since in this case the cross arch is struck with a smaller radius than the arch, its profile at this height is projected by r e farther than the arch, and it results here that the frequently occurring spandrel explained in Fig. 266

has a warped compartment spandrel above.

In the same manner will also be found the plan profile for the other cross rib s t at the same height f g, whose back is at the point u, so that the line p u here forms the back side.

When as here the joint is made radial to avoid too acute an angle, then the stonecutter do without the accurate construction of the plan projection e k w or r p x. He needs only to project the back edges e k and p r and to transfer them to his cut block; he works from this edge a radial bed and draws on this the actual cross section of the rig.

Since at the beginning the height of the springing is fixed by the length f g, now its length and height are determined by a rectangle y z a′b′ circumscribed about the upper surface. The projection of the cut block into the wall is naturally made by a projection of the line y b′ as large as the stone permits.

If it be desired also to cut the quickly separating side rib in the same block for its entire height, the width of the stone would be increased unnecessarily. Therefore it is cut as shown in Fig. 270 by the profile being radial at the edge of the stone, on it being made a cut in which is placed the back of the first stone of the side arch. Laying out is also similar here. The elevation of the side arch is revolved about d′e′ as base, and according to the width of the stone is erected a vertical to cut the inner side of the side arch at g. From this is drawn the radial joint g′h′ and the cut for the back runs in the arch line h′i′ to the top of the block.

The side arch of the little bay is stilted by the height of the springing, and therefore shows the same plan in the upper bed of the springing of the rib as in the lower one, so that also the face of the compartment is here formed by the continuation of the radial side surface of the profile of the diagonal arch a b. Fig. 270a exhibits a view of the springing of the rib of the smaller bay. In both views (Figs. 2707, b) the points corresponding to the plan are indicated by the same letters. The side arch of the smaller bay remains, as shown by F the Figs., entirely joined to the springing of the rib and first separates above its upper arch, so that on the surface C in Fig. 270a is set the cut stone represented in Fig. 270c, and on the radial surface D of the latter are placed the succeeding pieces of the side arch, just as those of the upper ribs

This little cut stone is set on the springing of the vault.

are set on the radial surfaces p x r .od e w k in Fig. 270a.

The construction of the cut stone drawn in Fig. 270c is the following. One draws above the line o′d′ as base the elevation of the side arch beginning abvoe the stilting. At the point r, where the side arch projects from the surface of the compartment is erected a vertical to cut the back at s′, and then o′s′ is the height of the block and s′t′ is the radial joint. In the perspective of Fig. 270c this joint is shown as the surface u″ w″. The cut stone must bond at both sides and therefore n needs corresponding projections, as a piece u″v″ at least equal to the thickness of the compartment, by which the cut stone is held in the nucleus, and then the piece w″x by which it extends into the wall, and that naturally has the length by which the side arch bonds in the wall.

This little cut stone is set on the springing of the vault, while at the other places the ribs are directly carried up. The interval between the rib and that block is filled by the masonry of the compartment with its backing extending to 1/3 or 1/2 the height of the vault. Where greater cost is not feared, it is particularly preferable for vaults of wide span to place the lower piece of the compartment as a cut stone in Fig. 270c into a great stone represented in Fig. 270d. The three arch surfaces E F C are set behind the backs of the ribs, while the surfaces H I K L are pieces of the compartment. The surface H shows by the hatching the warped form of the springing of the compartment.

If the height of the springing of the vault is very great, then it will be divided in courses, in this example into two, as illustrated in Fig. 270a and b. The form of the joint surface is easily laid off as before, it is sketched on the corresponding bed surfaces of the cut stone, that are then cut according to the requirements of the curvature of the arches. Thus is completed the construction of the springing of the ribs.

Restriction of the basal area.

Fig. 270 has shown that the construction of the side arch w with the body of the springing of the ribs in a single cut block in some cases brings with it certain difficulties, that c can be avoided in different ways. First then by a separation of the side arch from the other ribs.

Separate springing for the side arch.

This separation results of itself when for each rib is arranged a separate shaft. Fig. 271 shows the plan of such an arrangement from the choir of the church at Wetter. The shafts here stand so far apart, that the cross rib and tte diagonal rib s spring only in the straight sides of their profile, but the s side arch lies entirely free. But aside from the easier execution also the nature of the thing is thereby more perfectly i indicated; for the function of the springings of the arches only consists in connecting the separating arches at their beginning together and with the wall. But the side arch moves in toward the wall, is directly joined to it, and therefore requires no further connection with it. Thus in all cases a correct conception and easier execution occur together, and there exists a reason against every conception and every suspicios arrangement, whose execution would be excessively difficult, and only possible by concealed means. In the present case the shafts are connected with hollows and bear capitals of hexagonal plan according to the direction of the ribs. These capitals project strongly the shaft of the side arch a in Fig. 271. up to the higher base of the side arch, ending there with a round capital. Thereby is also avoided the stilting of the arch. On the hexagonal capitals b and c of the shafts in Fig. 271 then rest the cross and diagonal ribs, whose hollows adjoin, and thus the regularity of the entire design appears more clearly. But on the shaft caliptal d stands the side arch, whose profile corresponds to the plan of the shaft; hence the capital d is also round and merely serves to indicate the springing of the arch.

In more simply executed works and in smaller dimensions is sometimes found the separation of the side arch from the other ribs placed on only one shaft and effected by very thoughtful arrangements. Such is shown by the springing of the ribs of t the choir of the church at Immenhausen dating from the beginning of the 15 th century, Figs. 272, 272a, where the stilted side arch rib is set back below, thus leavinm free the springing consisting of a cross rib and two diagonal ribs. After the side arch has been thus stopped, there yet remains the rectangular body f g h before which the springings of the other ribs are placed, and that with the same rest on the octagonal capital. In like manner the side arches in the church of S. Maria

at Heiligenstadt terminate below like corbels. Such simple means contribute much to the high charm of the simplest old works, even of the later time, and distinguish them by their freshness and thoughtful invention from most modern ones.

Shortened springings. Corbellings.

Strictly taken all such shortened springings, so far as they are cut by horizontal joints, only form corbellings from the surface on which rest the ribs, and therefore may be replaced by corbels or even their work can be transferred. For static reasons strongly shortened springings require no support, since the thrust of the vault is transferred to the wall far above.

Fig. 273 shows the plan and Fig. 273. is the elevation of the springing of a rib consisting of two cut stones in height, and the hatched part of the plan gives the horizontal profile at the height of the joint a b. Hence the corbel indicated in the figure could support the upper stone, thereby raising the base of the vault the distance a c, which will be a real advantage in low rooms.

But by the arrangement the original line of the pure semicircle or pointed arch is changed into a segment of this form of arch, and thus the esthetic effect of it, the change from the vertical to the curve is destroyed. On the other hand by a complete development of the arch lines the loss of height caused is then only a disadvantage when it interferes with the placing of furniture against the wall surface concerned. Where no care is to be taken in this respect, there without injuring the effect of the whole the springing of the arches may rest on the floor. Thus it is found in the sacristy of the church at Wetter only two feet above the floor. But the wide span of the arches has the effect that at the height of a man they still project too little from the face of the wall to injure the convenience of the room. Figs. 274 and 275 show the contrast of the two arrangements.

For smalles pases a corbel is no longer necessary for structural purposes. But a simple horizontal ending is not satisfactory, since it does not show the transfer of the force to the wall, and it is better replaced by the arrangement shown in F Figs. 276 and 277. This form is recommended by its simple and cheap execution, and is generally found in old works, particularly in unimportant rooms. But it may also with advantage for

a single rib be substituted for the springing of the rib, especially if the limited plan of the capital offers no bearing f for the rib in question, so that all start with the others and must be joined to them for a considerable height. This height will be substantially reduced by an arrangement shown in Fig. 278. On some piers of the church of S. Elisabeth at Marburg is found an allied arrangement, so far as the extreme members of the ribs are corbelled instead of resting on the capital.

If it is desired to avoid by such means a too strong projection of the members, then men have not hesitated to compress together the mouldings at the height of the support, that only the bottom fillets of the profile remain free. In the vaults in the cloister at Aix-la-Chapelle these fillets directly form the wall pillar without a capital. Fig. 279.

Springing at a point.

At the springing of the ribs from the face of the wall or an angle, the plan of the ribs may be conversely so far compressed at their lower bed, that their middle line springs from a point lying in the surface of the wall, so that now the ribs spring from the wall surface and may now expand from their meeting at a point, as Fig. 280 shows in elevation and Fig. 280a in plan at a larger scale. In the last Fig. the plans of the separate ribs are given in the position behind the face of the wall, t that they would have in a free development from the point a. It need not be noted that this plan does not really exist, but is first developed as the ribs separate, as the horizontal section indicates at the different heights d and e in Fig. 280. The construction of the elevation from the plan is the same as shown by Fig. 281. The entire treatment represents in a certain sense an exaggerated result of the principle of all cogpressed springings of ribs. But their appearance in comparison to that of one resting on a corbel or capital, a springing of a rib in correct proportion to the size of the room, is dry and wearisome, since it even removes all body from the springing.

Therefore another arrangement, likewise belonging to the late Gothic, deserves the preference, by which each of the three r ribs springs by itself from the wall, so that the points in w which they spring lie beside each other as shown by Fig. 281 in plan and Fig. 281a in elevation.

The construction of this form is as follows:- all ribs are

struck with the same radius and their starts lie at the same height. First is then above a b as base is struck the arch a c etc. through the different angles of the cross rib, and the p projections of these angles are drawn as d e and f g, and then will each of these angles be visible in the elevation, where their arches intersect the face a k of the wall, thus for example the angle d at the height a i, the angle f at the height a k, etc., from which is the outline a d f l in Fig. 281a, in which the cross rib intersects the face of the wall is already determined. Now to find the same line for diagonal ribs their different arches are then struck above the line m n as base, the projection lines of the corresponding angles are drawn, at the points where the latter intersect the face of the wall, m making vertical sketches on m n for the corresponding arches, there results the length p r as the height at which the point u comes from the face of the wall, the length s t is the height at which the point v comes from it, etc.

If we now combine the last plan of the separate ribs coming from the face of the wall with that of the corbel there results the peculiar shape shown in Figs. 282 and 282a, an arrangement occurring in the south side of the cloister of the cathedral of Erfurt, where from each rib is arranged a separate corbel set in the direction of the rib. This form makes possible a broader design of the springing od the ribs and therefore it requires a smaller height for them. Such a wider springing of the ribs may indeed be placed on a common corbel, as for example occurs in the southern aisle of the church S. Blasien in Mühlhausen, yet it is not to be mistaken in the places mentioned, that the flat shape of this corbel has no favorable effect, and is inferior to that of Erfurt.

Usually the design of a combined corbel requires an animated projection of at least half the basal form, according to which it is shaped, but five sides of the octagon are better, four or five of the hexagon, two of the triangle, etc.

Crossing of springings.

We have even called silly the design of the ribs from a point as shown in Fig 280, yet in many works of the late Gothic, men sought to go beyond this, taking this in a literal sense. For with a fixed position of the cross rib, the springings of the diagonal ribs of a bay were extended over the cross rib into

adjacent bays, so that the springing of the diagonal rib on t the right side projected to the left, and conversely that of the left to the right, so that these diagonal arches crossed the cross rib near its springing. Very beautiful springings of ribs of this kind are found in the present Catholic church in Marburg, as well as in a side room of the church S. Maria there. The first is represented in plan in Fig. 283, in front elevation in Fig. 283b, and in diagonal elevation in Fig. 283a. Therein a is the projecting shaft, b b are the diagonal ribs that cross at c, d is the cross rib, that intersects the former ate, f f the side arch ribs which intersect the diagonal ribs at g, while attached to the wall. Here indeed is scarcely longer the assertion of any development of the entire form from the construction, and an utilization of the materials, of a demonstration of the structural principle, or in brief of logic, it is first a triumph of manual skill conscious of its certainty. Y Yet we cannot refrain from wondering at the precision with wbich these later forms are executed. It has the same effect as when one forgets the point of view of handicraft in viewing t the artistic perfection of manual skill. And in nowise may many modern artists justly think themselves superior to the point of view of those workmen, whose works have such an extremely wise consideration nf the effect of light and of the course of the lines, that they still possess the advantage of actually refreshing the eyes, more than many permeated by the pure art of the modern period. We may finally regret these works of the late Gothic.

Springings of ribs over isolated piers.

In like manner to the springings of ribs from the surfaces of walls previously mentioned are found those over detached p piers, excepting that naturally the connection of the cut stones with the wall is naturally omitted. An entirely regular springing of four ribs and four diagonal ribs is shown in Fig. 284. With larger dimensions would this consist of several cut stones laid on each other instead of a single blocg. The requirements before given for the regular springing of the members have been valid here as well as the rules for laying out the cut stones.

When from a smaller area of the abacus a too great compression of the members is to be avoided, then may be arranged

corbellings according to the method practised in the before m mentioned Fig. 278.

Expansion of arches above free piers.

Yet in most cases men have not feared the expansion of the s separate arches. A beautiful example early Gothic example from the beginning of the 13 th century is afforded by the springing of the ribs above the columns of the side aisle of the choir od the Cistercian church at Walkenried, whose lower cut block is represented in Fig. 285. In spite of the different cross s sections of the cross arch and the ribs, already given in Figs. 195, 196, there is produced a salutary springing of the members. Especially skilfully cut are the nail heads on the ribs, whose sides a below join the solid surface b.

Greater difficulties arise from hre aisle piers and bases, as shown by Figs. 286 from Notre Dame in Dijon. As seen in Fig. 286a, here the upper part of the dividing arch (designated on the plan by a b c) cuts through the vertical side surface of the diagonal archmarked 1. The line followed by the intersection will be obtained as in Fig. 281 in the following way.

There is drawn in the side Fig. 286b the side elevation of the part a b c of the dividing arch, and there is laid off for each point on the plan as for example g the obliquely measured distance g h as g′h′ in the side figure. The vertical h′ gives the point i′where the angle point g leaves the side surface of the rib. From the side Fig. the intersection i′ is easily transferred to the other elevations. Likewise are yet other points of intersection obtained. The upper bed of the springing of t the arch is drawn in the plan as the outline i k l m n o.

Strengthening the beginning by carrying down the compartments.

In later works the compression of the members is mostly carried still further. Where it comes to giving the pier and hence the springings of the ribs the smallest dimensions, there results at the springing a notable reduction of strength by the recession of the profile. This is avoided by filling the intervals, which is produced by extending down the surfaces of the compartments. See Figs. 287, 287a. But these parts of the compartments must be steeper in their lower parts, so that a break occurs at the height m,where the profiles of the arches become free. Above this place the compartments always lie concentrically on the backs of the arches. The arrangement represented

in the Fig. is found in the vaults of several rooms of the monastery of Haina from about the end of the 13 th century. Only the cross arch of these vaults is moulded as a rib, while the diagonal arches are merely groins.

The break in the surface of the compartment and here also in the diagonal groin would be at the height m m in Fig. 287a. Yet the diagonal arches might be formed with a true arch line, only the joining line of the compartment at the side of the cross arch forming that break. A warped surface would then form the transition. Fig. 287b.

The block obtained by extending down the compartments for t the springing is recognized in plan in Fig. 287. The area lying on the capital is limited by the outline k g l i, while o otherwise the line f g h i would be followed. Naturally also here the lower portion of the springing would be made of a single cut block.

The same arrangement would also be possible, if the diagonal arch also were formed of moulded bricks, then being cut in the same way from the mass of the springing of the ribs as in Fig. 287a.

Springing composed of vertical surfaces of the nucleus.

Besides that structural advantage, the entire arrangement has the esthetic one, that the proper function of the springing of the rib, the union of the different separate ribs in o one cut stone, i.e., of the various thrusts downward to the single piece produces a concealed appearance. It is capable of the most different variations, according to the inclination of the lower extension of the compartments. These may have a moderate connection with the greater radius, and finally may become a vertical surface. Such vertical surfaces of the nucleus as are shown by Figs. 288, 289, are found from the middle of the 14 th century down very frequently on detached piers, as well as on shafts and corbelled springings of vaults. Their origin is referred to the endeavor to beautify the mass of the springing as much as possible. The plan of the springing of the ribs is more or less accurately inscribed in some regular ground form, like the octagon in Fig. 288 as the circle in Fig. 289. In cutting the bottom surface of the stone must first be given this shape in outline. Next with regard to greater s strength the sinking of the hollows between the ribs was omitted,

Intersection of members of varied and of plier.

rather carrying the round or polygonal nucleus vertically upward, so that a penetration of this cylinder or prism into the various lines of the arches produced animated profiles of the ribs. The same result was attained in another manner. Let there be placed the hatched portion in Fig. 289, of the springing of the ribs on a round capital or corbel; but from the edge of the capital a wash extends inward between the intersecting springings of the ribs. As the wash becomes steeper must the form become similar to that shown in Fig. 289a and entirely p pass into that as soon as the inclination of the wash becomes 90°.

The construction of the lines by which this penetration occurs is already given in Fig. 281. The entire arrangement is capable of quite varied forms according to the proportion of the sections of the ribs to the nucleus. Thus can be avoided by a greater mass of the nucleus that intersection of the adjacent ribs, for example as it occurs in Fig. 289, while conversely it appears in a greater degree with a smaller nucleus. Thus further the ribs spring from the angles of the body instead of the sides as in Fig. 288, and they may meet the nucleus vertically or obliquely (the latter is assumed for the ribs a in Fig. 289). Finally the front edge of each rib may lie in the perimeter of the nucleus or even project from this, so that the rib concerned is cut from the nucleus above the base with a s slight break. But in this also lies a means for first giving the compartment an intended direction.

For example if the compartment in Fig. 289 intersects by its p perimeter the two ribs a and d at the same angle, is indicated by the horizontal section e f, then would one or the other rib be sunk in the nucleus so far as to attain this condition.

Intersection of members of vault and of pier.

The entire system of intersection made possible by these forms is already found in the works of the early Gothic, ewen if in the form shown at first belongs to the middle period. Thus occur intersections of members of arches by the surfaces of buttresses from which they spring, further those of gable mouldings with each other already on the oldest works. But it appears that men yet laid no weight on the lines in which the penetration occurred, but allowed them to shape themselves, while in the later period they first paid attention to their charming effect, and toiled to seek, heighten and finally carry them to

excess. Caught in this excess, they lost the real structural advantages of the forms given in Figs. 287, 288, 289.

The first began to form the polygonal nucleus with concave sides, also furnished indeed with sunk panels like tracery, so that the ribs came as through a window. Or the polygon was replaced by a member harmonizing with the springing of the rib and set diagonally with it as shown in plan in Fig. 290. Instead of the regular diagonal position one was also satisfied, for the projecting parts of the ribs should come from the recessed parts of the nucleus, and conversely, so that the rounds of the ribs formed an intersection with the hollows of the nucleus and those of the nucleus with the hollows of the ribs. Such an example is shown by Fig. 291, two cross ribs and two diagonal ribs. The upper hatched portions from a to b show the plan of the pier, the part from c to h as that of a cross and a diagonal rib, the part e to f being a division rib. Fig. 291a represents the front elevation of Fig. 291 is the side elevation of this capricious treatment. The development of the elevation from the plan is substantially by the procedure shown in Fig. 281.

Intersections of this kind, particularly of the members of ribs and arches with vertical members are found in the works of late Gothic chiefly in the form, that the latter member in its extension downward forms the pier and then stops on a base. A very rich example is afforded by the church S. Columba in C Cologne. On the contrary simply corbelled springings of ribs like Fig. 289 are not well shaped in this way, since the complicated members of the nucleus must have a certain length to become intelligible.

Those with the forms of Figs. 287 to 289 are at first made possible by utilizing the mass of the cut stone, so this principle leads in many early Gothic works to more ornamental works, charming in the highest degree. For example to execute the springing of the ribs (Fig. 288a) with the given joints f′ and f″, there is required the block a b f′f″, from which the part c f′f″must be cut off. But this mass invites in form the shaping of an ornamental treatment for it, and thus the original form of the block is later allowed to appear, as well as the structure of the whole. Very beautiful examples of this kind are shown by the springings of the ribs from the choir of the

Natural stone.

The compartments are constructed rather of natural or local stone (brick), and the latter very economically according to the local geological products of the region. For the more expensive stones are avoided as much as possible, especially are employed the different grades of limestone and sandstone. As exceptionally well suited for vaults is formed by the light brick. The conversion of light, the cross that the brick, and strictly distinguish calcareous, in brick, under brownish and poor brick. Good stone for vaulting was often carried to great distances in the middle ages. Because the local models, this has the peculiarity calcareous composition. From a its close surface coarse wall, and that be very porous stone hence the stone must be

collegiate church in Wetter, on which above the capital of the shaft projected the symbols of the evangelists in the manner given in Fig. 292, from the members of the ribs. A corresponding treatment may be very well given in connection with Fig. 288a, as for example is shown in Fig. 293. Instead of the separate leaves given here might occur continuous foliage, by which the effect would be still richer. An extremely beautiful form of this kind is shown by the piers of the choir aisle of the cathedral of Auxerre before the chapel of the Virgin.(See the illustration in Viollet-le-Duc, IV., p. 149).

Here is to be mentioned the wonderful series of canopies of figures that extend around the piers of the middle aisle of t the cathedral of Milan, but they bear less the character of the springing of a vault than of an inserted independent intermediate member.

9. Masonry of the Compartments.

Material.

Natural stone.

The compartments are constructed either of natural or artificial stone (brick), and the former vary substantially according to the local geological products of the region, yet the h hard and massive stones are avoided as much as possible, occasionally are employed the different slates, and especially often limestone and sandstone. An exceptionally valued material for vaults is formed by the light tufas, the travertine of Italy, the trass from the Rhine, the widely distributed calcareous tufa, that occurs among other places near Göttingen, Mühlhausen, in France, upper Bavaria and near Paris. Good stone for vaulting was often carried to great distances in the middle ages. Besides its light weight, tufa has the peculiarity deserving consideration, that to its rough surface mortar adheres well, and that the very porous stone keeps the rooms warm and dry.

Artificial stone.(Brick).

Now the prevailing material for vaults is brick, that already in the middle ages cheated its use for vaulting purposes far outside its limited home. It is light and porous, and has the advantage of uniformity of shape well adapted to vaulting purposes, it favors freehand masonry and permits a small thickness of vaults for great spaces. The usual thickness of a half brick or 15 cm can be employed for unloaded vaults up to 10 m or more

in span, it being assumed that compartments and ribs are correctly shaped. For natural stone the thickness of the compartments is mostly not under 20 cm, and only for especially suitable material does one go below 9 to 15 cm. An important quality of good stone for vaulting is light weight, and for this reason has been obtained a porous brick with good results, by mixing with clay a large quantity of sawdust or of similar combustible, which after burning leaves corresponding cavities. In this way is it possible to reduce the weight one half, without lessening the strength in a notable way. For the ribs and if necessary also for the spandrels of the compartments are employed other hard burned bricks. Hollow bricks are used with a certain foresight with ever increasing favor, and in all cases one should avoid the ever feared crushing of the mortar by allowing the direction of the holes to coincide with the principal direction of the pressure. An excellent material for vaulting with moderate stresses is the light and porous pumice stone found near Andernach on the Rhine and easily prepared in dimensions of 25 × 12 and 10 cm, but it cannot be used for ribs.

Mortar.

The prevailing binding material is a good and stiff lime mortar; cement in all cases must not set too quick and is less indicated for the compartments, but it can very well give good service in strongly pressed springings of vaults. With regard to the different setting of the two kinds of mortar, it should be avoided to carry the cement mortar too high, while spreading it sidewise favors the transmission of compression to large horizontal areas. Elsewhere for strongly pressed parts lead sheets find good use in joints of cut stone ribs, as well as other materials.

Method of execution.

If exceptional forms like pot vaults and the like are omitted, then three different methods of execution are to be distinguished.

1. Concrete vaults on centering.
2. Coursed masonry on centering.
3. Coursed masonry without centering, i.e., freehand masonry.

Although all three methods are sometimes employed beside each other, there appears in general a transition from the first to the second and from this again to the third.

Concrete work.

Concrete made of stone spalls and mortar was connected with Roman traditions, but lost its importance for vaults when men earnestly proceeded at any cost to reduce the thickness of vaults. For the interior of thick vaults concrete longer retained its ancient favor.

Coursed vaults on centering.

Masonry constructed in courses on a firm centering makes known the greatest change in care and perfection of execution, and three steps can be discerned, the first being an irregular masonry that was filled with mortar. The stones were of rough shape and laid more or less in courses on the centering, either in a full bed of mortar or even dry with a later grouting of mortar on top. A more perfect step is the regular rubble vault, where stones with surfaces more or less uniform and similar are set in courses with similar mortar joints. As the highest stage is to be regarded the vaults of cut stone with sharp edges and regular joints. The more imperfect the procedure, the more the strength depends on the goodness of the mortar, but on the other hand, the more developed the execution the more can the thickness of the vault be reduced. For the last reason here again in general is also recognized an increase in the quality nf t the technics' in the Romanesque period we find rude rubble vaults of great thickness, in the Gothic period better constructed and also thinner compartments. Thereby it is not stated t that there were not also Romanesque and early Christian times that were not rich in especially well executed vaults of cut stone, and as examples are mentioned only the domes of churches in western France at Perigueux, etc.. Fig. 294 gives a part of the latter near the crown and very finely allows to appear the hook-shaped bonding of certain stones there employed.

Freehand vaults.

As the highest expression of the perfected technics, even if very ancient (see p. 4), appears freehand vaulting, but which is joined with a suitable material, either brick or small and easily wrought stones, whether limestone, sandstone or tufa. Therefore they are restricted to brick countries and some vicinities with suitable cut stone, and among the latter is to be placed first Isle de France, where the thickness of vaults is generally only 10 to 15 cm, according to Viollet-le-Duc. For freehand vaults the position chosen for the separate courses

"Use of compartments and transverse or vaults."

It was stated above (p. [illegible]) that the development of vault pressure in the compartment, being flat acceptance, on the general form of the compartment and less on the [illegible] form of vaulting with those methods [illegible] tion, the results thereby obtained satisfactorily [illegible] can cause even to concrete vaults of greater [illegible]

The very general assumption that concrete vaults are [illegible] desired form. One fact to invest in spaces of the [illegible] [illegible], it [illegible] unity able to receive strong lateral pressure, but only in [illegible] case can occur a certain expulsion removal from the most [illegible] able line of pressure. But the greater the difference, greater also become the tensile stresses, and the greater must also be the resisting cross section, i.e., considerable deviation from the line of pressure requires greater thickness of the vault. A concrete vault will always exhibit the least of material when it follows the line of support. Further [illegible] by accidental stresses affecting the resistance to tension. by changes in temperature, settlements are not in [illegible].

Very conceivable is the assumption that concrete vaults [illegible] but however in good execution may serve as a model

are stated later in a full description.

In brick vaults almost generally a thickness of 1/2 brick or 12 cm is assumed, and this is considered as sufficient for unloaded vaults up to 10 m span, while for vaults of 10 to 14 m a thickness of the compartments of 3/4 brick made possible by special bricks is regarded as proper. On the other hand small and strongly swelled unloaded compartments may be made still thinner with a thickness of 10 cm or even of 1/4 brick. It is assumed that compartments and ribs are of statically correct form, and that the compartments are merely filling surfaces, that the ribs are the actural bearers of forces, and nothing opposes closing close meshed rib vaults of proper width by compartments of 1/4 brick in thickness. A limit of span is indicated by the stability of the ribs and not that of the compartments.

Form of compartments and pressure of vaults.

It was stated above (p. 47) that the transmission of the vault pressure in the compartment, aside from accidents, depends on the general form of the compartment and less on the courses of the compartment. Therefore a general consideration of the proper form of vaults will be made without regard to the execution, the results thereby obtained being applicable in a certain sense even to concrete vaults with mortar resisting tension.

Vaults with tensile stresses.

The very general assumption that concrete vaults can take any desired form, and that no thrust is exerted on the abutments by them, is only conditionally correct. When the concrete is fully able to receive strong tensile stresses, but only in this case can occur a certain capricious removal from the most favorable line of pressure. But the greater the difference, the greater also become the tensile stresses, and the greater must also be the resisting cross section, i.e., considerable deviations from the line of pressure require greater thickness of the vault. A concrete vault will always exhibit the least use of material when it follows the line of support. Further then by accidental stresses affecting the resistance to tension, as by changes in temperature, settlements are not in danger.

Very conceivable is the assumption that concrete vaults cause no thrust. Naturally they form a straight or vault-like slab, but however in good execution may serve as a substitute for it.

curved slab) are not fixed at their ends, produce no end thrusts and are stressed by flexure (compression and tension). It has fixed ends, produces thrusts, and therefore is not stressed by flexure only by compression, and can be made correspondingly.

If the curved line of sufficient thickness be made like a piece bowl and afterwards be carefully set on the abutments, while the mortar is soft for the entire perimeter, in spite of the contracting it for any reason easily appearing cracks [illegible] the vaults. [illegible] than the existing thrusts will be quite particularly large: not without reason the practical resistance supported the heavy vaults by very powerful abutments. Ion suoula be accustomed against all too thrustful use of wide and flat concrete vaults with only comparatively slender.

Like the commonly circular structures or all arcade and circuit. It is in vain to count on strict precision. And are well not certain hypothesis accurate, if [illegible] really not certain to be changed by reason. The latter [illegible] of vaults will case to notice ware, and then the [illegible] will be sufficient, even the corresponding tonnes small have a safe position in the relation of the compartment. Like common small the countersigned ensure the examinable account, one [illegible] to be the ordinary plate by the place of concrete [illegible] to be tested.

[illegible] favorably form of construction according to the c[illegible] of [illegible] is fully execute on p. 72 of say. one [illegible] to the ordinary means may be used further. [illegible] case there [illegible] according to values o conc[illegible] with their counter form manner for below the Allenbia I[illegible] on the latter may be answer the ordinary well tuscu [illegible] in time worse th ecout 7 wat not o oe, 3 to 5 fill for [illegible]. and s to 3 ind for tension system cuone. best [illegible] of cluscus in count corter can to accorded it or eyes I

But such slabs are then to be calculated for flexure like beams, whereby results a correspondingly greater thickness, especially for a heavier loading.

Beams and vaults are quite too much confused. Beams (also the curved slab) are not fixed at their ends, produce no end thrust and are stressed by flexure (compression and tension). The vault has fixed ends, produces thrusts, and therefore is not stressed by flexure but by compression, and can be made considerably thinner.

If the curved slab of sufficient thickness be made like a great bowl and afterwards be carefully set on the abutments, no thrust is to be expected, but otherwise it occurs at once while the mortar is soft for the entire perimeter, in spite of the centering if for any reason easily appearing cracks divide the vault. Since concrete work is usually very massive, then the existing thrusts will be quite particularly large; not without reason the practical Renaissance supported the heavy vaults by very powerful abutments. Men should be warned against all too trustful use of wide and flat concrete ceilings.

Vaults with only compressile stresses.

With the scarcely elastic properties of all stone and mortar materials, it is always risky to count on their unbroken resistance, and one will not assert mysterious accidents, if he entirely forbids them to be stressed by tension. The latter point of view will also be retained here, and then the requirement will be established, that the compressile forces shall have a safe position in the interior of the compartment, that nowhere shall the compression exceed the permissible amount, and that no slipping of the different parts by the effect of compression is to be feared.

The most favorable form of compartment according to the curves of pressure is fully treated on p. 52 et seq., and in regard to the compressile stress may it be added further, that in most cases the compression occurring in unloaded compartments with their correct form remains far below the allowable limit. As the latter may be assumed for ordinary well burned bricks in lime mortar at about 7 kil per q cm, 3 to 5 kil for porous bricks, and 2 to 3 kil for Rhenish pumice stone. Hard bricks or clinkers in cement mortar can be assigned 11 or even 14 kil per q cm, to natural stones according to their hardness and

ire scarce confined from 7 to 20 kil and more.

Slipping of earth on each other.

the mortar employed from 7 to 20 kil and more.

Slipping of parts on each other.

The slipping of the stones requires further explanation. When a stone exerts on its support an oblique pressure D (Fig. 295), then under some circumstances it will slip, and the easies the more obliquely the pressure acts, or in other words the greater the angle a between the direction of the perpendicular N to t the lower surface. The angle a is termed the angle of repose, and it varies much according to the nature of the surfaces of the bodies in contact. While two polished stones may perhaps slip already with an angle of 10°, the pressure of two rough stones may require an angle of 60° to 80° from the perpendicular before a slip results. For vaults a slip of one stone on another rarely comes in question, it rather concerns here the friction between mortar and stone, or just as commonly the slip of parts of the mortar on each other. Besides the rough surfaces of the stones it depends particularly on the nature of the mortar, whose resistance by friction varies between the widest limits according to the nature of its components, its mixing and quality. After settling and for moderately good mortar and moderately rough stone surfaces, the angle of repose seldom l lies under 60° or 70°, but on the contrary soft mortar shows the widest variations, when very fluid and movable, so that a slip may occur at less than 20° of inclination, and on the other hand it is possible that good stiff lime mortar makes it p possible to set a brick against a vertical wall.

For compartments vaulted freehand, whose construction is connected with the use of stiff mortar, one may usually count on an angle of repose of at least 45°. On the other hand cases h have occurred, when not yet closed compartment vaults have been caused to fall by pouring thin cement grout on the masonry. By the softening of the mortar joints the angle of repose became a minimum, and a statically unfavorable form of vault must have existed at the same time.

Location of the joints with reference to slipping.

The danger of slipping requires a consideration of the direction od the joints in section and in plan. Let Fig. 296 represent the section through a compartment or any single arch with the line of pressure drawn in, then must the angle a at the impost be not greater than the permissible angle (soft mortar be-

being assumed). When this case occurs, then the bed concerned must have a radial direction in the fnont part as indicated by the dotted line p q. Further the beds a b, c d etc., must not have too small an angle with the line of pressure. This possibility is scarcely to be feared with a radial portion of the beds of the vaults concerning us, and according to the measure of the angle of repose would even such beds be allowable, that converge somewhat at the top like m n or o r. Naturally will such irregularities be avoided, since the stones concerned might fall by accident without stress and without adhesion of mortar.

The same consideration is demanded by the friction of the beds of the separate courses of the vault appearing in plan, the short end joints coming in question less. The danger of slipping exists least of the joints are perpendicular to the direction of the pressure, and their position always remains ensured, if from this most favorable direction they vary less than the angle of repose.

Consequently the pressure in the vault is not affected, so long as the direction of the courses differs from a perpendicular to the direction of pressure less than the angle of repose. (In other words, so long as the angle between the direction of the pressure and the courses is not less than 90° - the angle of repose). Within these limits it is entirely indifferent how the courses may run.(see further the statements on pa 92 etc.).

If for a tunnel shaped compartment the mortar in the masonry or at least at the removal of the centering is so stiff that the angle of repose is 45°, then accordingly the angle between the courses and the axis of the vault must have this value at most. But if beds then have a greater angle as in Fig. 297, for example 60°, then would occur a movement of the courses on each other, or if this is hindered, at least a change in the pressure in regard to the abutment. For each the part A B C of the courses would load the side wall, indeed with a component K derived from the pressure D, and reduced by the resistance of friction. Certainly the force would not be very great, and the main thrust would always remain on the proper abutments. If the vaults were more hardened, so that the angle of repose was over 60°, so would the side walls be relieved even by settling or yielding, whereby the entire thrust again must be received by

lower temperature.

lower abutment alone.

The possibility that the position of the courses produces a different distribution of pressure will most easily occur while the mortar is yet soft, and therefore it is safer for the pressure to be transmitted according to the forms of the vault, so that it is well for the courses not to vary from the pressure perpendicular to their most favorable direction by more t than 45° (for greater security by only 30°) for classical vaults.

One position of the courses still needs special mention, that employed by the Byzantines and also in the later middle ages., recently again brought into honor by Möller, that at right angles to the ridge (Fig. 297, right). This especially makes easier the freehand masonry. In this the directions of the course and the pressure coincide, and each course transfers its part of the pressure to the abutment, and here naturally can occur any variation from the correct distribution of the pressure.

In most positions of the courses usual in practice no effect on the direction of the pressure by the direction of the courses is to be assumed.

Arrangement of the courses.

Position of the courses among the ancients.

If we must assume that the arrangement of courses is mostly without influence on the transmission of the pressure, yet this is the more important for convenience of execution. Therefnre in this direction were made many experiments in ancient and later times. So far as the ancients constructed their vaults in complete centering, the position of the courses was of slight importance, but when men built freehand vaults, then this immediately became of particular importance.

Generally the joints in tunnel vaults as well as in cross v vaults composed of tunnel vaults formed straight lines, that in early Romanesque vaults were horizontal in some parts and in others had the same direction as the tunnel. (Compartment I in Fig. 298). When men passed to raised vaults the boints could no longer have both peculiarities together. If they remained parallel to the axis of the tunnel, they rose toward the middle; on the contrary if they remained horizontal, they took a different direction in plan (II in Fig. 298). The first kind with the same direction as the axis of the compartment was employed in Germany and in eastern France, while in the then English

France proceeded in the second way --- always including all domes in horizontal rigs usual there. The same was followed also in Normandy and in England, and it led there to the adoption of the ridge rib, and formed the basis for the method of construction usual in the later net and fan vaults. But the more or less horizontal courses also occur early in other countries. With special influence on the direction and courses was the free-hand masonry, that in provinces with easily wrought small stones, and most generally acquired the mastery in brick countries. Freehand masonry required curved and short courses that men sought to attain as far as possible.

That in freehand masonry men did not adhere too closely to a previously assumed pattern, but sought to help themselves as best possible, is shown in an interesting way by the vaults in the cathedral cloister at Riga, which belong to the 13 th century. There are found directly beside each other the arrangements sketched in Fig. 299. Although the side and cross arches are pointed, the compartments are spherical surfaces whose vertices lie sidewise from the middle of the vault at the points marked by a cross. The ring-shaped courses lie about horizontal, but the closing of the compartment in almost every case has f found a different solution, since the regular annular arrangement drawn in Fig. 300 is badly carried out at an apex of the dome.

The most important positions of the courses may now be more closely examined.

Courses parallel to the ridge line.

1. courses parallel to the ridge line continued in those countries, where were large quarried stones were common until in the 15 th century, and men then vaulted without swelling on t the centering.

Most favored was the light tufa, and such was employed on h the Alexander church at Einbeck up to 60 cm long and 20 to 30 cm thick. In many provinces of Germany, for example in upper Hesse, have remained common compartments of rubble until now, the position of the courses parallel to the ridge is retained, to avoid an interlacing of oblique courses at the ridge, which is hard to execute with irregular rubble. The courses extend directly over the cross arches and join obliquely on the backs of the groin ribs.

Compartments without swelling or raising.

The simplest case occurs when the vault is neither raised n nor swelled, the joints run parallel to the ridge and are straight as in the simple tunnel vault, and it has no effect whether the compartments have a round or pointed cross section.

Freehand masonry of such a vault could only be executed for very small dimensions, for the upper steep courses must uphold themselves like straight arches, which is indeed not possible with their great length. The more pointed are the compartments the less this difficulty becomes, but still for the upper courses one could scarcely dispense with laths or planks for safe support of the upper courses, and generally the entire compartment has a centering.

If it is desired to lay out the joints, then the side arch is to be divided in equal parts according to the width of the stones, and the diagonal arches are divided in the same number of equal parts, where the lines are naturally longer. Connecting by straight lines the corresponding points gives each joint. For execution in brick or rubble no division is made baforehand, but the mason begins below with horizontal courses and continues till close to the ridge. The vault is to be executed in c cut stone, the surface is developed and the division is made on the development. (Fig. 301.

Each course extends straight and with unidorm width from the side arch. In Fig. 301 is laid off a course. Unimportant variations arise only if the projection of the cross arch differs from the side arch, for example when with a pointed side arch is employed a round groin with elliptical projection.

Compartments swelled and raised.

If a raised vault occurs with ridge rising in a straight line, then a course laid off in Fig. 302 II no longer has uniform w width, but is wider toward the groin. For swelled and not raised the course is made wider at the middle (Fig. 302 III), and the development of the surface caused in all directions is no longer possible, the compartment is both raised and swelled, thus a course is curved to the middle and also made wider to one end . Fig. 302 IV.

When the rise or swelling is unimportant, these variations for a separate course are so small as to be easily equalized by the joints. If greater, then in using bricks some of them

must be cut somewhat, on from time to time equalizing is made by a wedge-shaped course. Rubble can be sought with the necessary thickness, but cut stone with better construction must be arranged accordingly and for simplicity are fitted above on t the centering.

Pointed swelled compartments.

A particular irregularity occurs in pointed swelled compartments above at the vertex. If the courses are placed radial to the archas Fig. 303 shows in section and Fig. 304 in plan, at the top is a lenticular space to be filled by cut stones. It is closed in a somewhat irregular manner in the earlier vaults, and is more easil: completed with bricks than with rubble. In this mode of vaulting with radial joints and mostly done freehand, the joints appear as curved lines in plan. Fig. 304. Further illustrations are in Fig. 305 the internal view of a compartment and in Fig. 306 is represented the form of a widened course, and it is assumed there for more easily understanding, that the compartment forms a part of a spherical surface.

A substantially different procedure is given by Viollet-le-Duc in his Dictionary of Architecture. Vol. IV., p. 105. According to it the joints appear in plan as straight lines parallel to the ridge line. The beds are not radial planes as before, but are curved conical surfaces. For comparison this method of vaulting is compared with the former in Figs. 308 to 312.

If each course is supported until closed by a movable centering, that is best set each time under the top edge of the course to be set, then this centering will be set radially in the first method (Fig. 303, but on the contrary will always be set vertically in the method of Viollet-le-Duc (Fig. 308). Since the lengths of the courses contenually increase from below upward, Viollet-le-Duc recommends for their support two extensible planks held together by bolts and slots, that can be extended as desired. (Fig. 311). If the mason only takes care that the side of this centering is always accurately vertical, then will the joints be accurately arranged, and in each course will occur a small equalizing, since the ends are a little narrower than the middle. Thus the mason without assistance will give to each course its proper form until he comes to the middle, where a regular closing at the ridge occurs of itself. It is necessary to give to the mason nothing more than the rise of the longest

course at the ridge, or more correctly the radius of his centering, and all else then results of itself. It is recommended at the given place to construct the lower third as ordinary masonry without a centering, where the mason takes a line corresponding to each course on the centering, by laying a rod of the length of the course as a chord on the arch. A sketch is added to this explanation that shows the courses in the lower third, that have no bend upward but rather seem to have cut s sidewise. Viollet-le-Duc here makes a slight error, if the lower third is built in the way given by him, there originates w where the use of the centering begins, a lenticular opening a and a break in the direction of the compartment as shown in Fig. 310a in section and elevation. To fill this lack are here necessary small irregular blocks, that look heavy and ugly. T Therefore it must be more correct also to curve upward the lower courses, even when turned without the centering, as also assumed in elevation in Fig. 310.

If the two methods of vaulting are compared, it is obvious that in the second an advantage is to be seen, that the correct placing of the centering is easily watched and that it gives a regular solution for the crown. On the other hand the form of the bed joint is unfavorable, that in the first procedure lies in a plane, but is here a complicated and curved conical surface. The surfaces of the compartment also differ from each other; by the first construction results a spherical surface curved about the same at base and crown, while in the other case the surface is less curved than in the former case. The surface of the intrados of each separate course has in the second case approximately the form of an obliquely ascending cylinder (Fig. 312a). It represents a course like Fig. 312. If accurately cut stones were employed, then by the first procedure results a simpler form (Fig. 307), and by the second the more inconvenient form in Fig. 313. The cut stone in Fig. 307 has only two curved surfaces, namely extrados and intrados, while the beds and joints are plane; the cut stone of Fig. 313 has only plane end joints while the beds are curved. Although no accurately cut stones were usually employed, the latter more complicated form always expressed itself still in the difficulty of giving their correct positions to the ordinary rubble or bricks.

If a compartment is to be built of bricks to be plastered later, then may the centering be entirely omitted, as it can with sufficient swelling and corresponding prudence in even long courses can be turned freehand. With natural stone the centering can well be omitted, and it is advised for brick compartments on account of the regular appearance.

2. Horizontal courses of compartments.

2. Horizontal courses in compartments. The arrangement just described has the fault, that the quite long ridge courses are made rather heavy in freehand masonry. More tasteful are oblique courses as shown in plan in Fig. 298 III, that may take d different directions. Particular consideration is required by the position formed when the courses run horizontally, or more correctly if the ends of each course lie at the same height. On the ordinary cross vaults without raising this change has no effect, since the courses all terminate uniformly at the ridge horizontally (Fig. 298, compartment 9, and by a slight raising is no great advantage obtained (compartment II), the courses are only a little shorter at top, but extend in the ridge line O G with a tedious and pointed interlacing. First with a stronger raising will the horizontal courses for a simple cross vault become more tasteful, and the interlacing is more nearly rectangular. III in Fig. 298. Interlacing is entirely avoided in IV as in the vaults at Riga shown in Fig. 299. Though here the side and diagonal arches are pointed, but no angle exists at the ridge, the courses extend like the horizontal rings of a dome in regular curvature over the ridge, an arrangement that by the omission of the ridge rib is to be termed particularly tasteful, and is also to be strongly recommended for new structures with strongly raised vaults. If the swelling is moderate, the rings will be about concentric around the middle of the vault, but if it be raised very high, then in the vicinity of its crown must be a departure from the horizontal lourses, as shown by the vaults at Riga.

In many cases a prominence of the ridge line, whether as groin or hollow is not to be avoided, and then the use of a ridge rib is not only convenient for the erection, but is often required even for statical reasons. This owessits origin chiefly to the use of oblique courses.

More than for the simple cross vault have horizontal courses

...ding for a rich lay or pay vein, especially if all ...
...mpose one armour with the same curves. In this case the co...
...ge; no play are allowed to the line bespoke the ...
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...ually worst for such forms of veins.

...or the medium distance between the ribs of the two ...
...ng even be turned bespoke compartments without avoiding ...
...ses. The lower courses with their slight inclination lie sa...
...ly on each other, and the upper ones exercise scarce the po...
...rsity of straight upcast. If the upper course is very short,

...it is advisable to set a supporting prop under a course until ...
...the compartment is closed in. Such props sometimes reduce the ...
...ament and can act as rests in need, if the considerable thru...

...equilibrium. The last courses are to be set very firmly betwe...
...n the preceding, so that they can exert a sidewise stress ...
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...under the ribs are removed, then in case the mortar still ...

...tered, the effect of the horizontal arches is removed, and ...
...therefore the compartment exerts stresses from courses to ...
...It has the form of a section of a tunnel vault and acts ...
...cordingly. A slight settlement of the horizontal courses ...
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...Usually here also have swelled courses their prefer...
...before all others in the upper parts of the compartments, the...
...ause a different number of pressures to the ribs, but ...
...particularly lessen during vaulting the just described ...
...thrust of the separate courses against the ribs. For a ...
...construction will also the movable centering five good ...
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...forward.

...6. Courses perpendicular to the diagonal ...

...7. Courses much too flimsy correspondents to the ...
...it nearly square ...
...execution that they ...

a value for a rich fan or net vault, especially if all their arches are struck with the same radius. In this case the courses in the plan are perpendicular to the line bisecting the angle; in plan and elevation results a similar regular form (Fig. 314), and therefore this position of the courses was generally usual for such forms of vaults.

For the smallest distance between the ribs of the fan vault can even be turned freehand compartments without swelled courses. The lower courses with their slight inclination lie safely on each other, and the upper ones evermore assume the peculiarity of straight arches. If the upper course is very short, it is allowable to turn this between the ribs, but if longer it is advisable to set a supporting prop under a course until the compartment is closed in. Such props sometimes reduce the thrust and can act as rests in need, if the considerable thrusts of the horizontal straight arches are not sufficiently in equilibrium. The last courses are to be set very firmly between the preceding, so that they can exert a sidewise stress on all other courses. Now if the spporting props and centerings under the ribs are removed, then in case the mortar still possesses a little ductilit:, the compressile stresses are transferred, the effect of the horizontal arches is reduced, and therefore the compartment exerts stresses from course to course. It has the form of a section of a tunnel vault and acts correspondingly. A slight settlement of the horizontal courses downward is to be expected.

Naturally here also have swelled courses their preferences before all others in the upper parts of the compartments, they cause a different transfer of pressures to the ribs, but quite particularly lessen during vaulting the just described side thrust of the separate courses against the ribs. For a plain construction will also the movable centering give good service here, but men mostly omit its use and turn the courses entirely freehand.

3. Courses perpendicular to the diagonal groins.

3. Courses with bed planes perpendicular to the diagonal groins (Fig. 315)are particularly favorable for ordinary square or nearly square vaults. They have twofold advantages in the execution that they extend in a plane above the groins and that they lie close at the crown below 90°, the latter indeed only

for square vaults. The plane of the joints is perpendicular to the vertical plane of the diagonal groin and passes through t the middle of the latter. In the diagonal section (Fig. 315a) the joint plane therefore appears as a radial straight line. The separate course may be straight or swelled.

This position of the course is now the usual one in Germany, and we shall therefore show its graphical representation in t the following. Although the execution is not after such a drawing but is done according to the eye of a skilled master, and consequently there are always shown slight variations and irregularities, it follows the principle seen here.

Let a b c d in Fig. 315 be the plan if a rectangular cross vault without ribs. In Fig. 315a is drawn the diagonal section, which the here semicircular diagonal groin in actual elevation, but the two side arches are in projection, the actual form of half a side arch being given at the side. Fig. 316 represents the cross section at a smaller scale, the crowns of the side arches K and L may have the same or different heights as the middle crown C, the connecting ridge lines K C and C L may be straight or swelled. If the three points are at equal heights and the compartment courses are not swelled, the ridge lines are also straight, but otherwise are convex. If the middle of the vault lies higher than the crown of the side arch, it is simplest to draw a circular arc through the three points KCL as a ridge line. With very strongly swelled courses the ridge line is also made more convex (See C D L), so that the load of the courses here intersecting in a groin can be safely transferred. The curvature of the ridge line is also with reference to a fine development of the compartment with a certain relation to its swelled form. In any case the curvature of the ridge must not be carried too far, since otherwise an angle projecting downward might be formed in the compartment. In the present case let the ridge line be assumed as a circular arc in both directions, its projections being as given in the diagonam section of Fig. 315a.

It is next to draw the bedsof the vault. For this purpose t the division into courses is taken on the diagonal arch (for example in brick courses) and through the division points 1, n etc., are drawn radii to the centre. The prolongations 1 m, n o of these radii indicate the elevations of these beds, that even

if swelled appear as straight lines. The division points are the actual side arch at the side are easily found by horizontally transferring the points m, o, etc. to m′o′, etc.

Then the division points of both arches are also to be transferred to the plan. From those points on the diagonal arch (see l, n, etc. in Fig. 315a) verticals are dropped on the ground line and the divisions thus obtained are transferred to the diagonal on the plan. Likewise are found the points m, o, etc on the plan. If the two corresponding points are joined, then are found the plan of the beds l m, n o, etc. If the joints are really straight lines, they are so in the plan, if swelled, t they will also show a slight curvature in the plan, that can be obtained by the projection of separate points. The drawing of such curved beds will not be treated further, since it is a simple problem in descriptive geometry and is useless in practice.

The construction of the other half of the compartment corresponds exactly to this, and the compartments on the right and left of the diagonal arch will naturally be unsymmetrical in rectangular vaults.

If necessary, it is not difficult to draw the projection of the springing of the vault on a plane perpendicular to the diagonal, as done in Fig. 315b. From this is obtained the revolved plan of a course in Fig. 315c, which shows the angle at the groin and allows the end joints to be drawn in. If the courses are swelled, the curvature is carried into this revolved plan, Fig. 315d, and from there can be projected back to the elevation in Fig. 315e.

Entirely in the same manner will the course of the beds and end joints be found in a cross vault with ribs, where instead of the groin edge the two side lines of the ribs are to be taken as bases.

In case accurately cut stone does not come in question, such projections of courses are mostly superfluous. When the centerings are established for the groin and edge lines or the ribs are turned in ribbed vaults, then it is only necessary to tell a skilled mason that the courses are to run perpendicular to the diagonal arches and to further inform him whether and with what rise or radius the courses are to be swelled, and he will then be able to vault the compartments correctly. Swelled courses are naturally always more favorable than straight for free-

frequent visit of courses.

Fig. 2b and 2b$_1$ show the 2nd lower degree type of
combined arch in two view and elevations, so that the transverse
compress increases in wider bounds and also pushes the vertical
pos. Like [illegible] expresses in the vaults this increase in bound-
aryreactions and is combined in practice, mostly by erecting the
arches that are moved of coarsely the same thickness, but part-
ly in purely making the thick of another higher toward the
crowns. If the difficulties in which become decisive. [illegible]
the fields in certain cases be not necessary [illegible]
one end, or those can be accordingly lowered a reduc-e-
ments. But getting the bridge must be very nearly done,
unless central thickness must be avoided with regard to the
strength of the vault.

[illegible] case could in so great, that they cannot be made arch-
ordinary means in masonry. Then one decrease from the consist-
ant bearing of the high perpendicular to the bridge. [illegible]
be centered under a and β.

[illegible] be made wider toward the arch abutes. for even the compress a
may occur. Then the grain is very high but the mass of
[illegible]
it would lead to compress motion conversely climbing from the
grain toward the side angles.

4. Inclined parallel courses.

Only the longest course like in a plane perpendicular to the
diagonal one. The other courses running in crooked planes.
In the diagonal section of Fig. 3[illegible] the lowest two r s
[illegible]
beams are other particle to r s. Tangency are otherwise the
[illegible]-ponents of the courses. Motion can be transferred to the
[illegible] or other views. The lower courses rise obliquely like
diagonal to the side arch; it has amount of rise as given by
the meter for the lower courses. Then by unknown middle of

freehand masonry.

Unequal width of course.

Figs 315 and 315a show how the bed lines diverge from the d diagonal arch in the plan and elevation, so that the separate courses increase in width toward the side arches and ridge lines. With small stresses in the vault this increase is however unimportant and is obtained in practice, partly by sorting the bricks that are never of exactly the same thickness, but partly by simply making the beds of mortar thicker toward the cross arches. If the difference in width becomes greater, then may the bricks in certain courses be cut somewhat thinner toward one end, or there can be occasionally inserted a wedge-shaped course. But cutting the bricks must be very neatly done, and entire cutting throughout must be avoided with regard to the strength of the vault.

If in extensive vaults the differences in width of the separate courses is so great, that they cannot be made with the ordinary means in masonry, then one departs from the consistent making of the beds perpendicular to the groin. There can be chosen the variations from the prescribed construction to be mentioned under 4 and 5.

But it is not stated at all, that the courses are always to be made wider toward the side arches, for even the converse m may occur. When the groin is very high but the apex of the side arch lies very low, then the projection of the latter in Fig. 317 may fall at the point n, instead of n, i.e., it can lie b below the diagonal groin. But as a further drawing soon shows, it would lead to courses which conversely diminish from the groin toward the side arches.

4. Inclined parallel courses.

Only the longest course lies in a plane perpendicular to the diagonal arch, the other courses running in parallel planes. In the diagonal section of Fig. 317 the largest bed m n is drawn radial to the centre C of the circle. On the diagonal are then divided the widths of the courses and through the division points are drawn parallels to m n. Thereby are obtained the p projections of the courses, which can be transferred to the plan or other views. The lower courses rise obliquely from the diagonal to the side arch; if the amount of rise is given to the mason for the lower courses, they by uniform widths of ma-

masonry will naturally be obtained about the direction desired for the long courses. For simple cases it will be unnecessary to lay off the direction of the beds in the drawing, the lower courses approximately rise rather obliquely from the diagonal groin and the compartments are built with courses of equal breadth to the middle, a moderate swelling being given ho the courses.

5. Perpendicular parallel courses.

All courses are parallel in plan and are perpendicular to the diagonal. In the diagonal section are no longer radial joints, but they are all represented by parallel lines.

Let Fig. 318 be the plan and a part of a vault with ribs, so that a is the cross rib, b b′ shows the diagonal rib and c c the side arch. Now in Fig. 318a is drawn the elevation of the diagonal arch which also represents the elevation of the cross arch, if it is assumed that both are struck with the same radius. The diagonal arch is now executed to the height e, so that it is intersected by the radial line e f′. Over the point d on the plan, that has the same name in both Figs., the ribs separate from each other. Thus the point of separation lies in the vertical erected at d and consequently is at d′. Here may the beds of the springing of the ribs lie still horizontal, a and hence are cut obliquely by the arches as shown by d′d′. F From e is laid off downward on the back of the arch the divisions for the separate compartment courses according to the thickness of the bricks, and then the projections of these division points are transferred to the horizontals e f as f 1, 2, etc. In the plan these points are laid off forward from the p point d on the diagonal arches, and accordingly the beds are drawn perpendicular to the direction of these arches, so that they intersect the cross arches.

Accordingly the elevation of the part of the vault can be formed that Fig. 318b represents, where one diagonal rib is shown in front elevation and the cross rib is in oblique projection.

Indeed a very slight swelling results in this arrangement naturally for each course of the compartment, if each line is a straight one on the compartment. This may be more fully explained. The extreme bed in Fig. 318 goes from e to h. While t then the point e also lies at e in Fig. 318a, the point h lies

there on the cross arch similar to the diagonal and farther b back at h. If the horizontal lines on the compartment are always to be straight, then the projection of the rising beds in Fig. 318a is the curved line e h. The swelling of this is distributed over the entire length of the bed and accordingly is extremely small. The Fig. shows at the same time, that the course in question rises from the diagonal arch to the cross arch, indeed by the difference in height of the points e and h. According to the first construction shown in Fig. 315 this rise was smaller. Above the ridge of the cross arch ever becomes less and at last entirely disappears. If the courses of the c compartment are made straight, which is however possible, then will the horizontal lines on the compartment show a slight ugly curvature inward. By the use of compartments with a stronger intentional swelling, these will best be set off upward, w whereby according to the method recommended by Viollet-le-Duc a vertical movable centering can be used.

Summary of the positions of the courses.

In the preceding are described five different arraggements of the courses, that are all justifiable, and which is to be chosen is to be decided only in the case. To make the differences clearly prominent, they are placed beside each other in diagonal section in Fig. 319 for comparison. 1, shows courses parallel to the ridge of the compartment; 2, horizontal courses; 3, radial courses perpendicular to the groin; 4, parallel courses in oblique direction; 5, same in perpendicular direction. For completeness is added the arrangement 6, in which the courses lie perpendicular to the groin, to the cross arch, and to the ridge, and join in the middle of the compartment or interlace there.

Intersection of the courses of the compartment.

On vaults with projecting cross and rib arches whose backs are furnished by a projection entering the compartment, the courses do not join together generally but simply intersect against the projection (Fig. 320). The stones are to be cut so that they fit well. The direction of the courses is almost immaterial, and there is little value in making them perpendicular to the diagonal, the junction at the ridge line at the most is considered, except that the existence of the ridge rib.

Intersection.

It is otherwise with arches whose section without projection at back lies beneath the compartments, for here the courses intersect over the arch and their mutual directions are therefore of importance. If the solution described above and so much employed exists, where the courses cross the diagonal arches, h here one course goes directly into the other, but an intersection occurs on the cross arch, that by a rectangular cut is formed like Fig. 321 or 322, but with an oblique cut the bricks require cutting, while in unsymmetrical meeting as in Fig. 323 o occur offsets in the intersection. A junction exists at the r ridge line as shown in perspective in Fig. 325.

On the back of the diagonal arch the two arches belonging to the different bays lap most simply as in Fig. 324a, and therefore the springings of both arches must be cut separately and consequently weakened thereby. Hence it is advisable for great spans to seek a strengthening of the bond in such places, which is effected most easily in the way shown by g′ in the perspective of Fig. 324, so that both arches close alternately with a stretcher k and begin with a header l. On the back of the diagonal rib must be cut the separate bricks to obtain the same form of joining. But this cutting will be reduced if the backs of the ribs have the form shown in Fig. 324b.

On the back of the cross arch a in Fig. 318 meet the continuous courses of the compartments passing over both diagonal ribs b and b′ and are cut to the line m n o p. Hence when the separate vault bricks can no longer overlap, the position of each separate course is secured. But since the separate arches sink from m to g in Fig. 318b, and further each course has a bearing in the one next beneath, the adhesion of good mortar suffices to hold the brick at m in its place. A corresponding strengthening as the diagonal rib has by the brick l can also be received by the cross rib according to Fig. 324, when begun with bricks standing on edge.

Just conversely is the junction of courses when parallel to the ridge of the vault, as they then cross the cross arches a and intersect on the diagonal arches; the intersection at the ridge entirely disappears.

Vaults without ribs.

The erection of vaults without ribs occurs in the same way as for ribbed vaults, but here the construction of the groin

angles requires still more care. In rubble vaults of the early middle ages the construction of the groin was always a rather a miracle. In careful execution were employed at the groin stones more or less cut to fit. Men frequently were satisfied merely by angle headers, but otherwise the stones were allowed to meet with a joint at the angle. But the neglect of a supporting angle was conceivable, and its heavy construction contributed a good part to the introduction of the supporting and projecting rib arches. But moreover in the entire middle ages and especially in secular art were also executed many vaults without ribs, as also they have again found extended use in wooden buildings.

But in them the strengthening of the diagonal arch shown at 1 in Fig. 324 became a necessity, so that the bond was made entirely according to Fig. 318b, differing from this Fig. only in that the section of the rib is omitted, and on the contrary the triangle g r s cut off the brick z in Fig. 318b remains. But then this brick does not retain its rectangular shape, but it must be cut more obtuse nearer the ridge of the vault.

Cell (cloister) vaults.

Meanwhile that this cutting may also be avoided, there originated those native vaults particularly in the Baltic provinces, yet as a rule in cross vaults, but the cell vaults arranged according to a more complicated system, which are so formed that the groin angle shows a right angle in every section made perpendicular to it. Fig. 326b.

Between the groin angles that mostly represent rich star or net forms, the compartments rise like small pyramids or high troughs. The beds lie in a plane perpendicular to the arch (radial) and form a right angle at the groin, that makes possible a simple brick bond (Fig. 326c). Usually the beds lie symmetrically to a plane perpendicular to the groin, so that then form at each side an angle of 45° with this plane.

On the last assumption is based the construction of the beds shown in Figs. 326 and 326a. For simplicity is assumed an ordinary cross vault and straight beds.

On the revolved arch is first made a division for setting the bricks on it. Through the different points of the arch above a b are drawn radii of any lengths, and these are intersected by a concentric arch at any distance a b, and on ab are

drawn the line 1 1 parallel to a b at the same distance, the division points are then projected down on a b, which on the concentric are cut by the radii on 1 1, forming the points found on a b and 1 1, so that the ground projections of the beds of a portion of a compartment are found. Likewise are obtained thusefor the adjacent portion of the compartment, whereby the intersections above a d are also found.

Fig. 326. then shows the cross section of Fig. 326. It is n next to determine in this section the height of the point d as well as the location of the point f″ of the diagonal arch in Fig. 326. Through the latter is drawn the radius and thereon is laid off the distance of the point d in plan from the line a C as f″i. The distance of the point d from C̄ is laid off in Fig. 326. from C to d′, a perpendicular is erected at the latter and then is laid off the height of the point i above a C, when the height of the point d and the position of the bed d′f′ are found in d′f′. Likewise are all the other beds obtained, for example those of the beds i h k as i′h′k′. The beds obtained then determine of themselves the concave angle and the intersection therein.

On the laying off of a cell vault found in Cracow, see the study by G. Bizanz in Allg.Bauz. 1888. Vienna.

Fig. 326a shows how the point d rises high above the crown of the diagonal arch, so that a considerable loss of height occurs in general. This is lessened in a way, if the faces of the compartment rise from the diagonal arch rise in curved instead of straight lines. Such a vault then differs from the cross vaults shown above with flat swelling only in this, that here the segmental arch of the swelling is replaced by a broken pointed arch, that may also pass into a quadrant.

Yet for any considerable span the introduction of this curved line will however reduce insufficiently the loss of height, and only in the reduction in the size of the panels, i.e., in an increase in the curved lines dividing the ground surfaces, is to be found an effective means. Also there results thereby the transition from the plan if the cross vault to that of the star or net vault. But even retaining the cross vault the panels become smaller and the heights are reduced as shown in the perspective of Fig. 326b, as soon as also the ridge line is formed as a groin arch, which is then built like the others forming

a rect.ngular angle, whereby the entire plan is then divided in eight compartments.

The preference for smaller compartments is explained also, that with great compartments the beds would be too divergent.

A very substantial difference of this cell vault from the ordinary cross vault without ribs appears in the proportions of the cross arches where several bays join. While in the cloister vaults the cross arches may also be formed as groins, in which the compartments regularly intersect each other, this intersection occurs in other vaults without ribs by the intersection of the courses. The uneveness this makes, especially in vaults not plastered, causes it to be desirable to conceal by a projecting cross arch, on the back of which is made the junction of the courses. The arrangement of this cross arch is very common in works of secular architecture, where the vault w without ribs chiefly finds its place, where in an upper story are required walls placed on it, whose location is then determined by the division of the bays of the vault.

10. Centerings and Erection.

Preparation of the centerings for arches.

Centerings are called the wooden forms, whose backs are intended to support the vault ribs during the construction. Their manual construction is done in various ways. Here will be given briefly only some of those most common.

1. They can be formed like a pair of rafters and the tie in Fig. 327a, on the backs of which are either cut or placed the curve. Simpler is the construction employed in upper Hesse shown in Fig. 327, by which short struts a b of different lengths and each corresponding to an ordinate of the curve are set on the rafters. These struts are cut off exactly at the line of the curve and then covered by a strip nailed on, on the back of which the rib is built or set. If the span is considerable, exceeding 8 m, then must the rafters be further connected by a second tie, or for yet greater spans have a king post, a tie and two struts.

2. The centering can also be composed like a timber arch of doubled or tripled pieces of planks nailed together, whose back is cut to the required curve. Fig. 327b. In such arches with a heavy load the lower ends (in spite of contrary assumptions) tend to separate from each other to a certain degree. By this

can be produced an elastic setting of the middle in vaulting desired in certain circumstances. If a firm centering is wished, then the lower ends are to be held by strong ties, and besides is recommended a sufficient bracing to prevent deflections at the sides.

Which construction is preferable must be decided by circumstances. If the cost of material be least for the latter, on the contrary the cost of labor is greater, and the planks nailed together are later usable only for the most subordinate purposes, while the timbers framed as in Fig. 327 can always be readily used again.

Erection of centering.

The erection of the centerings first occurs when the vaults are to be closed, thus after the erection of the walls. Since now the springings of the ribs are to be undertaken at the same time with the walls, or are set if consisting of one or more cut stones, a centering is only needed from the upper bed of this springingof the rib joined with the wall as in Fig. 327. Since further the side and cross arches, if they have to bear upper masonry, are likewise already constructed with the wall, thus is needed for the erection of the vault proper centerings only for the cross arches and for other arches exclusively connected with the vault. The erectioo is on the basis of continuous purlins d, which are tied together by the transverse beams e in width. These beams are supported by posts h from which braces f support the purlins d. The transverse beams can only be placed beneath the middle lines of the bay, so that the upper ends of the groin centerings can be set on the struts g, which are again supported by a post h standing on the ground.

As shown in Fig. 327, all posts are set on wedges i, so that when the vault is closed only the wedges are driven out to permit the centering to sink, thus avoidind any shaking of the f freshly built vault.

Besides the ties given in Fig. 327a but omitted in Fig. 327, the separate frames may be held together by longitudinal timbers like purlins, that have notches for the rafters and are supported by struts from the posts g or beams e.

Certain modifications in the centerings are necessary for t the keystones usually extending below the underside of the rib, and necessary space must be left for them. In the construction

of Fig. 327 the short struts may be omitted where timber arches are cut to the curve at top. If their ends are weakened too m much somewhat deeper blocks may be spiked on them.

Otherwise is constructed the centering with the use of a fixed post. Where in Fig. 327 the middle support g (or h) is set on wedges, a through and firmly supported timber is arranged. this does not extend quite up to the keystone, so that it cannot hand on that when the centering is removed. The centering does not extend through above, but for each branch of the rib consist of a separate timber, whose upper end must be so fastened, that it can slide lown on the beam when removed. Each p pair of opposite rafters can be fastened together above and b below by ties including the timber. The lower ends of the rafters are most simply set on wedges, while the framework with the purlins d and their braces can be firmly constructed and be well utilized for the support of the scaffold for masonry.

Better than wedges, which even by careful removal of the centering easily produce shocks, are here as for other centerings sand pots or sacks, that by the correspondingly regulated escape of the sand make possible a quiet and regular settlement of the framework.

Vaulting is first undertaken when the building is brought under roof, only the springings are earlier executed at the same time as the walls. These consist of previously designed cut s stones, so that their erection presents no difficulty. For springings of brick are placed corresponding pieces of centering, not for support but as guides. In some circumstances it can be recommended to establish the final centerings, which remain in place until the later vaulting.

If springings of brick for any reason are not carried up at first, it would be conceivable that these might be erected later without bond of a small base with the wall. Then is rather made a correspondingly large recess in the masonry, which is set at the same time with the springing. The same is true for the later addition of cross vaults in old buildings.

Turning the ribs.

In constructions in brick at least three, in rubble even more (all if possible) adjacent compartments are to be centered at the same time, the vaulting of the compartments is to be carried up with regular breaks, and the centering of a finished bay

is again employed for one to be executed. After the centering is ready the construction of the ribs is first done. If they are of cut stone, the keystone will be set in place by plumbing down, and with these beginnings of the ribs it rests best on the corresponding ends of the centering. That the reb may be straight in plan a line is stretched on it and as many points are plumbed down from this, so that on the back of the rib may be accurately sketched. It is also advisable to sketch a line at one side of the rib which runs obliquely from the keystone to the springing. The sides of the rib are plumbed from this line, whereby at the same time is given supervision that the rib is not tipped sidewise. The cut stones of the ribs have a length of 1/2 to 1 m, they are set with cement, gypsum or lead sheets, and sometimes are connected by dowels or pins. The last may be made of iron, but copper, bronze or brass are better. Of the last material suffice pins of finger length and diameter, but best are brass tubes 2 to 3 cm diameter and 6 to 9 cm long. Dowels are first fixed in the upper piece of the rib and then slid into the lower piece in setting. If the last piece is doweled to the keystone, there is required a small art clamp with a string to its middle, that is entirely slid into the keystone and pulled halfway out after setting the rib.

Brick ribs may first be trussed separately like cut stone r ribs, but it is preferable to carry up brick ribs together with the compartments.

Vaulting the compartments.

The compartments are now to be vaulted freehand according to one of the positions of courses given in the preceding Chapter, so that the separate compartments are carried up together, a stiff lime mortar and very sharp sand being employed. If the mason employs no centering for the swelled courses, a sure eye is of value, and all humps that are easily recognized with later work must be rebuilt at once. In the ridge line in which the courses intersect is chiefly placed a centering. In closing one of the masons usually creeps through the middle of the vault to set the last courses from above. So that the compartments have a uniform shape, it is not advisable to employ too m many masons on the same vault, and on the other hand it is not favorable to the compressile stress to allow to occur too many interruptions for over night; on a great vault it is usual for

eight masons to be engaged at the same time. After the closing of the vault, a thin lime or cement mixture is poured over its upper surface so that the beds may not soften.

The centering remains under the ribs unchanged during the v vaulting of the compartments. Ribs of large cut stones are so rigid from their previously thin and strong cast joints, that their settlement and crushing is hardly worth mentioning, and the centering remains under them chiefly with regard to the considerable change of loading while the compartments are closed, for which the ribs are not calculated. After the completion of the compartments, nothing opposes the immediate removal of the centerings for cut stone ribs, yet they are mostly allowed to stand for some days.

For brick ribs it is advisable to leave the centerings somewhat longer, for otherwise aside from accidents the ccompressions will become so great as to be perceptible to the eye. Hase recommends 4 to 7 days according to the mortar used before removal, and in a wet late harvest to wait even 14 days.

It will be frequently observed, that after partial covering of the compartments a movement occurs in the ribs, so that their upper ends with the keystone rises from the support. This especially occurs with a yielding centering and is a natural consequence of the lacking load at the middle at first, and when this is added the keystone rises. But such great movements of the ribs are undesirable and should be prevented. This can be done by a careful propping of the keystone against the framework of the roof, but far better by a loading for which the bricks necessary for the compartments afford the natural means, and they may act directly on the keystone or be piled on planks enclosing it, indeed at first in an amount increasing with the increased height of the compartment. For the s same reasons can it be recommended to add heavy stepped backing. That the backing necessary for static reasons in the lower part is to be firmly bonded therewith and is to be carefully built in the layers added somewhat later, and not carelessly piled in is also emphasized here. A correct balancing of pendentive and crown loads will be considered already in vaulting. There appear broken joints after removal of the centerings, a and these usually warn one to regulate one of the two loads, which is to be effected according to the points of view stated

in Chapter V (Figs. 124 to 127). Slender pointed vaults are usually separated only so late from the resisting wall, that they require no backing of the spandrels.

The keystones of church vaults are frequently perforated, e either for exit of air or for fastening chandeliers, with reference to occasional lowering by suspension ropes. For the latter purpose also the compartments are often each perforated by a built-in tube of stone or metal, which is generally concealed by gilded stars suspended by copper or brass wires.

A protection of the vaults against injuries by water, which may arise from leaky church roofs seems worthy of mention. The upper surfaces of the compartments are made smooth so that the water rapidly collects in the spandrels by gutters plastered in cement, from their lowest part a channel leads to the exterior or a pipe through a vault conducts to the interior of the church. The latter is best made of lead pipe at least 3 cm clear diameter, and projects sufficiently far that occasional dripping of water is permitted. Naturally this means is useless as soon as the spandrel carelessly becomes a collecting place for dirt. The ancients in a way worthy of imitation often made the spandrels of their vaults accessible for better oversight, when they constructed the backs of ribs in the form of small stairs.

II. FORM AND STRENGTH OF ABUTMENTS.

1. General Form of Abutments.

Plan of buttress walls.

Simple solid walls.

As the nearest form of abutment for tunnel vaults and domes is presented the simple solid wall, and therefore it was entirely natural for men to take this first and also to retain for other forms of vaults, for example the cross vault. But solid walls require for great spans and height of abutments such an enormous quantity of building materials, that even at this point the not penurious Romas began to think of economy. More c consciously occurred the mastery of mass in Byzantine art, and it became the dominating endeavor in Romanesque and its completion was attained in the Gothic period.

Subdivided wall.

Already for the simple tunnel vault the continuous solid wall is notthhe best abutment even if the nearest. The building mat-

materials are already lessened by leaving large openings in t the wall (Fig. 329). The mass saved in such wise needs to be but partly used for thicker walls in order to restore their original resistance; for the power of resistance of a wall is to its length in a simple ratio, but for its thickness in its square. Yet more is attained by projecting piers, that allow a considerable thinning od the wall proper (Fig. 330). Finally the wall can be reduced to the minimum mass if it extends in arched form between the projections (Fig. 331), a form recently employed generally for retaining walls against the likewise continuous pressure of the ground. The transfer of the uniformly distributed thrust to separate piers is clearly expressed in the arched form of wall, but instead of a straight intermediate wall (Fig. 330), a straight arch must have a similar effect.

To such a control of the mass in plan can be added a similar one in elevation, when the masonry is not carried up in uniform thickness, but is distributed according to the course of pressure.

Thereby for the tunnel vault in place of the solid wall are found far more favorable forms, which certainly make rather t the impression of restraint and not of one derived from the p peculiarities of the vault. It is otherwise with the cross vault, for this directly produces changes as represented in Figs. 333 to 335. The thrust of the cross vault acts principally at certain points and requires its resistance at those points. T The parts lying between them can be reduced to enclosing the room, or if this problem is lacking may even be entirely wanting. For raised cross vaults must one indeed count on a greater part of the thrust falling on the wall, and this can only be transmitted downward and also sidewise to the buttresses, whereby again the curved plan in Fig. 335 presents advantages. But even under elevated vaults can the walls be opened so far as the thrust may be received by sufficiently powerful side arches. Such a side arch would contain a curved line of support in plan and elevation, which by the most difficult following of the curve of pressure would load again to the same result, to employ also a horizontally curved side arch after the form of Fig. 335.

The natural abutment for the cross vault is the divided and not the solid wall. Yet if men are left to employ the latter it is to be thought, that the parts next the springing can be

projected, and the more the wall is thinner, and therefore one can count with safety on a certain part of the length of the wall, perhaps half for average thickness, as a resistant mass. If the middle of the side wall is opened by great openings for doors or windows, this portion of itself is omitted as a buttress, just as in such a case the cross vault with right is opposed to the tunnel vault.

For a continuous abutment wall without important openings m may even the tunnel vault be preferable, aside from architectural respects, that will often decide for the cross vault on account of the freer development of the wall.

Buttresses outside or inside.

Commonly the supporting projections of the wall or buttresses lie outside the wall (Fig. 336), but nothing opposes moving t them toward the interior of the room (Fig. 337), or they may entirely inside it (Figs. 338, 339). In this case a wide side arch extends from one projection to another, a tunnel vault or an elongated cross vault. If the projections extend far into the room, they may afford opportunity for the formation of little chapels, connected with each other by openings. Finally by further opening can they attain the character of narrow side aisles.

As will be shown later, it is preferable to move the heavy mass of the abutment toward the interior as much as possible, for in this respect an enclosing wall moved outward is less f favorable. It can be proved useful so far as the desired spreading of the ground area of the buttress is brought to its outer edge. It is advisable for a wall lying outside to lead the upper loads to them less than to the internal abutments, so far as this can be done under the conditions. Under some circumstances a formal buttress system can be arranged in the interior of the church.

Elevation of the abutment walls and buttresses.

Resistance of abutment.

As stated, a solid wall requires a proportionally great abutment mass, that is true especially if no top load is on it. A Aside from accidental stresses by wind and the like, only three forces act on such a wall. 1, the weight G of the wall itself acting through its centre of gravity (Fig. 340). 2, the equal vertical resistance V opposed to the weight of the part of the

vault (half the vault) supported by the abutment. 3, the horizontal thrust H. of the vault.

The thrust H tends to rotate the body of the wall about the angle A or tip it over. The danger of overturning increases w with the amagnitude of the thrust H and with its height. The product H × b (force by lever arm) is termed the overturning moment. The overthrow is opposed by the vertical loads G and V. The greater are these and the greater their distances from the angle A (their lever arms), the more favorably they act. S Since these forces ensure the resistance and stability of the wall, the products of the force by the lever arm are termed t their moment of stability (resistance).

That the wall may stand the sum of all moments of resistance must be greater than the algebraic sum of all overturning moments. In the present case there must be; Gg + Vv be greater t than Hh. If this condition is not satisfied, the wall will then overturn. It will be later explained, that still other conditions come in question, for example that the pressure on the b building material must not be too great. -

From the requirements for stability directly result the most important requirements for the form of the abutment. The point of application of the horizontal thrust is to be kept as low as possible and the thrust is to be made as small as possible, which is especially attained by light and steep vaults. On the other hand it is of value to make the vertical forces as great as possible and to place them the farthest possible from the outer angle.

The resisting weight can be increased by the use of a heavy material, its lever arm by external stepping or battering. The weight of the vault itself increases the stability, yet as a rule it must be kept as small as possible, since the unfavorable thrust increases with it. At most can a heavy backing of the spandrels come in consideration as useful.

Effect of top loads.

Of the greatest effect can be a rightly applied top load on the wall, which is itself more efficient, the larger it is and also its lever arm.

One must not depend too much on the loading effect of a roof construction, beam ceiling or even a paneled partition. Aside from the variations in weight, for the usual location on a long

plate is hard to assume that the pressure is uniformly distributed everywhere, and it is quite conceivable that just above the springing of the vault the woodwork does not rest on it, so that the wall can yield without hindrance. Besides there easily occurs a temporary lack of such construction during repairs, while rebuilding or in conflagrations.

As a useful top load on the other hand may be a solid wall, yet this depends much on its position. Its centre of gravity must be as far as possible back from the outside angle of the resisting wall (Fig. 342). If a thin and heavy wall is placed at the outer face of the wall (Fig. 341), the moment of stability will then be increased but little, while on the contrary the compression on the outer side may be increased very unfavorably. Even when in the course of time a certain leaning outward appears, the centre of gravity may be mysteriously near the outside angle.

On old works the abutment walls without buttresses have often yielded considerably, particularly of the original conditions of loading have changed, which can frequently be observed in churches without piers and monasteries. As an example are mentioned the vaults in the cathedral cloister at Riga (Fig. 341). Although the vaults are favorably constructed, their springings lying close above the ground and the widths thickness of the abutments amounts to nearly a third of the span, according to the evidence of statical results the walls are at the limit of stability. Here the raising of the upper floor and the addition of other injurious loads have had this result.

It follows from these considerations that a solid continuous wall can only be recommended as an abutment for vaults, and p particularly cross vaults where only small thrusts occur, favorable top loads exist and for other reasons thick and solid walls are already required, for example in cellars or lower s stories of high houses. In other cases the arrangement of buttresses will always lead to great economy of materials.

Elevation of buttress.

Since the resistance of a buttress increases according to t the square of its width or simply as its thickness, it seems advisable to make it as thin and wide as possible. But the limits will be fixed by the possible difference of the thrusts in the two adjacent vaulted bays, by the danger of sidewise

overturn or bending, and finally by the fact, that elongated development in plan a good distribution of the pressure on the cross section is doubtful and accordingly shearing is to be feared. Usually the width varies between twice and thrice the thickness, when the portion extending through the wall is taken with the width. It will be generally recommended to make the buttress project as thick as the wall, its projection from that being as great as the diagonal of a square constructed w with the thickness of the wall; but it should be noted, that too rigid rules for the dimensions of such parts of the building are idle and were unknown to the early middle ages.

A buttress may dominate or be subordinate to the wall, and accordingly is arranged its importance as a resisting body, b but the greater problem chiefly falls to the buttress. If the wall is only thin, then its entire length is not reckoned with the buttress as an abutment, but only the adjacent parts, perhaps at each side a square piece of wall of its thickness (Fig. 344). If the wall is yet thinner, it is advisable to count not at all on its aid, or at most to apportion to it for high vaults the thrust from the corresponding parts of the compartments.

In the elevation the buttress may rise to the springing of the vault, to the main cornice or still higher, it can extend straight up or have offsets at front and sides, finally may a have also continual changes of cross section.

The theoretically best form will a buttress have, when it exactly follows the line of support (Fig. 343). This will always lie in the middle and the cross section gradually increases downward according to the increase in compression. Whether the beds are made perpendicular to the direction of the pressure or merely horizontal is mostly almost immaterial. The internal part C D E of the wall could be entirely omitted, as far as it is unnecessary to keep the pier upright before the addition of the vaults.

In fact the buttresses on old works approximate quite near this ground form, that naturalln does not appear so directly in consequence of the entire architectural development. Even the defects of the lower superfluous part C D E are compensated by gradually corbelling out the members of the vault. Such piers are naturally constructed with the theoretically least use of materials, but require a rather deep extension in the

direction of the thrust. If this be limited, there remains nothing further than piling up a greater mass in the direction of the height.

The straight piers of the first Gothic hawe no great thickness but required quite a large mass (Fig. 344). The trapezoidal pier (Fig. 345) is somewhat thicker in plan, but saves little in mass. Instead of the trapezoid might be considered a triangular projection of the pier, especially when the enclosing walls are already quite strong (Fig. 346). The trapezoidal or triangular outline need not appear in its plain form, as it can rather have a suitable solution, when too abrupt changes in cross section are to be warned against, which of themselves easily lead to cracks and sliding.

The advantage of the pier diminished upward in Figs. 345,346, against theat of Fig. 344 are recognized by a comparison with that of the form of Fig. 343, but it also results more clearly as soon as the moment of stability shows, that the wide distance of the centre of gravity behind the angle of rotation is of advantage. In this respect yet more can be attained if the rectangular cross section is abandoned and for it are introduced different cross sections below and above, for example two triangles reversed to each other (Fig. 348). It is favorable below to make the dangerous external side a b as long as possible, but it is better at top to move the mass back as far as possible. Even this advantage the middle ages did not neglect, There frequently occur plans like that of Fig. 349, where the exteroal angles are strengthened by angular projections, while heavy finials at top close to the face of the wall load it in the most favorable manner. It is evident that the possibility of manifold treatment was lacking to the buttress in neither static nor architectural respects, but its further development will be treated elsewhere.

In Fig. 343 was it shown, that a space beneath the buttress could be entirely omitted, particularly notable for very high piers. A further advance and according to Fig. 347 the forces acting at the springings of the vault were resolved in two directions. One component in a pier A B could pass vertically downward, the other consisting of the thrust and also any part of the vertical loads was carried down following the line of support in a curved mass of masonry A C. The latter becomes

thinner but projects farther outward than the buttress (Fig. 343). The space C B between the outer and inner piers could be taken inside the building, whereby also in this way was developed the basilican church plan with its buttress system. According to the manner in which the forces are distributed to the two masses of masonry, and the mode of arrangecent in them men were able to deduce the most diverse forms for such a buttress system. How the condition of equilibrium in these can be tested will soon be described in a special Section.

Middle pier.

Thrust equal on all sides.

If vaults occur beside each other in several rows, middle p piers will become necessary for their support. Utilization of the interior demands for them generally as small dimensions as possible., to obtain which equilibrium in the thrusts from all sides contributes most efficiently. If all horizontal thrusts are mutually neutralized, the pier only requires to be so large, that under the load of the vaults resting on it, it will be neither crushed nor bent, but it would usually have only a very small cross section, which for safety with regard to accidental oblique or unequal pressures during the process of vaulting are somewhat increased.

If a wide hall or a church with several aisles was covered by vaults of equal dimensions and height, then naturally resulted generally an equilibrium of the thrusts, but such could also be covered by adjacent vaults of unequal adjoining spans, for example in a church with aisles of unequal widths, yet it might be attained in part or entirely by suitable construction.

Equilibrium of thrusts of vaults of different spans.

If there adjoin two vaults of equal thickness and heights b but of different spans, then the thrusts are very different, being related about as the squares of the spans (Fig. 350). With the ratio of 2 to 3 for the spans, for example the thrusts will be as 4 to 9, and with a difference of 1 to 2, the larger thrust would be 4 times the smaller, so that after equilibrium at the pier must remain an excess of thrust equal to 3/4 of that of the greater vault.

It is better to balance the thrust when the rise of the small vault is so reduced that the ratio of its height or rise (f to b) equals that of the great vault (F to B), and then the thru-

thrusts will be in the direct ratio of their spans (Fig. 351).

If the thrusts are to be entirely equilibrated, the rise of the smaller vault must be still less (see the dotted line in Fig. 352). By sufficiently flattening the small vault equilibrium of the thrusts is always made possible, but seldom is this solution possible in architectural respects. In any case so far as possible, the rise of the little vault is to be reduced instead of being increased, for one is to be particularly warned of very pointed lancet arches, that they are always statically disadvantageous (See p. 54 above) as especially conceivable in this case. If the narrow vault must be carried to the same height as the wide one, in place of a slender pointed arch it is best to employ a flatter stilted arch as shown by Fig. 352. Therefore is increased the thrust of the pointed a arch and it is applied higher, both having a favorable effect.

If the thrust cannot be equilibrated by a suitable rise, then must one pass to an artificial increase of the weight of the narrow arch, best attained by extra masonry on the cross arch (Fig. 353).

If the middle vault springs higher, its thrust exceeds more, then the excess masonry on the cross arch may even produce a stiffening by which the thrust is partly carried above the smaller cross arch (Fig. 354).

With a greater difference in height a solid wall over the c cross arch would be too heavy and would too much increase the thrust of the little cross arch. One must then place openings in the resisting wall to reduce its weight, but over it must be received the thrust of the principal vault,(Fig. 355). An inclined arch is most suitable for this. Thus entirely of itself it forms the buttress system, which soon after its adoption was further perfected in a wonderful manner.

Determination of width of buttress.

Just as in great constructions the ground forms of the buttress are placed beside each other, in the further treatment a and development of the walls, the buttress and flying buttress will find in the proper place their justification in connection with the entire development of church architecture; here is at first concerned to learn to know the required width or depth of the abutment and the stresses occurring in it. The correct dimensioning of the thicknesses of the walls and piers is a g

question for mediaeval architecture of such importance, that several successive Chapters are to be devoted to it.

So far in the lack of something better, men have directed t themselves according in rules of construction which were taken from the traditions of the latest middle ages, or established with much acuteness by modern gasters. (On this see later the development of the plan of the church). For moderate proportions these are mostly good, but naturally they lose their value, as soon as particular cases arise, that may even lead to conceivable errors. Never allow to such rules a feeling of safety, a circumstance that has perhaps alienated many young men from mediaeval architecture. But one has confidence in his constructions, when he first clearly realizes the action of the forces and can directly work with them. (Note). The simple composition and resolution of forces, which have recently been carried to such high importance by Graphic Statics, affords an extremely convenient and easily understood means, that for present p purposes is so much more valuable, since it gains access to m matters almost unknown to mathematics, it assumes as preliminary knowledge nothing more than the theory of the parallelgram of forces, which proves that the diagonal of a parallelogram represents the cagnitude and dierection of a resultant, or that may be resolved into two components represented by the sides of the parallelogram, or conversely may replace two such compcnents. (Note). In the course of our times, rather from ignorance or convenience employing a more costly substitute for a cheaper sonud construction, modern architects have not been a ashamed to erect seeming ribbed vaults with a complicated network of lattice girders and wire netting covered by mortar.

2. Magnitude and Location of the Thrust of the Vault against the Abutment.

If it be to obtain the forces or more correctly the stresses in an abutment, then must one first know the thrust exerted a against the abutment by the vault. Although this results from the previously described static properties of the vault, it w will be described here, so as it comes in question for the abutmenton account of the better connection.

Every vault exerts an oblique pressure against its abutment, more nearly horizontal the flatter is the vault. (Figs. 356, 357). This pressure W against the abutment may be resolved

into a horizontal component H and a vertical component V, the first being called the horizontal thrust and the second the abutment load. There may be taken at pleasure in the calculation either the oblique thrust W or its two components.

The abutment load V always equals the weight of that portion of the vault resting on this abutment.

The horizontal thrust varies not alone with the magnitude a and distribution of the weight, but very particularly with the ratio of the rise of the vault. In Figs. 356 and 357 V is assumed the same, but on the contrary H becomes very different on account of the unequal inclinations, that naturally have the greatest influence on the depth of the abutment.

To obtain the thrust against the abutment are given several methods, which so far as already mentioned for vaults, will again be repeated briefly here.

Obtaining the thrust against the abutment. 1. By the aid of the line of support.

1. By constructing the line of support, which is more fully explained under vaults (p. 52), one obtains the clearest and most reliable representation of the course of the compressile stresses in vaults, at the same time the end forces of the p pressure line directly give the oblique pressure against the abutment in magnitude and direction.

For the tunnel vault the line is obtained for a strip of perhaps 1 m wide, for the cross vault each line of pressure is separately obtained for the ribs and the cross arch and the common thrust is composed of that at the springing of the vault.

In every arch or vault is possible a great number of lines of support (Fig. 358). As most favorable is first to be designated as I, that deviates as little as possible from the middle line (more correctly stated, that has the least angle pressures --- on this see later). Besides this are steeper lines of support, the former giving a smaller and the latter a greater pressure against the abutment. If the mortar cannot resist tension, then neither of the lines of support broduced by the loads to be expected must leave the vault anywhere, but it is better to establish the condition that the line must remain within the kern (middle third). As permissible limits are to be regarded on the one hand the line of support II in Fig. 558,

the steepest lying within the middle third, on the other hand being III, the flattest.

If it is desired to make a truly certain investigation for the depth of the abutment, this is to be made separately for the two limiting positions II and III. The steeper one will require a rather weaker, and the flatter one a somewhat stronger abutment. For thin and high vaults both values usually coincide quite closely.

It is usually advisable to determine the depth of the abutment according to the flatter line III, for one is then certain not to make the abutment too weak in any case.

2. Approximate graphical procedure.

2. An approximate graphical method of obtaining the thrust of the vault is very simple, if one employs not the entire line of support but only its end forces. These end forces may be approximately obtained, for they must always be the components of a resultant of all external forces acting on the vault. The latter consist only of its own weight with any extra loads on the vault.

If one has a vault symmetrically shaped and loaded, only one half is considered (Fig. 359). The upper end force at the crown must be horizontal in this case and besides must pass through the kern of the cross section. There is drawn a horizontal to obtain the location and direction of the upper end force H, b but not its magnitude. The weight G of one half the vault is computed, that must act through its centre of gravity, and it cuts the horizontal in the point O. Through this point O must also pass the force W against the abutment, whose direction is obtained by assuming its intersection e with the abutment; as such is here taken the outer limit of the kern (at one third distance from the external angle of the bed). Besides the locations thus obtained, also to find the magnitudes of the forces H and W the weight G is laid off downward from o at a definite scale (for example 100 kil = 1 cm), and through its ends are drawn parallels to the components, by which the parallelogram O i c b is produced, the lengths of its sides O i and O b being the desired magnitudes of the forces H and W at the same scale. If an unsymmetrical vault exists, then the corresponding method is employed for the entire vault instead of for one half, (Fig. 360).(Also see the preceding p. 57 and Figs. 128, 129).

3. Approximate method of calculation.

3. The approximate mode of obtaining the thrust against the abutment is nearly allied to the preceding. First is calculated the magnitude and location of the weight G acting in one h half the vault (Fig. 361), then assuming the probable intersection of the end forces at d and e. For the lower point e is established the equation of moments. This is based on the fact, that a structural part (here on half the vault) can only be in equilibrium, when the moments (force × lever arm) of all existing forces about any point are neutralized. Here comes into consideration only the three forces G, H and W, the last of which disappears since it passes through the point e and its lever arm = 0. Thus the moment equation is $G \times a = H \times h$, by which the upper horizontal thrust is computed as $H = \frac{G\,a}{h}$.

Since all forces must be statically equilibrated in vertical and horizontal directions, and as in every vertically loaded vault the horizontal thrust at top and bottom must be the same, at the same time is thus found the horigontal thrust acting on the abutment. The vertical load V on the abutment is also known, being just as great as G. Having the components H and V, then one also has their resultant W.

Location of the pressure at the crown and at the abutment.

It is recognized that obtaining the pressure against the abutment is a very easy matter, there being only a certain difficulty in choosing the most suitable points of intersection d and e. If their locations were accurately known, then would o one have not an approximate but an exact procedure. But an exact obtaining of the forces is now not possible everywhere for a vault, since many accidental things concur, and therefore t the given procedure can be regarded as entirely adequate for practice. If one is in doubt, how the points d and e are to be assumed, a clear conclusion may be obtained by the construction of one or more lines of support (first method). In most cases will it be advisable to assume the point d at the crown toward the intrados, and on the contrary the point e toward the extrados.

If the springing of the vault is backed with masonry and bonded into the wall, then it is difficult to assume a definite bearing area on the abutment. In some cases this can be placed at the first oblique bed in which is assumed a point of intersection e′ (Fig. 361). But it will generally be simpler in such

cases to take the point e in the vertical face of the wall M M, then taking care for safety that this is taken rather too high than too low. The case may very easily occur, that the backing can be utilized in transmitting the thrust of the valut and a much flatter line of support be formed than the first view of the vault makes probable. The most probable point e usually l lies about 1/5, 1/4 or even 1/3 of the rise above the impost.

With an unsymmetrical or loaded vault (Fig. 360 or 360a), obtaining the forces by calculation is also again similar to that my drawing, the vault being regarded as a whole and the magnitude and location of its total weight G being first computed. Then are assumed the points e_1 and e_2 with the approximately tangential direction of the end forces W_1 and W_2, their magnitudes then being determined. By the graphical method this was done by constructing the parallelogram of forces, but here is first established the equation of moments about the point e_1 to find the force W_2; then the equation of moments about e_2 to obtain the force W_1 against the abutment. It is then to be considered that this is not the pressure against the abutment, but is the counter pressure obliquely upward of the abutment that is to be used in the calculation. (Reaction of the abutment. Fig. 360a).

General requirements for equilibrium.

Forces in the interior of the body or at the surface of contact of two bodies are known to exist in pairs, and thus the pressure that one body exerts on the other always causes an equal and opposed counter pressure in the other body. If one undertakes statical investigations in a body or a part of it, it is regarded as being taken from its surroundings and for them is added to each cut surface the counter forces acting there. Then all forces must be in equilibrium, but this is the case when the three following conditions of equilibrium are satisfied.

1. From any point as pivot must the sum of all moments with right hand rotation equal the sum of all moments with left hand rotation.

2. the sum of all forces acting vertically downward must equal the sum of all forces acting vertically upward.

3. The sum of all forces acting horizontally to the right must equal the sum of all forces acting horizontally to the left.

To apply the two last conditions to forces with oblique direction, they must first be resolved into their vertical and horizontal components.

By the aid of these three requirements Statics is known to solve most problems, also for the present simple obtaining of the forces in the abutment in Fig. 361 are they applied, but it is to be added thereto, that the end forces H and W are not to be inserted with the direction shown in Fig. 361a, but as shown as counter pressures in Fig. 361b. Id the case be less simple, for example if the weight G represents a greater number of external forces, then their course as found is to be always regarded as the same.

In describing the three ways for obtaining the thrust against the abutment, it is not stated what species of vault is assumed, but they apply directly to the tunnel vault, yet may be directly transferred to the cross vault.

Thrust of cross vault with straight ridge.

For a simple cross vault with straight ridge and without swelling as easily seen, Figs. 362, 362a, it gives about the same force acting on the abutment, as for a tunnel vault of the same cross section and with the same ground area. In both vaults act the same three forces G, H and W. The resulting load G is nearly the same in both in magnitude and location. (in the cross vault it is often somewhat smaller, on account of less backing masonrw, but its lever arm is somewhat greater --- in coated vaults the difference is more marked). In both vaults the horizontal thrust at top must lie at the same height d in the upper bed. The height e of the intersection of the resulting lower thrust of the vault likewise only exhibits slight variations. The only essential difference consists in this, that in the t tunnel vault the thrust is distributed along the entire width m p of the buttress in the plan of Fig. 362, while in the cross vault it is transferred to a single point at Ca

Thrust of a swelled cross vault.

If a strongly swelled cross vault exists with projecting ribs and cross arches, which at the same time also shows a raising of the crown, one is to proceed in the same manner, only that it is more difficult to fix the height of the upper horizontal thrust. Fig. 363 shows such a vault in cross section and plan. The thrust will be distributed over the entire length r n of

the cross section through the crown. A portion will be transmitted by the compartments and another portion by the cross section of the cross arch. Now on the longitudinal section is assumed an average height for the horizontal thrust as the horizontal line x x, where then the cross arch receives a relatively large portion, especially when the crown is strongly raised. Generally will the average location of the thrust at the crown be assumed rather low than high for greater security. When one has thus equilibrated the thrust at the crown and has assumed the lower intersection for the thrust, the vault is again considered as if it were a tunnel vault. It is conceived that instead of the cross vault is a corresponding tunnel surface with the average direction of the pressure and the same distribution of the weights, which is indeed usually termed the ideal tunnel vault. By its aid can very quickly attained the aim, though t this is exposed to the reproach of a certain superficiality, that one cannot avoid in general by mathematical accuracy in vaults, and that secondly one is able to pursue farther the investigation as desired by thoroughly following the transmission of the pressure.

Thrusts of raised cross vaults.

For a very strongly raised vault (Fig. 364a in section and Fig. 364 in plan)., an ideal tunnel vault d e may likewise be assumed for it, and the thrust be found by the aid of the computed weight G. There the point e is to be placed higher than elsewhere, since it is to be assumed that a certain part C D s inplan of the compartment transfers its thrust above the side arch (see p. 50). For a greater raise such a use of the ideal tunnel vault in fact may become somewhat fanciful, and it is therefore better, at least to consider separately the portion of the compartment pressing above the side arch. For this is drawn the little ideal tunnel vault d_1e_1 with the weight G_1, and introducing for the remaining part of the half bay a second and greater ideal tunnel vault $d_2e_2$with the corresponding weight G_2. In this way is first separated the thrust that falls on the side arch or the solid wall, from that referred to the springing, which is often desirable for the further investigation of the abutment.

If in important cases one is not satisfied by this, it is p permissible to follow the previous indications with increased

accuracy as desired.

Explanations of the Tables of Weights and horizontal Thrusts of simple tunnel and cross vaults.

Although according to the preceding it is right easy to compute the thrust of the vault with the required accuracy, it still appears desirable for yet further simplification of the most common species of vaults to prepare a Table of the rise and thickness of vaults and building materials (Table I). The Table is obtained on the basis of constructed lines of support and by the use of the simple formula, H h = G a (Fig. 365), a and it is correct for symmetrical cross vaults of square or s slightly rectangular plan that are but slightly raised. it is applicable to vaults with any size of bay, since the weights V^o and H^o are given in units per q m of the plan. These numbers are multiplied by the area (in q m) of the part of the vaults loading the abutment considered, in order to give the vertical load and horizontal thrust on the abutment.

To estimate the weights and thrust by the ground area may at first appear rather venturesome, since in vaults of different dimensions may arise certain variations in the amount of backing masonry and in the arch members; but an investigation shows that these differences fall within very narrow limits in the average forms of vaults, and by giving the thrust as a calculation made by the assumption of a second value. For vaults of different form, which for example have masonry on the cross a arches or separate additional loads, the Tables are naturally not applicable.

The lengths given (lever arm of weight etc.) are expressed in ratios to the span or rise. The span is taken as the clear distance between the walls, or when such are considered, are to be understood as between the side and cross arches, but on the contrary the height of rise is taken from the basal area (top of capital if no stilting exists) to the underside of the compartment at the crown. If the vault is raised, an average rise is to be taken.

The Table separates vaults according to their height into five groups:- I to V with a ratio of rise to span of 1:8, 1: 3, 1:2, 2:3, 5:6. Each group has the same subdivisions from a to f, containing the material and thickness of vaults, For vaults not exactly falling within the group or divisions may be inter-

interpolated values.

Vo = weight per q m of ground plan including the compartments, the projecting moulded arches, a moderate masonry backing, and a plastering of the underside 1 to 1 1/2 cm thick. As dimensions of bricks is assumed the German normal standard of 25 × 12 × 6 1/2 cm, as the unit weight of bricks and mortar is assumed per cu. m 1600 kil for ordinary bricks, 1200 to 1800 kil for very light porous bricks (when arches and spandrels a are calculated for hard bricks, 2000 kil for sandstone and 2400 kil for rubble in lime mortar.

For vaults filled over with floor under f, for average weight of brick compartments, filling and floor is assumed 1600 kil per cu. m (the weight of vaults filled over varies according to their size and therefore only can be given for vaults of fixed dimensions (see examples in last line f).

a = lever arm of the resultant weight passing through the centre of gravity of the part of the vault resting on the abutment (for example one half the vault). This lever arm varies from 1/6 to nearly 1/4 of the span, according to the steepness of the vault, and is to be measured from the face of the wall or of the side arch.

b = lever arm of the horixontal thrust or the rise of the line of support or of the ideal tunnel of support. By this is to be understood the difference in height between the horizontal thrust at top and the lower transfer of the pressure of t the abutment. As the limit of the abutment is considered the face of the wall or the vertical plane tangent to the face of the side arch. This length h is at least to be carefully found, since in the same vault are possible flatter and steeper thrusts, and for safety the rise of the line of support is not taken too great, and as a rule this height becomes notably less than the rise of the vault; h varies in the Table between 3/4 and 9/10 of the rise of the vault.

x = height at which the abutment reaction cuts the face of the wall or of the side arch. This height is measured upward from the basal area if the vault, i.e., in vaults not stilted from the top of the capital or impost moulding. For the determination of the depth of the buttress is required this height, and concerning its accuracy, see what is said under h.

Ho = horizontal thrust per g m of basal area of the part of

the vault resting on the abutment, for example a half bay. With regard to the variations possible, two values are given here, the larger being rather for small and the smaller for great bays.

It is interesting to compare the thrust H_o and the weight V_o for half vaults of different heights.

According to this Table these average:-

Rise 1 : 8, H_o is to V_o as 2 : 1.
Rise 1 : 3, H_o is to V_o as 3 : 4.
Rise 1 : 2, H_o is to V_o as 3 : 7
Rise 2 : 3, H_o is to V_o as 1 : 3.
Rise 5 : 6, H_o is to V_o as 1 : 4.

For superficial estimates these ratios may be noted; for pointed cross vaults of average heights and about 2/5 rise is to be expected a thrust about equal to 1/3 the corresponding weight of the vault (one half), and which passes into the wall at about 1/4 the height of the rise.

In the last column of the Table are given as examples the weights and thrusts for two cross vaults, calculated for dimensions of 4 × 4 and 8 × 8 m, with the assumption that two adjacent bays meet at one buttress (see C in Fig. 366). the loading area m n p r then has the area of a half vault.

The thrust at an angle D of the wall (Fig. 366) is produced by a small part p r q D of the vault and accordingly is notably less. It is sufficiently safe to take in each of the two directions D k and D g the thrust half as great as that at C. Instead of the side thrust D k and D g may naturally be introduced the diagonal thrust D d in the direction of the arch. This is always less than the thrust at C (7 : 10).

For rectangular bays (Fig. 367) the thrusts at the point C and E differ. At both points indeed rests a half bay m n p r or r t q u, but the spans C F and E F are unequal, and consequently of vaults with equal rises, the short direction have a smaller ratio of pier and therefore a smaller thrust. At the angle D in vaults not swelled and for the rectangle, the direction of the thrust falls in the diagonal. The Table gives no accurate values for very much elongated bays, weights and thrusts are then a little too small in the longitudinal direction and abundantly great in the transverse direction. But if rectangular bays do not differ too much from the square, then the Table can always be applied to them, and for the ratio of

the rise must always be taken in the direction of the thrust considered.

Table I. Weights and horizontal thrusts of vaults. Fig. 365). Upper headings of the columns. From left to right.

Classification of vault.

V_o = weight of one q m ground area.

a = lever arm of resultant of weight.

b = lever arm of horizontal thrust.

Z = height of pressure on abutment above springing of vault.

H_o = horizontal thrust per q m of plan of loading portion of vault.

Example I. Vaults 4 × 4 m.

V = weight of one half. H = thrust of one half.

Example II. Vaults 8 × 8 m.

V = weight of one half. H = thrust of one half.

(Left vertical column).

I. Ratio of rise 1 : 8.

a. Compartments 1/2 porous brick thick.

b. 1/2 hard brick or 3/4 porous brick thick.

c. 3/4 hard brick or 1 porous brick thick.

d. 1 hard brick or 20 cm sandstone thick.

e. Rubble 30 cm thick.

f. Brick vault filled over with floor 32 cm thick at crown.

II. Ratio of rise 1 : 3.

a. Compartment 1/2 porous brick thick.

b. 1/2 hard brick or 3/4 porous brick thick.

c. 3/4 hard brick or 1 porous brick thick.

d. 1 hard brick or 20 cm sandstone thick.

e. Rubble 30 cm thick.

f. Vault with filling and floor 32 cm thick at crown.

III. Ratio of rise 1 : 2.

a. compartment 1/2 porous brick thick.

b. 1/2 hard brick or 3/4 porous brick thick.

c. 3/4 hard brick or 1 porous brick thick.

d. 1 hard brick or 20 cm sandstone thick.

e. Rubble 30 cm thick.

f. Brick vault with filling and floor 32 cm thick at crown.

IV. Ratio of rese 2 : 3.

a. Compartment 1/2 porous brick thick.

b. 1/2 hard brick or 3/4 porous brick thick.

c. 3/4 hard brick or 1 porous brick thick.
d. 1 hard brick or 20 cm sandstone thick.
e. Rubble 30 cm thick.
f. Brick vault with filling and floor 32 cm thick at crown.

V. Ratio of rise 5/6 to 6/6.

a. Compartment 1/2 porous brick thick.
b. 1/2 hard brick or 3/4 porous brick thick.
c. 3/4 hard brick or 1 porous brick thick.
d. 1 hard brick or 20 cm sandstone thick.
e. Rubble 30 cm thick.
f. Brick vault with filling and floor 32 cm thick at crown.

Table I. Weights and horizontal thrusts of vaults.

Kind.	V_o	a	b	Z	H_o	$V^{Ex.I}$	H	$V^{Ex.II}$	H	
I. a.	200	0.22 - 0.23 s	0.90 f	1/8 - 1/6 f	360 - 400	1600	3200	6400	11500	kil.
b.	270	Aver. 2/3 s	or	,,	500-- 550	2160	4400	8600	16000	
c.	370	,,	1/10 s	,,	700--750	2960	6000	11800	22400	
d.	500	,,	,,	,,	950 -1000	4000	8000	16000	30400	
e.	850	,,	,,	,,	1600 -1700	6800	13600	27200	51000	
f.	---	0.20s = 1/5 s	,,	,,	---	5800	11000	26000	46000	
II. a.	230	0.19-0.21 s	0.85-0.75f	1/6-1/4f	160- 180	1840	1440	7400	5100	
b.	310	Aver. 1/5 s	Or	,,	220 - 240	2480	1920	9900	7000	
c.	420	,,	3/10 . 1/4 s	,,	300 - 350	3360	2640	13400	9600	
d.	570	,,	,,	,,	420 - 450	4560	3600	18200	13400	
e.	1000	,,	,,	,,	710 - 750	8000	6000	32000	22700	
f.	---	0.17 = 1/6 s	,,	,,	---	7300	5200	37500	23000	
III. a	260	0.17-0.20 s	0.80-0.70 s	1/5-1/3f	110 - 120	2080	960	8300	3500	
b.	350	= 1/6-1/4 s	Or	,,	140 - 160	2800	1280	11200	4500	
c.	480	,,	2/5 - 1/2 s	,,	190 - 220	3840	1760	15400	6100	
d.	700	,,	,,	,,	280 - 320	5600	2560	22400	9000	
e.	1200	,,	,,	,,	480 - 550	9600	4400	38500	15300	
f.	---	0.16 s	,,	,,	---	8000	3800	41600	17600	
IV. a.	290	0.17-0.20 s	0.80-0.72f	1/5-1/4 f	90 - 100	2320	800	9300	2900	
b.	380	= 1/5-1/8 s	Or 1/2 s	,,	110 - 130	3040	1040	12200	3500	
c.	530	,,	,,	,,	160 - 180	4240	1440	17000	5100	
d.	750	,,	,,	,,	220 - 250	6000	2000	24000	7000	
e.	1300	,,	,,	,,	400 - 430	10400	3440	41500	12800	

Kind.	V_o	a	h	Z	H_o	Ex. I. V	Ex. I. H	Ex. II. V	Ex. II. H
V. a.	340	0.16-0.19 s	0.80-0.75 f	1/5- 1/4 f	80 - 90	2720	720	10900	2600
b.	450	,,	or	,,	100 -110	3600	880	14400	3200
c.	650	,,	Aver. 2/5 s	,,	150- 160	5200	1280	20800	4800
d.	900	,,	,,	,,	210- 230	7200	1840	28800	6700
e.	1500	,,	,,	,,	350- 370	12000	2960	48000	11200
f.	---	0.15 s	,,	,,	---	13000	3000	77800	17500

3. Obtaining the Line of Support and Stresses in the Buttresses.

Security against sliding, overturning and crushing.

When by calculation, construction or by Table I the thrust of a vault has been found, or which is the same, both its components H and V (Fig. 368), it is then to investigate the resistance of the supporting body. That must be ensured against sliding, overturning and crushing.

Sliding of the abutment.

A sliding or slipping of the abutment is seldom to be feared with the usual materials of construction. It may occur if with soft mortar the angle between the direction of the pressure a and bed is less than 45° to 60°, or with hardened mortar if t this angle is under 30° to 45°. Slipping can be prevented by a changed position of the bed, or less effectively by dowels. Care is to be taken not to make the damp-proof bed of soft pitchy material, since this has already permitted the sliding of an entire mass of masonry. Such isolating beds must only be arranged where the compression acts almost perpendicular to the bed, and further care is to be taken by choice of material and accessories, that the isolating material remains neither too soft nor too smooth.

Overturning:

Safety against overturning is easily proved. The equation of moments is established about the outer dangerous angle (A in Fig. 340). It must then result that the sum of the moments of rotation in a favorable direction (force × lever arm) is greater than the sum of those with unfavorable rotation (overturning). For a simple case was already described the investigation for overturning on p. 124 (Fig. 340). --- For that drawin in Fig. 368, the mass of masonry pushed from both sides cannot overturn about the angle A; so long as $G_1 \times a_1 + G_2 \times a_2 + W_2 \times n$ is greater than $W_1 \times m$.

If it is desired to examine whether overturning cannot occur about the other angle B, the moments can be found for that case.

Instead of the forces W_1 and W_2 acting on the abutment, there may naturally be taken their horizontal and vertical components as in Fig. 340.

Overturning can most easily occur at the bottom of the foundation (surface I in Fig. 369 and pivot A), then at the surface

of the masonry resting on the wider footing (II, pivot B), and finally at each abrupt recession of the cross section (III, p pivot C). The stability at these places must be examined. If the overturning moment exceeds at any point, then the overturn of the masonry above it can only be prevented by special means, and to this belongs the clamping of the wall at the back. Likewise the strong adhesion of a mortar able to resist tension m may hinder overturning, and in fact many dangerous walls are thus held. But the resistance of the masonry to tension cannot be safely counted on even with cement mortar, since perhaps already invisible hair cracks produced by the mode of execution, chushing, temperature stresses, etc., exist, and the connection may fail. A firm adhesion of the wall to the footing generally does not exist, if the overturning moment is too great, at most the earth beside it may be useful, but this is a scarcely reliable factor.

Resistance and allowable compression.

Safety against overturning does not alone suffice; compression must nowhere exceed the allowable limit for the material. By investigation of the materials coming into consideration c concerning their resistance have been obtained the following values.

Material.	Kil. per cm q. Crushing.	Shearing.
Granite, diorite	500 - 1800	60 - 100
Limestone, dolomite	300 -1000	a. 30 - 50
		b. 50 - 70
Sandstone	180 - 900	a. 13 - 40
		b. 15 - 40
Limestone tufa, light	80 - 200	30
Clinker bricks	250 - 700	40 - 60
Good wall bricks	100 - 200	15 - 30
Porous or hollow bricks	40 - 100	---
Cement mortar	100 - 200	18 - 30
Lime mortar, hard	50 - 90	---

Note. A = resistance in direction of bed; b = same perpendicular to bed.

With regard to defects in materials (cracks and crevices) a and imperfect bedding of the compressed surfaces, the allowable stress must remain far below the ultimate resistance to compression. Particularly, stones with small resistance to shear

should not be too strongly loaded, since by bad transmission of the compression may easily occur crushing. For the same reason the resistance to shear or thrust is placed in the Table. Since the resistance to shear is less in the direction of the grain, it is bad to set some kinds of stone on bed, still one need not be too anxious concerning selected blocks without defects, as proved by numerous examples from the middle ages.

The resistance of mortar and stone for masonry must be cared for at the same time. For limestone or sandstone set with cement or lead, according to the stone it is usual to allow 16 to 30 kil per q cm, for rubble in lime mortar 5 to 7 kil or up to 10 kil after hardening, for brickwork in lime mortar 7 kil, f for bricks in cement 11 kil, and for very good materials 14 kil. Since piers and walls of churches are carefully built, when the load of the vaults is only added after hardening of the mortar, good bricks in lime mortar may be stressed to 10 kil without hesitation and to 20 kil in cement, it being assumed t that official decrees do not place lower limits. Old works often show far higher pressures, 20 to 30 kil on bricks and 30 to 50 kil on cut stone are not uncommon.

Moderately good ground like loam or sand is loaded with 2 1/2 or 3 kil per q cm, and likewise here in old works (for example the tower at Ulm) are shown far higher pressures of 10 kil and more on yielding soil it is of greatest importance to equalize the areas of the foundations, so that the soil beneath all parts of the building may receive the same pressure if possible, since otherwise settlement is unavoidable.

Location of line of support.

When the resultant compression falls at the middle of the a area of cross section, it is uniformly distributed over that. The stress per unit area is then found directly by dividing t the pressure by the area. For example at the middle of a pier with 1/2 q m or 5000 q cm area rests a load of 11,500 kil, its own weight being 6000 kil, there is at the bottom of the pier a compression = $\frac{11{,}500 + 6{,}000}{5000}$ = 3 1/2 kil per q cm.

But in the abutments of vaults the pressure seldom falls exactly at the middle of the cross section to be examined, and therefore approaches more or less to one side. The nearer the centre of pressure is to one side, the more is the pressure increased there, while it is reduced in like proportion at

the opposite side.

Location of the resultant compression in a cross section.

That the distribution od the stresses may be obtained, it is necessary for the intersection of the section concerned by the line of support be found, which is easily done by calculation or by the graphical method.

Obtained by drawing.

1. Graphical procedure (Fig. 370). To find the location and magnitude of the pressure on the surface A B, the weight G of the mass of the abutment above it with the thrust W of the vault are combined at their intersection O by the parallelogram of forces. Thereby are obtained the magnitude and direction of the desired resultant pressure R and its intersection P with the surface A B. Only the vertical component D comes into consideration as the actual compression on the bed, while the horizontal component S is opposed by the friction of the courses on each other.

2.Location by calculation.

2. Method by calculation.(Fig. 371). Not the thrust of the vault but its components H and V are introduced, and the equation of moments is established as follows for the point P sought, which is at the unknown distance x from B:-

1. $V x + G(x - m) = H k.$

From this is obtained the distance x and thus is fixed the position of the centre of compression P. The magnitude of the pressure R is composed of those of its components D and S, but these are easily obtained. D must equal the sum of all vertical loads, thus is here:.

2. $D = G + V.$

S must equal the algebraic sum of the horizontal forces, here being only H:-

3. $S = H.$

If more forces occur than in this example, they are to be c combined in the same way by the graphic or analytic method. T The course is always the same, whether a wall, buttress or middle pier is to be examined, whether a single vault or any larger number of vaults act at different heights and at different sides.

Example. A prismatic buttress 10 m high, 1 m wide and 2 m deep in the direction of the thrust is built of rubble weighing 2400

kil per cu. m., and receives at the height of 8 m the pressure of a vault, whose thrust H = 3000 kil and its weight V is computed at 1-00 kil. The centre of gravity is 1 m from the inner side. The equation of moments for the point P sought is:-

$$9600\ x + G(x - 1.00) = 3000 \times 8.00.$$

Weight of pier $G = 10.00 \times 2.00 \times 1.00 \times 2400 = 48,000$ kil.

$$9600\ x + 4800\ x = 48,000 = 3,000 \times 8.00$$

$$57,600\ x = 72,000. \quad x = 1.25 \text{ m}.$$

The centre of pressure is thus 1.25 m from the inner edge, 0.75 from the outer one, and 0.25 m from the centre of gravity.

The magnitude of the vertical compression is :-

$$D = G + V = 48,000 + 9600 = 57,600 \text{ kil}.$$

Horizontally:- $S = H = 3000$ kil.

The horizontal portion S is relatively very small, and it will be opposed by the resistance of friction with entire safety. The vertical part D is the pressure under consideration. If it passed through the middle, the compression everywhere would be $= \frac{57,600}{20,000} = 2.88$ kil per q. cm. But in the present transfer of pressure it would be greater at the external edge, as will be shown somewhat later.

Course of line of pressure.

In the manner described can be determined the location of the compression in any cross section. For vertical piers or walls it suffices to examine the bottom area on the foundation or the under surface of the footing. If the supporting body recedes or has offsets upwards (height III in Fig. 369), it will then be necessary to test the location of the compression under each of these. It it is desired to represent the middle line of pressure in its entire course from top to bottom, the mass is divided in horizontal layers as in Fig. 372, and for each area are combined all pressures acting above it into a resultant compression. If the intersections of the compression are connected by a curve, this represents the line of pressure.

With a greater depth of the abutment is advisable a vertical subdivision instead of the horizontal (Fig. 373) and the pressure of the vault is combined with the weights of the steps.

In simple cases can one already conclude from the location of the line of pressure whether the depths of the abutment are sufficient or not so. If the latter seems too weak, it can be made deeper and the line of pressure be found anew. For impor-

important cases must one take into account the magnitude and distribution of the stresses.

Distribution of the stresses; Kern of cross section.

If we return again to a single cross section for which the location and magnitude of the resultant compression are determined in the method described, two cases are to be distinguished, for the pressuere may lie either within the kern of the cross section or outside it, and what this means will be explained immediately.

If the pressure passes through the middle or more correctly through the centre of gravity of the cross section, it is uniformly distributed over the entire area, which is indicated in Fig. 375 by the little arrows of equal length directed upward (which do not represent the pressures downward, but the equal reactions exerted by the support). Each q cm receives the pressure $p = \frac{D}{F}$., where D = the total pressure in kil and F = area of cross section in q cm.

If the pressure D is moved somewhat from the centre of gravity to a point nearer A (Fig. 376)., the pressure increases at A, while it diminishes at B. It retains at the centre of gravity the average pressure $p = \frac{D}{F}$.

Kern of the cross section.

If D moves still farther, the case must finally occur, where the pressure at B = 0 (Fig. 377). This position of the pressure is important, since in most cases it is not willingly passed, for if D is moved farther, the pressure is no longer distributed over the entire area. For a square or rectangle (plan, Fig. 378)., this limit lies at 1/3 of the entire length A B. If the pressure conversely approaches the edge B, the pressure at A would become = 0 when D reaches a. For a transfer sidewise would result the limiting points f and g in the same manner. If the points a p f g are joined a quadrilateral is formed, which is termed the kern of the cross section. The length and breadth of the kern is 1/3 the length and breadth of the rectangle. Only when the resultant pressure falls in the kern is each part of the area under compression, and if this is required, then the pressure must move only within the middle third. If it is diagonal, its possible location is still smaller, for it is to be considered that diagonally the width of the kern is only 1/8 the length of the diagonal.

The kern of a circle is also a circle with a diameter 1/4 of that of the larger circle.

The kern of a triangle is a similar triangle with a length of 1/4 and an area of 1/16. The angles of the triangular kern lie on the centers of the lines joining the middles of the sides (Fig. 380).

When the pressure is at the limit of the kern, then for the rectangle and circle the maximum pressure at the edge is twice the average pressure p; for the triangle it is on the contrary only 1 1/2 times the average pressure.

Two other plans may come into consideration by the combination of wall and buttress, and are represented in Figs. 381 and 382, with the insertion of the principal measures for the dimensions of the kerns.

If it is desired to find the limiting points of the kern for any other plan, for example the point P in Fig. 382, then is employed the formula:- 4. $w = \frac{J}{F z}$.

Here w = distance of the point sought from the centre of gravity; J = moment of inertia about the gravity axis Y Y; F = area of the entire surface; z = distance of the neutral fibre (without pressure) from the centre of gravity. With this formula c can be found the limiting points of the kern figure for any cross section.

Pressure within the kern.

If the pressure D lies neither on the border of the kern nor at the centre of gravity, but is at any other point of the kern --- see Fig. 376 --- the neutral fibre must be conceived to lie at a point O outside the area. If the location of this point can be determined, then can be found the entire distribution of the pressure, for it is only necessary to lay off the average pressure p above the centre of gravity s at any definite scale (for example 1 kil = 1 mm or 1 kil = 5 mm), and to draw through the end of p a line to O. The height of this line from the base A B denotes at each point the magnitude of the pressure per q cm.

The location of the neutral fibre O is known if its distance z from the centre of gravity is known, and can be found by equation 4, solved for z:- 4a. $z = \frac{J}{F w}$.

Here again J = moment of inertia; F = area, and w = distance of the force D from the centre of gravity. The moments of inertia

about the axis Y Y are for the plans considered as follows.

Rectangle (Fig. 378) $J = \frac{b\,h^3}{12}$.

Square (upright or diagonal) $J = \frac{h^4}{12}$.

Circle (Fig. 379) $J = \frac{\pi\,D^4}{64} = 0.049\ D^4$

Triangle (Fig. 380) $J = \frac{b\,h^3}{36}$.

Octagon (regular) $J = 0.055\ d^4$.

Plan in Fig. 381 $J = 1.083\ a^4$ (about axis X X, $J = 2\ 1/3\ a^4$

Plan in Fig. 383 $J = 3.619\ a^4$ (about axis X X, $J = 2\ \frac{5}{12}\ a^4$)

Example. For the example described on p. 140 --- pressure on the area of the cross section of a buttress --- there was obtained p = 2.88 kil as the average pressure per q cm. The transfer of this pressure toward the edge was not yet sought, but is now to be found by the given formula la. The point of intersection P (Fig. 370) was given at a distance x = 1.25 m from the inner side E, which is 25 cm from the centre of gravity or middle; and thus w = 25, and further the area F = 200 × 100 = 20,000 q cm, $J = \frac{b\,h^3}{12} = \frac{100 \times 200}{12} = 66,666,667$, so that $z = \frac{66,666,667}{25 \times 20,000} = 133$ cm. This distance z is laid off to the right from the middle, and from the end point O is drawn in the given way the oblique line O K, and now can be measured the pressure at every point.

If it is desired to omit the drawing, the pressure at any point can be found directly by the use of the next formula for the calculation.

5. $p_1 = \frac{D}{F} \pm \frac{D\,w\,c}{J}$.

Here again D = the resultant pressure, w = its distance from the centre of gravity, F = area of the surface, J = corresponding moment of inertia, and finally c = distance of the point examined for its pressure from the gravity axis. The sign + is to be used for the side most strongle compressed, the sign - for the side less strongly compressed.

Example. The preceding example will again be used, where the rectangular area has b = 100 cm breadth and h = 200 cm length, with a total pressure of D = 57,699 kil. That falls at a distance w = 25 cm from the centre of gravity. The moment of I has alrea-

already been computed as J = 66,666,667.

If the maximum pressure p_1 is to be found for the outer side, then for the distance c = 100 cm from the centre of gravity:-

$$p_1 = \frac{57,600}{20,000} + \frac{57,600 \times 25 \times 100}{66,666,667} = 2.88 + 2.16 = 5.04 \text{ kil.}$$

Hence the maximum pressure at the outside = about 5 kil per q cm, which is regarded as permissible for the intended construction of the buttresses in brickwork with lime mortar.

The pressure at the inner edge is found just the same by using the negative sign and $p_2 = 0.72$ kil. (Also $p_2 = 2 \times 2.88 - 5.04 = 0.72$). It is unnecessary to compute other values of p, since these are known to increase uniformly from the inner to the outer edge.

Pressure outside the kern.

When the resultant pressure falls outside the kern, the line of 0 pressure lies within the cross section (0 in Fig. 383). There result compressile stresses on the side with the force, but on the opposite side are tensile stresses. At the centre of gravity is the average pressure = $p = \frac{D}{F}$ as before, for the symmetrical plans (rectangle and circle). The maximum pressure = 2 p greater than the maximum tensile stress at the other side. To obtain the neutral fibre and the distribution of the stresses there remains in force formulas 4 (or more correctly 4a and 5).

Masonry with tensile stresses.

If the masonry is in condition to resist tensile stresses, then will in any excentric position of the pressure, the distribution of the stresses be obtained in the same way. The pressure may even lie outside the masonry (Fig. 384), whereby the compression and the tension at the edges are both increased, until with an infinite distance of the force D they pass into infinitely great values.

Masonry without tensile stresses.

Now for the reasons given earlier the masonry can resist no tensile stresses. The parts not compressed have no part whatever in the transmission of the force, they rest on each other without stress, and under the circumstances even a more or less open joint may be formed here. The transfer of the pressure o occurs just as if this part of the cross section did not exist. For example on a rectangular plan, Figs. 385, 386, on which the resultant pressure D acts at the point P outside the kern, the stress is distributed as if a surface with the length A L existed

with zero pressure at the point $\frac{}{r}$. If this zero pressure is at L, then the centre of pressure P must represent the limit of the kern, and it follows that for rectangular or square cross sections, the length A P is to be laid off thrice from the location of the point L.

The average pressure d acting at the middle of the compressed area (b n) must equal the pressure divided by area, thus $d = \frac{D}{bn} = \frac{D}{3\,b\,m}$.

The maximum pressure at the edge is twice as great:-

thus 6; $d_1 = \frac{2\,D}{3\,b\,m}$. 7; $N = 3\,m$.

These formulas apply to square and rectangular sections of masonry with the breadth b, where the compression D acts outside the kern and at a distance m from the outer edge. By equation 6 is found the maximum corresponding d_1 per q cm, and by formula 7 results the length n to which continues the compression on the area.

For a triangular area n = 2 m if the compression approaches the vertex. For other compound sections the relations for the position of the compression outside the kern are less simple, so that their demonstration here must be omitted.

It is to be emphasized, that in masonry which can resist no tension, the resultant force (or the line of pressure) can never be close to the outer edge, since there the pressure would soon approach an infinite value, thus pulverizing the material would result. Passing beyond the edge would be followed by ruin. Only for masonry able to resist tension can the line of pressure pass outside the surface so long as the material resists.

For compression in the next Table are collected the edge pressuresfor different locations of the line of pressure, indeed for rectangular plans of masonry with or without resistance to tension. The values are referred to the average pressure p, t that each q cm would receive by an uniformly distributed compressi

Thus $p = \frac{\text{compression}}{\text{area}} = \frac{D}{F}$ or $\frac{D}{b\,l}$.

Table of magnitudes of edge pressures on a rectangular area of masonry with different positions of the resultant compression.

m = distance of resultant from outer edge.

d_1 = edge pressure at front. Masonry without tension.

d_2 = edge pressure at rear. ,, ,, ,,

n = distance of zero line from front edge. ,,

p_1 = pressure at front edge. Masonry with tensile resistance.
s_1 = tension at rear edge. " " " "
n = distance of zero line from front edge. " "

m =	d_1	d_2	n	p_1	s_1	n
Resultant acts within the kern.						
1/2 1	p	p	Infinite	Same values as at left.		
5/12 1	1 1/2 p	1/2 p	1 1/2 1	"	"	"
1/3 1	2 p	0	1	"	"	"
Resultant acts between kern and edge.						
1/4 1	2 2/3 p	---	3/4 1	2 1/2 p	Tens.1/2p	5/6 1
1/6 1	4 p	---	1/2 1	3 p	" p	3/4 1
1/12 1	8 p	---	1/4 1	3 1/2 p	" 1 1/2 p	7/10 1
0	Infinite	---	0	4 p	" 2 p	2/3 1
Resultant acts outside section.						
- 1/2 1	---	---	---	7 p	" 5 p	7/12 1
- 1	---	---	---	10 p	" 8 p	5/9 1

p = compression per q cm with uniform distribution.

Application to abutments of old buildings.

When it concerns the repair or rebuilding of old structures not trustworthy, it is particularly indicated to compute weights and thrusts as far as possible and to determine the compression. There the abutments require far more attention than the vaults. For a vault not that is supported by centering is seldom dangerous after hardening, even when large cracks appear, so long as the abutments remain immovable.

Cracks appearing later in such vaults indeed are produced by the yielding and settlement of the abutments.

If the vault has received many coats of whitewash or color, these can usually give an acceptable indication, whether the yielding of the abutments occurred on a particular occasion or continually. In the last case a further progress of the movement is to be feared. In repairing the vault, only the principal arches, the springings and the backing of the spandrels merely require closer attention, cracks in the compartments, particularly in the swelled parts, are less dangerous.

If the claims of the abutments are conceivable, where with otherwise good condition of the masonry much larger values can be allowed than in new structures, there come in consideration anchors, clamps, increased foundations or the addition of sup-

supporting masses (buttresses). If several vaults combine, equalization of the thrusts may be useful (p. 127), yet alterations in loads and thrusts in old works always demand special oversight.

Neutralizing the thrust by tie anchors is generally cheapest, but on account of flexibility and perishableness of iron it is only reliable from necessity. The strength of the anchor is computed according to the magnitude of the thrust of the vault, which may be approximately found by the statements of the preceding Chapter, ind in proper cases also by Table I (p. 135). to each q cm area of cross section of the iron may be assigned a tension of 700 to 1000 kil.

When the stress obtained shows that stability is only due to the tensile resistance of the mortar, then must repairs or rebuilding be undertaken with particular care. If thorough relief cannot be created by removal of the injurious thrust, then a carefully inserted iron clamp at the dangerous place would be proper, that can receive tensile stresses on the loosening of the mortar. The strength of the clamp is obtained according to the preceding from the magnitude of the tensile stresses existing. Also here can be safely assigned to iron 700 to 1000 kil per q cm, or about half as much to bronze.

Insufficient foundations.

In most cases the yielding of the abutments is to be referred to the condition of the soil, and therefore attention must be especially devoted to the bottom of the foundation. Tensile stresses between earth and masonry are entirely excluded. If the compression falls near the outer edge (Fig. 388), there arise at this very considerable pressures. But it is yet more conceivable than in a mortar bed, where after the hardening of the mortar no further crushing occurs. A yielding ground only comes to rest late or even not at all, and the more strongly pressed edge b: alternate yielding of the ground as well as by each change of loading or vibrations of the masonry (for example by wind) will be pressed deeper, which will result in a continual sinking of the masonry at one side, until its fall can be no longer postponed. By suitable distribution of the footings in new structures can this danger be prevented almost always without additional masonry, as shown by comparing Figs. 388 and 389, that require no further explanation after

the preceding statements relating to the distribution of the pressure. In old works may be required a subsequent extension of the foundations in the given direction, but this must always be regarded as a very difficult work, to be undertaken with t the same care as the addition of large masses of masonry to be described.

If sinking walls are to be supported by buttresses placed before them, this addition is to be made with special care, if it is to fulfil its purpose properly. Both in the middle ages (especially in the 15 th century) as well as in modern times were built numerous subsequent supports, partly with very good and partly with quite doubtful results. By the observation of such conditions is it recognized, that usually one of the three events shown in Figs. 390, 391 and 392 has resulted.

In Fig. 390 the pier has sunk into the ground by settlement of the mortar and by pressure, being entirely separated above from the wall. The pier is useless and the wall stands in consequence of its own resistance, perhaps it would be even better without the addition.

Fig. 391. The pier has se settled like the preceding with the wall pressing against it at the same time. Each of the two masses has carried out a rotation, while at the line of contact the bonding stones are sheared off. After the hardening of the new masonry and the comppresion of the ground under the front edge, the movement may entirely or nearly cease, and the wall finds a certain bearing on the supporting body.

Fig. 392. The bond between the pier and the wall is sufficient, so that neither loosening or shearing is possible, so that these act as a common body. The two preceding results are presented, yet the buttress in settling forms a part of the wall as an overhanging weight, until at last a condition of rest occurs, and then this construction has an effect far more allowable than the two first mentioned.

A certain movement of the wall to be described last is generally opposed with difficulty. If the wall were actually in m motion, then with the addition of the stress this would continue a little farther, till a condition of rest appears. In this is also no difficulty, but it is important that the subsequent movement does not exceed a reasonable amount. For this purpose care is to be taken, that the ground beneath the foun-

foundation is not loosened unnecessarily, that the footing of the latter is as hard as possible, and that masonry shrinking least or not at all be employed. Most consideration is usually required by the soil, that has been compressed under the older parts, but must first experience this compression under the new ones. In the circumstances it is feasible to previously compress the ground somewhat by loading and perhaps also by careful tamping. That the dangerous wall is to be shored before removal of the earth scarcely requires mention.

For particular cases may be advisable constructions like Fig. 392a. (A similar arrangement was made for the securing of a part of the cloister of the cathedral at Riga). The earth before the old masonry may remain undisturbed, the new foundation be carefully constructed independently and carried somewhat deeper, the earth under it can previously or after the completion of the foundation, be compressed by loading. The supporting mass exerts an allowable counter pressure, it can be built against it in its lower parts without bonding in the beds a d, and after settlement the upper part is to be ensured against shearing, best by bonding with a cut block of tough granite or limestone.

4. Thickness of Walls and Buttresses.

Obtained by trial.

The preceding Chapter gives the means for ensuring to a buttress a given form and depth the degree of this safety or for obtaining its resistance. If it be to first sketch for a definite vault, the depth of the abutment will depend on statical investigations, then may an abutment be assumed and the resistance found for it. According as it is weak or unnecessarily strong will be made other assumptions, until a suitable depth is found.

Direct calculation of the depth.

Instead of this assumption in the circumstances may the aim be reached directly by calculation as shown in the next example.

Example. The forces H and V of a vault acting at a height k above the ground are known. (Fig. 393). The vault is to be supported by prismatic piers t m in height, whose depths are twice their widths. These sides x and x/2 are calculated on the assumption that the line of pressure passes exactly through the limit of the kern, so that A P = x/3.

The equation of moments for the intersection P is established, which is in this case;. $\frac{2}{3} V x + \frac{G x}{6} = H k$.
This weight G is the volume of the pier × its unit of weight g per cu. m., thus:- $G = \frac{x^2}{t} g$.
Inserting this weight in the previous equation, there results:- $\frac{2}{3} V x + \frac{1 t g x^3}{12} = H k$, or $x^3 + \frac{8 V}{t g} x = \frac{3 H k}{t g}$.

Thus is obtained an equation of the third degree, that can be solved by Cardan's formula or more simply by repeated assumptions of a value for x.

Quite similar is the procedure for other forms of abutmentsa

If the compression does not pass exactly through the limit of the kern, then can be obtained another determination of its location, for example the intersection P is assumed to fall at x/12 distant from the middle or perhaps 0.30 m from the outer edge. The equation of moments for P again gives an equation of the second or third degree, that is to be solved for the desired depth of plan.

In this manner were computed the depths of the abutments in the following Tables II, III and IV.

Explanations of Tables II, III and IV for calculating the depths of abutments. (Also see Table I, p. 135).

The Tables contain the depths to abutments in metres. For continuous walls as well as for vertical buttresses and those diminished upwards, with definite forms of plan and heights. They do not make unnecessary the otherwise sufficiently simple obtaining of the depths of abutments for special cases by the aid of the line of support etc. (See the preceding p. 140 et seq.), but they afford a preliminary value only for the designer, and will serve yet more to give a clear view of the great variation of depths according to rise, kind of vault, span and height of the resistance to the vault.

The values are obtained gy computation on the basis of the weights and thrusts of Table I. They give only the depths required by the vaults. Particular conditions must still be considered, and thus the perhaps existing wind pressure against h high roofs may make advisable a relatively small addition to the depth of the abutment.

Kind of vaulta In the first column are given the different vaults according to rise and thickness of the compartments, the existence of the projecting ribs and backing of the usual

are assumed. (Otherwise is applicable what has been already s stated in regard to Table I). The calculation is made for a square bay 4 × 4 m = 16 q m, and one 8 × 8 m = 64 q m areas; for other sizes are to be interpolated the values.

Rectangular vaults. If the rectangle does not differ too much from the square, the depth of its abutments equals that of a square bay of the same area in plan and the same rise. For the rectangle is the ratio of the rise ($\frac{\text{rise}}{\text{span}}$) to be measured in the direction of the thrust under consideration, and it is less (flatter) in the long direction than in the short one, so that the Table gives for the longer direction of the rectangle a relatively deeper abutment than for the shorter one.

Row of vaults, separate vaults, corners. The depths of abutments are computed for a row of vaults, so that at each abutment two adjacent vaults meet (see C and C_1 in Fig. 366). With a single vault enclosed by abutments (for example a tunnel vault) the depth may be somewhat less, and likewise at ends of rows of vaults (D in Fig. 366). This reduction in abutment walls may then amount to 1/4 the depth, if the walls are high a and the vaults are light; if the vaults are heavy and the abutments are low, then the tabular values are taken for separate vaults. The same saving up to 1/4 of the depth occurs for buttresses, if only their depths are reduced, but if widths and depths are both reduced, then this reduction may amount only to 1 or 2 tenths of the width and depth. If at the angle a diagonal buttress is employed instead of two buttresses, then it is to be made as the Table gives for a continuous wall.

Height of abutment. The height of the abutment wall or buttress is assumed equal to the height of the top of the keystone, since in churches the masonry usually ends at this height. If the buttresses are lower, then the difference will be abundantly compensated by the weight of the connecting wall. --- The depths are calculated for a lower height of abutment (from footing of foundation to top of keystone is 5/4 of span, for an average one (2 1/2 span), and one is desired one with an infinite height. By an infinite height of abutment the depth does not become infinite, but it approaches a not excessively great limit. This is to be taken for the interpretation of values for lofty vaults (of towers etc.). That the height of the abutment has another limit by the crushing of its materials by its own

[illegible] of [illegible] at the [illegible]. The numbers given in the [illegible] for this table are lower limits, to which one must [illegible] appropriate, since the abutment of this depth (without [illegible] to [illegible]) would certainly fail.

[illegible] of the pressure within the beam. The values given [illegible] compression is not too great (see somewhat [illegible] below). When too great also pressure is proper a small [illegible] of the depth of the buttress, but on the contrary if the edge pressure is very small, then may the depth be reduced somewhat in any case, yet must still be kept sufficiently far from the limit of failure.

[illegible] of the compression at the middle stage into consideration only in Table II for buttresses diminished upwards. If the deeper depth be employed for this location of the compression, a uniform distribution of the pressure over the section is assumed in the most favorable way.

[illegible] through the limit of the beam, the pressure on the inner side, [illegible] at the opposite edge attains the maximum pressure, for which

1. Maximum pressure per q cm. 4 to 4 Kil.
2. " " " " " 5 to 7 Kil.
3. " " " " " 7 to 11 Kil.
4. " " " " " 11 to 21 Kil.
5. " " " " " 21 to 38 Kil.

[illegible] the location of the arc out to the middle (7a to 7b) the given pressure extends over the entire cross section. If the construction is too dear [illegible] the intended material, the [illegible] must be made somewhat deeper.

If the [illegible] for windows is not exceed [illegible] to [illegible] but a solid piece of wall remains [illegible] side, then [illegible] of those in Tables III and IV [illegible] 11 to 30 per cent.

[illegible]

N' = [illegible]. Resultant at edge of section.

N' = [illegible]. Resultant at edge of base.

N' = [illegible]. Resultant at edge of section.

weight is self evident.

Location of pressure at the edge. The numbers given in the Table under this title are lower limits, to which one must not approximate, since the abutment of this depth (without resistance to tension) would certainly fall.

Location of the pressure within the kern. The values given under this designation afford sufficient depth of abutment, in case the edge compression is not too great (see somewhat farther below). With too great edge pressure is proper a small increase of the depth of the buttress, but on the contrary if the edge pressure is very small, then may the depth be reduced somewhat in any case, yet must still be kept sufficiently far from the limit of falling.

Location of the compression at the middle comes into consideration only in Table IV for buttresses diminished upwards. If the proper depth be employed for this location of the compression, a uniform distribution of the pressure over the cross section is obtained in the most favorable way.

Magnitude of maximum compression. If the pressure passes through the limit of the kern, the pressure on the inner side = 0, and at the outside edge exists the maximum pressure, for whose approximate statement are given the little numbers, that mean:-

1. Maximum pressure per q cm, 0 to 4 kil.
2. ,, ,, ,, ,, 4 to 7 kil.
3. ,, ,, ,, ,, 7 to 11 kil.
4. ,, ,, ,, ,, 11 to 14 kil.
5. ,, ,, ,, ,, 14 to 21 kil.
6. ,, ,, ,, ,, 21 to 28 kil.

With the location of the pressure at the middle (Table IV) the given pressure extends over the entire cross section. If the compression is too great for the intended material, the abutment must be made somewhat deeper.

If the openings for windows do not extend from buttress to buttress but a solid piece of wall remains at each side, then must the widths of piers in Tables III and IV be reduced by 10 to 20 per cent.

(Additional explanations of the Tables.

A′ = cut stone. Resultant at edge of section.

A″ = cut stone. Resultant ad edge of kern.

B′ = brick. Resultant at edge of section.

B″ = brick. Resultant at edge of kern.

a. 1/2 porous brick vault.
b. 1/2 hard or 3/4 porous brick.
c. 3/4 hard or 1 porous brick.
d. 1 brick or 20 cm sandstone.
e. 30 cm rubble vault.
f. Brick vault with filling and floor above.

	A′		A″		B′		B″		A′		A″		B′		B″		A′		A″		B′		B″
V. Rise 5/6 (u = 2.80 m.																							
a.	0.26	1	0.44		0.31	1	0.51		0.40	2	0.70		0.50	1	0.83		0.55		0.95		0.68		1.20
d.	0.36	1	0.57		0.40	1	0.63		0.62	2	1.05		0.71	1	1.20		0.88		1.55		1.10		1.85
f.	0.36	1	0.59		0.41	1	0.64		0.71	2	1.80		0.84	1	1.50		1.10		1.95		1.40		2.40

B. Vault 8 × 8 m = 64 q. m.

	t = 10 m.					t = 20 m.					t = infinity.				
III. Rise 1/2 (u = 3.30 m).															
a.	0.63	2	1.05	0.73	1 1.25	0.75	3	1.30	0.90	2	1.55	0.85	1.50	1.05	1.80
d.	0.91	2	1.56	1.10	1 1.80	1.15	3	1.95	1.35	2	2.30	1.35	2.35	1.70	2.90
f.	1.20	2	2.00	1.40	2 2.30	1.55	3	2.65	1.85	2	3.10	1.90	3.30	2.35	4.05
IV. Rise 2/3 (u = 4.50 m).															
a.	0.45	2	0.80	0.58	1 0.95	0.65	3	1.10	0.78	2	1.30	0.80	1.35	0.95	1.65
b.	0.48	2	0.90	0.60	1 1.00	0.70	3	1.20	0.83	2	1.40	0.85	1.50	1.05	1.80
c.	0.60	2	1.00	0.70	1 1.20	0.83	3	1.40	1.00	2	1.75	1.05	1.80	1.25	2.20
d.	0.68	2	1.15	0.78	1 1.30	0.95	3	1.60	1.15	3	2.50	1.20	2.10	1.50	2.55
e.	0.85	2	1.40	0.98	2 1.60	1.25	3	2.10	1.45	3	1.90	1.65	2.85	2.00	3.45
f.	0.95	2	1.55	1.05	2 1.70	1.40	3	2.40	1.65	3	2.80	1.90	3.30	2.35	4.05
V. Rise 5/6 (u = 5.70 m).															
a.	0.38	2	0.64	0.45	2 0.75	0.57	3	0.98	0.68	2	1.15	0.73	1.30	0.90	1.55
d.	0.53	2	0.86	0.60	2 0.98	0.86	3	1.45	1.05	3	1.70	1.20	2.05	1.45	2.50
f.	0.70	3	1.10	0.72	2 1.20	1.25	4	2.10	1.45	3	2.45	1.90	3.35	2.35	4.10

Note. Resultant at edge causes overthrow! Resultant at edge of kern ensures abutment if not too great. This is given by the separate single figures. (See explanation of Tables on p. 153).

Table II. Thickness of straight abutment wall.

Length half occupied by openings. Height of wall taken to keystone of vault.

Thickness of wall given in metres.

	Small height of wall.				Aver. height of wall.				Infinite height of wall.			
	A′	A″	B′	B″	A′	A″	B′	B″	A′	A″	B′	B″
A. Vault 4 × 4 m = 16 q m.												
I. Rise 1/8 (u = 0.60 m).												
I.	t = 5 m. height of wall.				t = 10 m. Height of wall.				t = infinity.			
a.	1.00	1 1.75	1.25	1 2.10	1.10	2 1.90	1.35	1. 2.30	1.15	2.00	1.45	2.45
d.	1.55	1 2.65	1.85	1 3.15	1.70	2 2.90	2.05	1. 3.50	1.85	3.15	2.25	3.90
f.	1.80	1 3.10	2.15	1 3.60	1.95	2 3.35	2.35	1 4.05	2.15	3.70	2.65	4.55
II. Rise 1/3 (u = 1.25 cm)												
a.	0.60	1 1.02	0.70	1 1.20	0.70	2 1.20	0.83	1 1.40	0.78	1.35	0.95	1.65
d.	0.89	1 1.50	1.05	1 1.75	1.05	2 1.80	1.25	1 2.15	1.25	2.15	1.50	2.60
f.	1.00	1 1.70	1.15	1 1.95	1.25	2 2.10	1.50	1 2.50	1.50	2.55	1.80	3.15
III. Rise 1/2 (u = 1.60 m.												
a.	0.44	1 0.74	0.53	1 0.88	0.53	2 0.92	0.65	1 1.10	0.63	1.10	0.78	1.35
d.	0.65	1 1.05	0.75	1 1.25	0.86	2 1.45	1.00	1 1.70	1.05	1.80	1.30	2.20
f.	0.76	1 1.25	0.87	1 1.45	1.00	2 1.70	1.20	1 2.00	1.25	2.30	1.55	2.65
IV. Rise 2/3 (u = 2.20 m).												
a.	0.35	1 0.59	0.40	1 0.67	0.46	2 0.79	0.55	1 0.91	0.58	1.00	0.71	1.25
b.	0.38	1 0.64	0.44	1 0.72	0.51	2 0.89	0.62	1 1.05	0.66	1.15	0.81	1.40
c.	0.43	1 0.71	0.49	1 0.81	0.60	2 1.05	0.71	1 1.20	0.78	1.35	0.95	1.65
d.	0.48	1 0.79	0.54	1 0.88	0.70	2 1.20	0.83	1 1.40	0.92	1.60	1.10	1.95
e.	0.55	1 0.91	0.62	1 1.00	0.86	2 1.45	1.00	1 1.70	1.20	2.05	1.45	2.55
f.	0.57	1 0.92	0.63	1 101	0.87	2 1.55	1.05	1 1.75	1.20	2.10	1.50	2.60

Table III. Depth of uniform Buttresses.

The Table contains depth X in m. Width is half the depth.

Pier rises in outline without offsets to height of keysone of vault.

	A′		A″		B′		B″		A′		A″		B′		B″		A′		A″		B′		B″	
	Small height.								Average height.								Infinite height.							
A. Vaults 4 × 4 m = 16 q m area.																								
Rise 1/2 (u = 1.60 m.	t = 5 m.								t = 10 m.								t = infinity.							
a.	0.81	1	1.20		0.89	1	1.30		1.00	2	1.45		1.15	1	1.65		1.20	1.70			1.35	1.25		
d.	1.00	1	1.50		1.15	1	1.60		1.35	2	1.95		1.50	1	2.20		1.65	2.35			1.85	2.70		
f.	1.10	1	1.65		1.20	1	1.75		1.50	2	2.20		1.70	1	2.45		1.85	2.70			2.15	3.05		
Rise 2/3 (u = 2.20 m).																								
a.	0.64	1	0.94		0.69	1	1.00		0.89	2	1.30		1.00	1	1.45		1.10	1.60			1.25	1.80		
b.	0.67	1	0.98		0.72	1	1.05		0.96	2	1.40		1.10	1	1.55		1.20	1.75			1.40	2.00		
c.	0.71	1	1.05		0.75	1.	1.10		1.05	2	1.50		1.20	1	1.70		1.35	1.95			1.55	2.20		
d.	0.74	2	1.10		0.78	1	1.15		1.15	2	1.65		1.30	1	1.85		1.50	2.15			1.70	2.45		
e.	0.79	2	1.15		0.82	2	1.20		1.30	2	1.90		1.450	2	2.10		1.80	2.60			2.05	2.95		
f.	0.80	2	1.20		0.83	2	1.25		1.30	2	1.95		1.45	2	2.10		1.80	2.60			2.05	3.00		
Rise 5/6 (u = 2.80 m.																								
a.	0.47	2	0.70		0.50	1	0.73		0.80	2	1.15		0.90	1	1.30		1.05	1.55			1.20	1.75		
d.	0.51	3	0.75		0.53	2	0.78		1.00	2	1.45		1.10	2	1.60		1.45	2.10			1.65	2.40		
f.	0.49	4	0.72		0.50	3	0.75		1.10	2	1.60		1.20	2	1.75		1.70	2.50			1.95	2.85		
B. Vaults 8 × 8 m = 64 q m.																								
Rise 1/2 (u = 3.30 m.	t = 10 m.								t = 20 m.								t = infinity.							
a.	1.30	2	1.90		1.45	1	2.10		1.55	3	2.25		1.75	2	2.25		1.80	2.60			2.05	3.00		
d.	1.60	2	2.35		1.75	2	2.55		2.05	3	3.00		2.30	3	3.35		2.50	3.55			2.85	4.10		
f.	1.90	2	2.75		2.05	2	3.00		2.50	3	3.65		2.80	3	4.10		3.10	4.45			3.55	5.10		

	t = 5 m.				t = 10 m.				t = infinity.			
Rise 2/3 (u = 4.5 m).												
a.	1.00	2	1.50	1.10	2	1.65	1.40	3.	2.00	1.55	3 2.25	1.70	2.45	1.95	2.80
b.	1.05	2	1.55	1.15	2	1.65	1.45	3	2.10	1.65	3 2.25	1.80	2.60	2.05	3.00
c.	1.15	2	1.70	1.25	2	1.80	1.65	3	2.40	1.85	3 2.65	2.05	2.95	2.35	3.40
d.	1.20	3	1.75	1.30	2	1.90	1.80	4	2.60	2.00	3 2.85	2.30	3.80	2.60	3.75
e.	1.35	3	2.00	1.40	2	2.10	2.10	4	3.05	2.35	3 3.40	2.80	4.00	3.20	4.60
f.	1.40	3	2.05	1.45	3	2.15	2.25	4	3.30	2.50	3 3.65	3.10	4.45	3.55	5.10
Rise 5/6 (u = 5.70 m).															
a.	0.78	3	1.15	0.83	2	1.20	1.10	4	1.60	1.20	3 1.75	1.65	2.35	1.90	2.70
d.	0.87	4	1.30	0.90	3	1.35	1.60	4	2.35	1.80	3 2.65	2.25	3.25	2.55	3.70
f.	0.91	6	1.35	0.94	5	1.40	2.00	4	2.90	2.20	3 3.20	3.10	4.45	3.55	5.10

Note. Resultant at edge causes overthrow! Resultant at border of kern ensures buttress if edge pressure is not too great. This is given by the prefixed figures.

Table IV. Depth of trapezoidal diminished Buttress.

The Table contains depth at bottom in m. Depth at top = 5/11 of depth at bottom.

Height of buttress is taken to the keystone of the vault.

	Small height of Vault.				Average height.				Infinite height.			
	A′	A″	B″	B″	A′	A″	B′	B″	A′	A″	B′	B″
A. Vault 4 × 4 m = 16 q m area.												
	t = 5 m.				t = 10 m.				t = infinite.			
Rise 1/2 (u = 1.60 m).												
a.	0.90	1 1.20	0.98	1 1.35	1.15	2 1.50	1.30	1 1.70	1.35	1.75	1.55	2.00
d.	1.10	1 1.50	1.20	1 1.65	1.50	3 2.00	1.70	1 2.20	1.90	2.40	2.15	2.75
f.	1.20	2 1.65	1.30	1 1.80	1.70	2 2.25	1.90	1 2.50	2.15	2.75	2.45	3.15
Rise 2/3 (u = 2.20 m.												
a.	0.70	1 0.95	0.75	1 1.03	1.00	2 1.35	1.10	1 1.50	1.30	1.65	1.45	1.90
b.	0.73	1 1.00	0.78	1 1.08	1.10	2 1.45	1.20	1 1.60	1.40	1.80	1.60	2.05
c.	0.75	2 1.05	0.79	1 1.15	1.20	2 1.55	1.30	1 1.75	1.55	2.00	1.75	2.30
d.	0.78	2 1.10	0.83	1 1.20	1.30	2 1.70	1.40	1 1.90	1.75	2.20	2.00	2.55
e.	0.83	2 1.20	0.85	2 1.25	1.45	2 1.95	1.60	1 2.15	2.05	2.65	2.35	3.05
f.	0.83	2 1.20	0.85	2 1.25	1.45	2 1.95	1.60	1 2.15	2.10	2.70	2.40	3.05
Rise 5/6 (u = 2.80 m.												
a.	0.50	2 0.70	0.52	1 0.75	0.90	2 1.20	1.00	1 1.35	1.25	1.60	1.40	1.80
d.	0.53	3 0.76	0.55	3 0.80	1.10	2 1.50	1.20	2 1.65	1.70	2.15	1.95	2.50
f.	0.50	5 0.73	0.50	4 0.75	1.20	2 1.60	1.25	2 1.75	2.00	2.55	2.25	2.90

B. Vault 8 × 8 m = 64 q m area.

	A′	A″	B′	B″	A′	A″	B′	B″	A′	A″	B′	B″
Rise 1/2 (u = 3.30 m.												
	Height of buttress t = 10 m.				Height t = 20 m.				Height t = infinite			
a.	1.40	2 1.90	1.60	1 2.10	1.75	3 2.35	2.00	2 2.65	2.05	2.65	2.35	3.05
d.	1.75	2 2.35	1.90	2 2.60	2.30	3 3.05	2.60	2 3.45	2.85	3.65	3.25	4.20
f.	2.05	2 2.80	2.20	2 3.05	2.80	3 3.75	3.15	2 4.20	3.55	4.10	4.05	5.25
Rise 2/3 (u = 4.50 m.												
a.	1.10	2 1.55	1.20	2 1.65	1.60	3 2.10	1.75	2 2.35	1.95	2.50	2.25	2.85
b.	1.10	2 1.55	1.20	2 1.70	1.65	3 2.20	1.85	2 2.45	2.10	2.70	2.40	3.05
c.	1.20	3 1.70	1.30	2 1.85	1.80	3 2.40	2.05	2 2.70	2.35	3.05	2.70	3.45
d.	1.25	3 1.80	1.35	2 1.90	1.95	3 2.65	2.20	2 2.95	2.60	3.35	3.00	3.85
e.	1.40	3 2.00	1.45	3 2.15	2.35	3 3.15	2.60	3 3.50	3.20	4.10	3.65	4.70
f.	1.45	4 2.10	1.50	3 2.20	2.50	3 3.40	2.75	3 3.70	3.55	4.60	4.60	5.25
Rise 5/6 (u = 5.70 m.												
a.	0.83	3 1.15	0.88	3 1.25	1.40	3 1.90	1.55	2 2.10	1.90	2.40	2.15	2.75
d.	0.90	4 1.30	0.93	3 1.40	1.80	3 2.40	1.95	3 2.65	2.55	3.30	2.95	3.80
f.	0.93	6 1.35	0.95	6 1.40	2.20	4 3.00	2.35	4 3.00	3.55	4.60	4.10	5.25

Note. Resultant at edge causes overthrow! Resultant at border of kern ensures safety of buttress, if pressure at edge is not too great. This is given by the left extra figures. (On importance of this Note see Note on Table III and explanation on p. 153). Resultant at middle produces on entire cross section the pressures given by left extra figures.

5. Thickness of Middle Pier.

The statical requirements for abutments treated in the two preceding Chapters apply in their full extent to middle piers of all kinds. Since the danger of sliding is scarcely to be feared here, three conditions relate to them.

1. The pier is safe from falling in any direction.

2. Nowhere is the permissible limit of compression exceeded.

3. The middle line of pressure must remain within the kern of thecross section, if possible.

The first condition is satisfied always as soon as the secoNd is fulfilled. The third condition may also be neglected if the edge pressure remains small. Especially with good execution in cut stone, the crushing of the beds at one side and the opening at the other is so little to be feared, that a slight movement of the pressure outside the kern is generally permissible. Not to occupy the free space, men in middle piers go to the limit of stress judged allowable, yet in such cases they should not neglect to spread the foundation quite safely. To very slender piers will be given an increase with regard to the danger of buckling or crushing (more exact statements concerning safety from buckling cannot be given for lack of basis for masonry.

The most common cases of loading of the middle pier are already represented in Figs. 350 to 355. If it is desired to follow distinctly the course of the pressure from top to bottom of the entire pier, the graphical method is best employed, if it be only necessary to find the distribution of the compression on the ground area or on any other cross section, then the aim is reached just as well by calculation. (See p. 140 and t the example below). The graphical procedure frequently gives such acute intersections of lines in slender piers, that even for greater accuracy calculation is to be preferred in such cases.

Middle pier of a hall church.

Since the stress in the middle pier of a hall church is tolerably simple, it is especially suited for explaining the procedure for obtaining the compression. A definite case is therefore assumed.

Location of the copression in the pier.

Example 1. (Figs. 394, 395). A hall church with middle aisle 9 m wide and side aisles 6 m wide, with bays 6 m long, is divided by square piers set diagonally, 12 m high and 1.25 m squ-

square, thus being 1.77 m in diagonal length. Dividing arches 0.70 m wide and backed in spandrels to 2 m above the capital. Pier and dividing arch consist of sandstone weighing 2300 kil per cu. m. Vaults with sandstone ribs and cross arches are 1/2 brick thick and are built of ordinary bricks. (Weight 1600 kil. per cu. m). The cross section of both aisles shows a rise averaging 2/3 of the span.

Now the location of the centre of compression and the maximum stress in the area of the pier at the height of the floor is to be sought.

The forces in the vaults may be assumed by Table I (p. 135) and indeed (according to line IVb) the weight per q m area of plan is 380 kil and the thrust for each q m of plan is 120 kil. On the pier acts from each side half a vault of 27 or of 18 q m area, and accordingly the forces result:-

For the vault of the middle aisle $V_1 = 27 \times 380 = 10,260$ kil.

$H_1 = 27 \times 120 = 3,240$

For the vault of the side aisle $V_2 = 18 \times 380 = 6,840$

$H_2 = 18 \times 120 = 2,160.$

The height of the point of application of the forces above the capital may be taken at 1/4 rise, thus in the middle aisle being 1.50 m, im the side aisle 1.00 m.

The dividing arch with its backing may have a volume of 5 cu. m, and thus a weight of $5 \times 2300 = 11,500$ kil $= G_1$

The weight of the pier is computed at $G_2 = 1.25^2 \times 12.00 \times 2300 = 43,125$ kil.

The equation of monuments is then established for the sought point x for the middle axis, $G_1 (x + G_2)(x + V_1)(x + 0.35) + H(12.00 + 1.00) = V_2 (0.35 - \mathcal{C}) + H_2 (12.00 + 1.50)$.

$$x = \frac{V_2 \times 0.35 + H_1 \times 13.50 - V_1 \times 0.35 - H_2 \times 13.00}{G_1 + G_2 + V_1 + V_2}$$

Inserting the values assumed above for V_2, H_1, etc., $x = 0.20$ m. i. e., the centre of compression lies 0.20 m = 20 cm sidewise from the middle. The kern measures only 1/3 of the side or 1/6 of the diagonal, and thus ithas in the direction of the latter only a breadth of 29.3 cm or its half is only 14.7 cm. Thus t the centre of pressure P lies 5.3 cm outside the kern (see plan in Fig. 395a).

This little distance from the kern causes that on the inner

side of the pier a part remains without compression, may very well be termed permissible, in case the compression on the extreme edge is not too great. With a central pressure the compression on the unit area = total weight divided by area, the former being $G_1 + G_2 + V_1 + V_2 = 71,700$ kil and the area = $1.25^2 = 1.56$ q m = 15,600 q cm. Hence $\frac{71,700}{15,600} = 4.6$ kil per q cm if pressure acts at the middle. If it falls on the edge of the kern, the maximum compression on the outer angle is twice as great and = 9.2 kil. It will now be somewhat greater, yet as may be estimated, there can be only 12 kil per q cm, which for good sandstone masonry is not too high compression, and therefore the pier can pass as sufficiently safe.

The foundation will properly be extended outward, so that t the middle of its footing is moved about 20 cm from the middle of the pier, for the pressure to become central (see Fig. 395). If the footing weighs about 13,000 kil, there is a pressure t transmitted to its bottom = 71,700 + 13,000 = say 85,000 kil. If the soil can be loaded with 2.5 kil per q cm, there will be required an area of $\frac{85,000}{2.50} = 34,000$ q cm = 3.4 q m, which is properly arranged as in Fig. 395a, shown in plan and in Fig. 395 in elevation. For an entirely reliable soil it is better to spread it still more and give it the form of the dotted rectangle (Fig. 395a).

In the example just given the line of pressure fell outside the kern of the area. If an entirely similar pier were employed instead of this set diagonally, with its sides parallel to the axes of the vaults, the pressure would lie within the kern and accordingly the edge pressure would be less. From this point of view, it is advisable to compare the current plans of piers.

Central plans of piers.

Most piers can be reduced to the four forms of plan, I and IV in Fig. 396, indeed the square (or rectangle), regular octagon, circle, and square set diagonally. It is assumed that the four plans have equal areas, and then their diameters in the direction of the thrust will be as 1.0, 1.10, 1.13, 1.41. The converse proportion is found in the dimensions of the kerns in the same direction, being as 1.00, 0.88, 0.85, 0.71. But it follows from this that for a location of the resultant pressure in the kern or near it, plan I is best and IV is most unfavorable, but conversely for the location of the compression near the outer

edge, I is most unfavorable and IV is most advantageous.

One is thereby convinced that it is best in all plans to examine two similarly located points of pressure P and P_1. If the first point P falls just on the border of the kern, it lies within I and II, but is outside in IV. The edge pressure will be greatest in IV, but a part m n o is at the inner corner and will remain without pressure.

The point P_1 lies at the outer edge in I, so that here destruction certainly follows, but in the other plans it still lies inside, although even in these the edge pressure is so great, that it is doubtful whether it will resist. Even in IV wi,, t the compression be distributed only over the area s t u, which scarcely contains 1/6 of the total area, and the maximum compression at s will be nearly 18 times as great as if the pressure were equally distributed.

However it is possible, that with an unfavorable location of the compression on pier IV, it may still maintain its resistance, when I has already failed and it is especially possible , when there occur unforeseen variations of the loads, that may occur by transfer of the wind etc. Certainly such important transfers of the compression toward the edge, even when they occur but occasionally, are already injurious to the durability of the pier on account of the loosening of the beds to be feared.

Extended and unusual plans of piers.

A plan combining the advantages of I and IV is that of a rectangle with the same direction as that of the thrust (Fig. 397), already employed in Romanesque churches. Similar advantages has also a Gothic round pier, that has projections or rounds in the direction of the thrust, but not in those of the dividing arches (Fig. 398); thus the pier from Mantes that is represented later in Fig. 426. The pier in the Market church of Hanover likewise has below rounds only in the direction of the aisle, while the rounds for the dividing arch are corbelled out above. But too great extension lengthwise disturbs the connection of the side aisles, and therefore men preferred yet more a central plan, so that the circular pier with four little shafts in Fig. 399 accordingly lies by its static effect between the circle and the diagonal square. Men frequently sought to reduce even the height of the pier, when the rounds of the middle aisle

did not extend down to the floor. This is also justified by the predominance of the thrust of the middle aisle, since already the inner side received little or no compression in this case.

It is generally an advantage to move the centre of the bottom section as far as possible toward the side aisle, but to place the upper load and particularly the centre of gravity of the dividiyg arch more toward the middle aisle, so as to oppose the predominating thrust of the middle aisle. Thus for example a pier unsymmetrical on the two sides as in Fig. 400, where t the architectural treatment permits, may be statically very suitable. It would have only one round at the slightly compressed side, but would have two rounds on the strongly pressed side and there form a broad base, besides being in this case moving the dividing arch of unsymmetrical form toward the middle aisle. By such treatment would one be able to even attain to lead the compression through the centre of gravity of the plan.

A wall on the dividing arch may be advantageously employed in balancing the forces, if its principal mass can be moved toward the middle aisle. If it serves to carry the roof framework with it, the effect of wind comes in question, for which see a special Chapter.

Equalization of thrusts of the vaults.

Most advantageously treated are always the statical conditions of a pier, when the variations in the loading are obviated and the thrusts of the vaults can be equalized on all sides. What method is to be followed in aisles of different widths is already indicated in Figs. 350 to 355. To more fully represent the great influence of a suitable equalization of the thrusts on the dimensions of the pier, let an example be inserted in addition to that previously treated.

Example II. In the hall church assumed in example I (Fig. 394) the middle pier of sandstone with an allowable cropression of 20 kil per q cm with a circular plan is to be made as small as possible, that the space may be least restricted. To equalize the thrusts of the vaults the cross arches of the side aisles are to be backed with masonry, and it is to be determined how heavy their backing is to be made and what cross section is to be given to the pier.

First is to be found the weight V_3 which is to be placed on one half the cross arch. It is assumed that the backing masonry will be so distributed that besides the sought vertical loading V_3 of the abutment, this shall exert a thrust $H_3 = \frac{V_3}{3}$ on the pier at the height of 1.20 m above the capital and thus 13.2 m above the bottom of the pier. The pier will be about the smallest when the resultant compression passes through the middle of the bottom area, and if such is tha case, then the equation of moments is to be established for this middle point.

$V_1 \times 0.35 + (H_2 \times 13.00) + (H_3 \times 13.2) = V_2 \times 0.35 + (V_2 \times 0.35) + (V_3 \times 0.35) + (H_1 \times 13.50)$.

According to the previous example are to be inserted:-- $V_1 = 10,260$; $V_2 = 6,840$; $H_1 = 3,240$; $H_2 = 2,160$, and $H_3 = \frac{V_3}{3}$.

$10,260 + (2160 \times 13.00) + 13.20\frac{V_3}{3} = (6840 \times 0.35) + (3240 \times 13.5) + (V_3$

By this the unknown quantity V_3 is found = 3,559 kil.

If the masonry backing be made of sandstone with unit weight of 2300 kil, then to obtain this load are required $\frac{3559}{2300} = 1.55$ cu. m. The entire cross arch will require twice this load or 3.16 cu. m of rubble backing. The area of the pier will now be sought. With the location of the pressure at the middle, 20 kil comes on each q cm so that the total weight divided by the area must = 20 kil.

The total weight consists of the loading and its own weight; the loading is $G_1 + V_1 + V_2 + V_3 = 11,500 + 10,260 + 6,840 + 3,559 = 32,159$, and the unknown weight of the pier = $\frac{D^2}{4} \times 3.1416 \times 12.00 \times 2300$.

Likewise the unknown sectional area = $\frac{D^2}{4} \times 3.1416$ or $\frac{D^2}{4} \times 3.1416 \times 10,000$ q cm.

Thus $(38,185 + \frac{D^2}{4} \times 3.1416 \times 10,000 = 20$.

Or $\frac{D^2}{4} \times 3.1416 \times 10,000 \times 20 - \frac{D^2}{4} \times 3.1416 \times 12.00 \times 2300 = 32,185$.

$D^2 = 0.238$, and $D = 0.49$, i.e., the pier requires only 0.49 m. in bottom diameter. In the upper part of the pier the compression is not exactly central, so that here in spite of the small loading there would be necessary a slight increase in size, w which can be taken into account by seeking the point of intersection at the height of the capital. But there also occur other considerations.

For piers of this slenderness (diameter scarcely 1/24 of the height) there must already be taken into account the danger by bending or buckling, and besides in view of accidental variations of the loading, and finally on account of the architectural

appearance, a greater diameter is held to be advisable, so that the diameter is to be incriesed to at least 70 cm.

Thus there occurs with central compression a pressure of 11 kil on each q cm. The foundation is to be lconsiderably extended from the pier, since it has to transfer about 50,000 kil including its own weight. If the soil can be loaded with safety to 2.5 kil per q cm, a ground area of $\frac{50,000}{2.50} = 20,000$ q cm or 2 q m are required, which would yet be somewhat increased on a not entirely suitable soil. Just where one builds boldly on the ground should he not neglect a good foundation, since by such neglect occur most injuries.

Interesting is a comparison of the size of the pier now obtained with that assumed in the first example for the same church. While the stress is equal in both, the square pier with 1.25 m side requires 18.7 cu. m of masonry, while the round pier 0.70 m diameter contains only 4.7 cu. m, so that there is a saving of about 14 cu. m of cut stone for each pier. There is certainly an addition to this of 3 cu. m of rubble for loading the cross arch, and further the external buttress must be somewhat deeper, since the backing of the cross arch increases the thrust. But this is trivial in comparison to the saving in size just where this is so strongly required.

One recognizes by this example how justifiable was the endeavor in the middle ages to arrange all forces as advantageously as possible, and he will further see how valuable for the execution is even a merely approximate (though sufficiently uncertain))determination of the statical conditions of the construction.

Basilica without the buttress system.

The course of the investigation to be established is the same, that has just been shown for the pier of the hall church, except that here besides the pier to be considered is also the p part of the clearstory wall carried on it. We shall first take no account of the roof and the wind pressure, taking the forces acting aside from these, when these particular stresses will be taken up later and the procedure be repeated.(See the following Chapter on roof load and wind load).

The forces are best combined graphically from above to below, in order first to obtain a clear view of the entire course of the stresses, and then is taken up the most dangerous cross s

section for closer examination, when for accuracy calculation can be employed, (See examples on p. 155, 157), as well as the explanations on p. 140.

As a rule there comes in question the cross section at the height of the springing of the vaultof the aisle (I in Fig. 401), then the bottom of the pier (II) and finally the bottom of the foundation (III).

By a skilful estimate of the masses in the clearstory wall, the pier of the vaults, for which the graphical method offers indications in a speaking way, one is able to lead the line of pressure within wide limits as may be advisable in each case. By backing with masonry under the side roofs, there may be obtained particularly favorable results. All the numerous refined and instructive experiments that the middle ages made in t this direction, can we appreciate graphically and thereby our less trained constructive feeling may be strengthened, and even elevated to the same height as that of the old masters.

When the middle aisle is not carried so high, then may be m made possible solutions very satisfactory statically without the certainly more perfect but always more costly system of t the flying buttresses.

Basilica with flying buttresses.

Forces at ends of the flying buttresses.

Like all other masonry arches the flying buttress exerts a thrust at each end, whose magnitude depends on the corresponding height of the point of support, as well as the weight, span and form of the arch. (See Figs. 402 to 405). If we wish to f find the line of support of the arch to test the suitability, the arch is then divided by vertical lines into slices (Fig. 402) and on the load line are laid off their weights in the usual way (see preceding p. 52). The line of pressure furnishes then the end forces, that are especially concerned. Otherwise there are also found approximately the thrust forces by t the frequently described and simplified methods by calculation or graphics (p. 130). The latter in which the direction of the end forces are assumed by estimation and their magnitudes are determined from the weight of the arlh is inserted in Figs. 403 to 405 for illustration.

Figs. 403 to 405 show how greatly varied are the effects of the flying buttress according to the form chosen. The lower

pressure B_2 is important for the depth of the buttress, and t the upper pressure B_1 has the problem to entirely or partly neutralize the thrust of the vault.

In the arch in Fig. 403 (S. Ouen at Rouen) the pressure B_1 is directed obliquely downward, so that besides the horizontal thrust H_1 a portion of the weight V_1 of the arch is thrown on the bearing.

The most commonly occurring arch of Fig. 404 is almost or quite horizontal above and accordingly exerts a horizontal pressure B_1.

The flat curved and steeply inclined arch of Fig. 405 (Halberstadt) exerts a considerable end force B_1 obliquely upward, i.e., it does not load the upper bearing, but even tends to lift it upward. In consequence this arch is suited to receive a part of the weight of the clearstory wall and to conduct it to the external buttress, thus relieving the middle pier.

For all this and yet other varied forms of arches the middle ages offer manifold examples, and which of them is to be chosen depends in each case on the effect desired. Usually at the upper end will be produced neither a loading nor a lifting effect, but only a horizontal thrust opposed to the thrust of the vault. In this case a quadrant is suitable (arch in Fig. 404).

In early Gothic the quadrant was already preferably replaced by half a pointed arch, thereby obtaining a statically arch c curve. Generally the centre of the pointed arch was placed but little beyond the middle line (point a in Fig. 404), so that the arch is nearly horizontal at top and also exerts a quite or nearly horizontal force. But if it be desired to receive a part of the load and to reduce the depth of the supporting buttress, they were employed very steep arches (for example at H Halberstadt, Regensburg and Notre Dame at Semur). Therefore if it be to design a flying buttress that exerts an entirely definite counter pressure, then must one start with the upper end, sketch approximately the desired line of pressure and then distribute the weight of the arch, so that the desired line of p pressure results from the graphical construction. There is assumed at first that the usually existing line of pressure with a quiet loading has the same or a somewhat flatter curve than the middle line of the arch.

Stiffness of the flying buttress.

An important requirement for every flying buttress is a sufficient degree of stiffness, i.e., the arch must not only be able to offer the ordinary counter pressure corresponding to its form, but in variations of loading it must also be able to transfer other and especially greater forcer without destruction. Such variations can first be produced by very considerable wind pressures on very high middle aisles.

In Fig. 402 are drawn two lines of pressure,,the more strongly curved one offering rather small counter forces (O N in the added force diagram of Fig. 402a), but the pressures of the f flatter line for the same weight have those much more important. (O′M′O′N′ in Fig. 402a). The flatter is the line of pressure, the greater become the end forces and naturally also the compression forces which the arch can receive. It results from this that in the arch, according to whether a small or great force is transferred to it, there is found a more curved or a straighter transfer of the pressure. The arch form must be so created, that it can safely contain all possible lines of pressure for the loadings that may occur.

The chief requirements for a good flying buttress may be sumgarized in this, that it is not too heavy, for generally only a moderate thrust is exerted, but in particular cases an important counter thrust may be offered.

If from this point of view the construction of the ancients is examined, one cannot be sufficiently surprised by the refined art with which they correctly understood all requirements, if certain types of arches are taken.

Figs. 402 and 404 show the most common forms, consisting of an arch with complete coping, the latter suited to receive flat lines of pressure. The coping must therefore by well jointed; men soon came to place special value on reliable construction of their roofing, that was particularly utilized for transmitting compression. The intermediate masonry could then be made lighter and even resolved into tracery.

Fig. 406 (Amiens) shows a resolution of this arfh into a lower curved and an upper straight cross arch. The lower curved part transfers the usually acting thrust, but on the contrary the upper straight cross arch has the task to receive any variable forces. Since each one of its cut blocks is supported

from beneath, the coping is always in equilibrium, whether a great or a small longitudinal force acts in it. It can be compared to a shore, whose stress may change from 0 to the border of buckling. If the stress be too great, then would occur buckling, which is prevented downward and it made difficult sidewise but is possible upward, yet where the weight of the cut b blocks oppose it.

Some also sought to prevent the upper coping from buckling upward by curving it somewhat downward, and brought it into direct connection with the lower arch as a counter arch by means of tracery (Fig. 407). The manifold shapes of flying buttresses are thus not alone designed for architectural effect or for a better water channel, but they first serve important structural purposes.

With high middle aisles men passed to two flying buttresses over each other, Partly as Viollet-le-Duc opines, to create a great base for the wind pressure distributed over a greater surface, and chiefly probable to give the clearstory wall greater stiffness against the important effect of the wind.(On the architectural treatment of the flying buttress see farther on under the development of the elevation of the church).

Line of pressure in the middle pier under the influence of the flying buttress.

If the static conditions of the middle pier of a basilica w with flying buttresses are investigated, there are at first o omitted the roof load and wind pressure, and only for the mass of the masonry with its vaults is made the examination in the frequently described manner by graphics or by calculation. Most favorable will be the course of the force when the middle line is always closely attached to the middle axis of the wall or of the pier. A careful estimate of the masses of the wall and pier as well as of the thrusts of the vault, but particularly the insertion of a correctly determined thrust of a flying buttress at the proper place will lead to the aim. Fig. 408 r represents a favorable transfer of force under ordinary conditions.

The thrust of the flying buttress is somewhat less than that of the vault of the middle aisle and is applied somewhat above that . The counter pressure of the flying buttress B_1 is combined at the point 1 with the weight P_1 of the upper part of the wall to the resultant force R1, that is turned toward the inside

of the wall, until at the point 2 the oblique thrust W_1 of the middle vault joins it. The thrust of the vault is greater than that of the flying buttress and therefore turns the resultant R_2 again towards the outside, and at the point 3 combines with the weight P_2 of the part of the clearstory wall concerned into the force R_3, which at the point 4 unites with the thrust W_2 of the vault of the side aisle. It is now of importance, w whether the difference between the horizontal thrust of the middle aisle and the flying buttress is greater than the horizontal thrust of the side aisle, or smaller. If the remaining thrust above were greater, then would the resultant R_4 tend to the outside, but if smaller as assumed in the Fig., then R_4 again tends to the inside and finally combines with the weight P_3 of the pier with the division arch to form the pressure R_5, which at the point U enters the foundation, and after receiving the weight P_4 of the foundation at last passes at the point E into the ground.

Just in the lower part of the pier, where the load has become greatest and the mass is usually most restricted, is desired a position of the pressure as nearly central as possible.

What changes occur by varied attachement of the flying buttress are explained by sketches in Figs. 409 and 410. In Fig. 409 the thrust of the flying buttress nearly equals the thrust of the vault of the middle aisle, and consequently the force R_2 passes almost vertically downward, in Fig. 410 the thrust of the flying buttress of the side aisle together are as great as that of the middle aisle, which leads to making the resultant R_4 vertical below. Farther in Fig. 409 the flying buttress is dropped far down with the result, that the resultant R_2 tends outward, while the high lying flying buttress in Figs. 408 and 410 force the point 2 of intersection back against the vault of the middle aisle.

Likewise as by the position and form of the flying buttress the line of pressure can be pushed in or out, the weight of t the different parts of the wall and its corbelling toward the interior or exterior, and further the weight and ratio of the rise of the vaults, may exert the greatest influence. There a are so many inexhaustible possibilities of leading the line of pressure, that even apparently very developed conditions by t the existence of galleries, triforiums and external passages

may be mostly mastered by correct judgment without difficulty. The flexibility and freedom of the mediaeval method of construction appears in no other place more strikingly.

6. Roof Load and Wind Pressure.

Permanent weight, snow load and wind pressure on roofs.

Since the roof load is exposed to great variations in consequence of wind and snow pressures, and since further it sometimes is lacking during repairs for a time, one cannot regard it is a favorable loading, but must first take into account t the stability of the building without regard to the weight of the roof and the wind pressure, then adding both.

Weight of the roof alone.

The weight of the roof is composed of the weights of the trusses, rafters, strips or sheathing and the covering.

The trusses without the rafters but with the roof beams belonging to the construction weigh for light construction 20 to 30 kil, for heavy trusses 30 to 50 kil, and the weight of iron trusses can be taken the same. If complete floors and temporary loads are to be expected, they are to be considered.

The rafters weigh per q m of inclined surface 10 to 15 kil, the strips 5 to 10 kil, and a sheathing of boards 2 1/2 cm thick 20 kil, or one of 3 1/2 cm thick 30 kil.

For one q m of covering material (without sheathing or strips), may be calculated as follows:-

Double tile roof, 75 to 100 kil or an average of 90 kil.
Single, pan or grooved tiles, 45 to 65, averaging 60 kil.
Slate roof, 30 to 45 kil, averaging 40 kil.
Metal roof, 8 to 16 kil, averaging 10 kil.

Accordingly the total weight of the roof construction with covering is:-

Covering.	Per q m of roof surface.			Per q. m of plan. Inclinat.		
Kind.	Kil.	To	Aver.	30°	45°	60°
Doub. til.	120 k	175 k	140 k	---	200 k	280 k
Sing. tile.	85	140	110	---	155	220
Slate	75	120	90	105	130	180
Metal	60	95	75	85	105	150

Snow load.

The snow load is computed per q m area of plan and is usually assumed at 60 to 75 kil. But the snow so seldom adheres to steep roofs, that this assumption requires justification there, so

that for roofs over 45° there is only 30 kil per q m of plan, and for roofs of nearly or over 60° no snow load is generally considered. On the contrary on very flat roofs, especially where snow drifts are to be expected, more is to be taken (perhaps 90 or 100 kil in northern Germany.

Wind pressure against a vertical surface.

The maximum wind pressure on a vertical surface is usually taken at 120 kil per q m in Germany. For example this pressure will have to be assumed for vertical walls, gables, and walls of towers.

For particularly exposed places, towers and gable walls, this value is to be increased, perhaps to 150 or 180 kil. C. W. Base by reference to definite cases warns against too low an assumption of wind pressure. Particularly may a gust of wind produce vibrations of a high gable wall, which lead to its overthrow.

Wind against roof surfaces.

The pressure against an inclined roof surface is smaller. T The wind that strikes against a q m of the surface of the roof (120 sin angle of inclination in kil) is resolved in one direction perpendicular to the roof, and in another parallel to it. The last component is considered as ineffective. The pressure perpendicular to the surface is alone considered and has the magnitude of 120 $\sin^2$ a, when a is the angle of inclination of the roof. For flat roofs instead of a is taken an angle of a+ 10°.

The roof must be constructed with sufficient strength to receive this wind pressure and to transfer it to the support. At present the wind only concerns us so far as it loads the support, and for this purpose it is advisable to resolve it again into two components, indeed into a wind load directed vertically downward and a horizontal wind thrust. These forces are calculated for different inclinations and are collected in the Table.

Vertical Wind Load and horizontal Wind Thrust for 1 q m of inclined surface of the roof in kil per q m.

Incl. a.	Both sup. Vertical wind load on supp.	Wind. sup.	Leew.supp.	Hor.Thrust.on both Supports.
Up to 20°	28 k.	20 k	8 k.	10 k.
30°	43	29	14	25
40°	544	31	23	45
45°	57	28.5	28.5	57

50°	58	23	35	69
55°	57	14	43	81
60°	53	0	53	92
70°	40	-45	85	110
80°	21	-152	173	118
90°	--	---	---	120

The distribution of the horiontal wind thrust to the supports cannot be stated in general, since it depends on the character of the construction.

For iron construction is usually fixed one support, the other being made movable (by rollers etc.); in this case the fixed support must alone receive the thrust for both directions of the wind, while the free one at most can receive a portion corresponding to the resistance of friction. For firmly anchored roofs it may be assumed for greater safety, that the wind thrust is in the most unfavorable way received by the left or the right support. Otherwise one would also not err for similarly fastened and moderately steep roofs in dividing the horizontal wind thrust between the supports in the same proportion as the vertical wind load.

In the distribution of the vertical wind load to the supports as given in the Table is assumed a gable roof; if this is flat the load on the windward support predominates; at 45° both supports have an equal share, and then the leeward one has more, until for more than 60° of inclination, the windward support is relieved and the roof must be prevented from overturning by its weight or by anchoring.

For shed roofs both supports receive equal vertical wind loads if the higher end lies directly above on the plate. On the contrary if the upper end is supported by intermediate construction, so that both supports are at the same height, then already at 45° of inclination the windward support has no longer to receive any vertical wind load, and with a greater inclination an overturning moment is formed under the wind pressure, opposed to which is the moment of stability of the weight of the roof.

Example. The middle aisle of a basilica with 12 m width and 7 m length of bay is covered by a slate roof of 50° inclination. The loads and thrusts of the roof on the side aisle piers are to be determined with and without wind.

Preliminary soon pier supports only the permanent weight of
a surface area of 6.4 × 7 = about 33 q m. The weight of the
7 × 3 × 30 = 1890 kil.

The wind is taken a perpendicular pressure of each q m of
exposed surface of 30 to 35 kil (see Table), and thus the wind-
ward pier receives 64 × 28 = about 1820 kil, and the leeward
pier receives 48 × 28 = 2810 kil. The maximum roof load with
snow and the strongest wind (that moreover can scarcely occur
together) would then be for the leeward pier of the middle si-
ngle increase to 6640 + 1890 + 2810 = 9810 kil, while the wind-
ward pier might receive 5040 + 1890 + 1820 = 8750 kil.

More important is the horizontal wind thrust, and according
ded, so that only about 5500 kil needs to be taken for one sin-
tude is not entirely unimportant at this height, and it depen-
am for it.

If we consider the wind pressure obliquely to roof of a hall
encounter one of a single pitch, then the rise raised at the w
minished since will partly neutralize the thrust of the vault,
also produce the load on the supports, but at the leeward sup-
ver more thrust is added to the thrust of the vault, and there-
fore the leeward pier exerts more thrust, this is to be taken into accor-
ers an determination and thickness of the wall. Method
proposed by Schinkel.

When the middle pier of a hall cruciform vault
does not support any roof load, then it will not be neutrali-
ly affected by the wind thrust. See it a sum of the roof reac-
on the middle pier, it depends entirely on the mode of constru-
ction how that distribution is received the wind roof
loads. If it be known by static investigations (according to
the earlier examples on p. 194 and 195), that the middle pier
receives the wind thrust sometimes from the right and sometimes
from the left and cannot remain without to overturning

Ordinarily each pier supports only the permanent weight of the roof over half a bay; with 9.4 m inclined length this has a surface area of 9.4 × 7 = about 66 q m. The weight of the framework and covering of the roof is 90 kil per q m (p.162); then on a pier rests a permanent load = 66 × 90 = 5940 kil.

If a snow load of 30 kil per q m of plan be added, this = 7 × 6 × 30 = 1260 kil.

The wind causes a perpendicular pressure on each q m of exposed surface of 23 to 25 kil (see Table), and thus the windward pier receives 66 × 23 = about 1520 kil, and the leeward pier receives 66 × 35 = 2310 kil. The maximum roof load with snow and the strongest wind (that moreover can scarcely occur together) would then be for the leeward pier of the middle aisle increase to 5940 + 1260 + 2310 = 9510 kil, while the windward pier might receive 5940 + 1260 + 1520 = 8720 kil.

More important is the horizontal wind thrust, and according to the Table it amounts in this case to 69 × 66 = 4554 kil. E Even if it be assumed that this thrust is almost equally divided, so that only about 2500 kil needs to be taken for one side, this is not entirely unimportant at this height, and it deserves consideration in the static investigation of the pier, unless as hereafter shown, care is taken to add a buttress system for it.

If it concerns the wind pressure against the roof of a hall churchor one of a single aisle, then the wind thrust at the w windward side will partly equilibrate the thrust of the vault, thus reducing the load on the support, but at the leeward side the wind thrust is added to the thrust of the vault, and therefore for large and steep roofs, this is to be taken into account in determining the buttress or thickness of the wall, which presents no difficulty.

When the middle pier of a hall church or two aisled church does not support any roof load, then it will not be essentially affected by the wind thrust. But if a part of the roof rests on the middle pier, it depends entirely on the mode of construction how this participates in receiving the wind and roof loads. If it is shown by static investigations (according to the earlier examples on p. 154 and 157), that the middle pier receives the wind thrust sometimes from the right and sometimes from the left and cannot resist without an undesirable increase

in dimensions, it is very advisable to build a stiffening of masonry over the cross arch in a transverse direction between the walls, with this transfer of the wind thrust to the external walls.

Pressure of the wind against the walls of the basilica.

Very powerful is the pressure of the wind against high wall surfaces, for single-aisled or hall churches it is usual tn build the external walls with their buttresses so strong that the side struck by the wind can receive the pressure itself. Only for very great heights will one have to take in consideration, to transfer part of the wind pressure above the vaults to the other external wall and can join it to the thrust of t the vault.

Wind pressure and vault thrust

But for the clearstory walls of the basilica, that are to be supported on as slender piers as possible, the mastery of the wind pressure belongs to the most important questions, and as will be shown later, it may even become more important than that of the thrust of the vault, it is striking that the importance of the buttress system for the movements of the wind has received so little consideration before.

The clearstory wall of the great cathedral rises 15 to 20 m or more above the side aisle. With 7 m width of bay and 20 m free height, for example there would be presented to the wind 140 q m of surface in each bay, which would receive a pressure of 140 × 120 = about 17,000 kil, aside from the wind thrust of the side roof, which perhaps also exerts 2000 kil on the clearstory wall and the thrust on the middle roof, which with the magnitude of 5,000 to 8,000 kil carries a greater or smaller part to the wall considered. Therefore the surface of each bay of such a basilica receives a wind thrust of 20,000 to 25,000 kil, and even a greater amount is calculated for the cathedral of Cologne. But nov a vault over the middle aisle of 70 to 100 q m area in plan (7 m length of bay and 10 to 14 m span) with compartments vaulted one brick thick or with a like weight of natural stone, according to Table I (p. 135) only exerts a thrust of 15,000 to 20,000 kil on each wall. The vaults must therefore be knite heavy to produce a thrust that equals the maximum wind thrust to be expected.

For the mastery of the wind thrust are

For the mastery of the wind thrust are two possibilities, e either the stability of the clearstory wall concerned or of its piers suffices to receive the thrust, or the wind thrust must be entirely or partly transferred in or over the vaults to the other wall.

Wind pressure on the basilica without flying buttresses.

The first case, the reception of the wind by the wall concerned itself must occur in a basilica without flying buttresses, since a transfer to the other wall would increase the thrusts of the vault, to oppose which in the basilica without flying buttresses would already cause great difficulties. The leeward wall would already be stressed sufficiently more if it could safely receive the part of the wind thrust of the roof falling to it.

If in a basilica of medium size and without flying buttresses each middle pier receives a vertical load of 300,000 kil, and the entire wind pressure against the clearstory wall with its roof is calculated at 10,000 kil with an average height of the application at 16 m above the base of the pier, then this wind pressure would notably move inward the line of support in the pier at bottom, indeed by a distance x, which is very simply calculated by the equation of moments:-

300,000 x = 10,000 × 16.00, so that x = 0.53 m.

Accordingly by the wind is to be expected a movement of the pressure of about 53 cm at the bottom. If this movement occurs within the limits of the kern, then can the pier ordinarily , i.e. without wind, can receive the pressure in the outer angle of the kern and have dimensions for a rectangular plan of 3 × 0.53 = 1.59 m or of 4 × 0.53 = 2.12 m for a circular plan.

Then would the edge pressures already be quite important, so that for weaker material these dimensions would have to be still increased. (Without wind pressure by balancing the masses the pressure to be conducted through the middle of the pier, and thus would remain within lower limits, and also the dimensions of the pier could be made correspondingly less.

It is evident that for moderately high basilicas with low middle aisle in all cases the reception of the wind by the wall struck by it is yet possible; but when in the 12 th and 13 th centuries the proportion of height was considerably increased without increasing the loading mass of the wall, or it

was even sought to lessen the latter as much as possible, then the middle wall could no longer satisfy this requirement, and otherwise in consequence of the vibrations of the wind the lower dimensions of the pire must receive an enormous increase, that men sought to reduce first of all.

Wind pressure on the basilica with flying buttresses.

For example of a basicila with a load of 300,000 kil on the pier receives a wind thrust of 20,000 kil applied at 25 m above the floor, then would the wind produce a change in the line of pressure of $x = \frac{20{,}000 \times 25.00}{300{,}000} = 1.6/$ m. But in such a basilica one could not make the entire pier so thick. For this example based on only average proportions it is evident, that the middle pier of a very high basilica could receive only a very small part of the wind pressure, that the principal portion must be transferred in or above the vaults to the other side and must be received there in a suitable manner. But since the wall can even less receive in itself, the addition of the flying buttress becomes a necessity. There may be established here the statement, that the introduction of the flying buttress was required just as much by the wind thrust as by the vault thrust. Only from this point of view can one understand fully and entirely the construction of the ancients, and first in it is recognized the purpose of the flying buttresses set doubled over each other, the upper one being usually applied much higher than is required by the wind thrust.

To illustrate the effect of the wind thrust it is drawn in Fig. 411 (cross section of the minster at Strasburg), the position of the lines of pressure with and without wind pressure, the first being dotted and the last in broken lines. The course of the action of the wind from the left is as follows.

1. The wind exerts on both walls a thrust directed to the right. 2. In the clearstory wall struck by the wind and its p piers the line of pressure is moved somewhat to the right. 3. The left clearstory wall then leans somewhat to the right. 4. In consequence of this leaning of the clearstory wall the flying buttress at the left side is somewhat relieved (line of pressure is more curved). 5. By this leaning the left clearstory w wall lies against the vault of the middle aisle and causes a greater transverse stress in this, which is transferred in flatter lines of pressure through the cross arch and the upper

part of the compartment. By the greater pressure of the vault the right clearstory wall is somewhat inclined, thereby moving the line of pressure in it and in the pier beneath it somewhat to the right. 7. The flying buttress at the right side receives greater stresses by the leaning of the walls, which produce straighter lines of pressure. 8. In the external buttress at the right the line of pressure is moved to the right in consequence of the greater thrust of the flying buttress.

One must regard the entire system as movable, in spite of t the rigidity of the materials there exist little elastic movements, even if they only measure mm, which cause no injury in the joints of the masonry within these limits. The weaker parts are first moved somewhat and the stronger parts less; if a stronger and a weaker part of the construction have to participate in this work under similar conditions, the stronger will accordingly take the greater part of the work upon themselves.

For example if the middle pier below is very slender or is placed on a ball joint, then the middle wall would swing with the least excess of thrust from right to left, and would lean against the vault or the flying buttress, transferring there the entire thrust without carrying any downward. Conversely if the middle pier were very strong and the upper pressure were very weak or entirely absent, then would the pier carry the greater part or the entire thrust to the ground. Accordingly within very wide limits were men able by skilful arrangement of the construction ., to cause either the middle pier or on the other side the external buttress with its entire system of transverse pressure, to carry more of the thrust.

But they would make the middle pier as thin as possible, and therefore they transferred to it as much as possible only vertical loads of moderate size, and on the contrary must the thrusts and especially the varying side movements be kept from it as far as possible, and for these entered a buttress system so much stronger.

But the resistance of the buttress system is produced less by piling up masses, than by their correct arrangement. Already it is shown by the flying buttress that its stiffness can be obtained by a corresponding form, and that it may otherwise be quite easily obtained.

Transfer of wind pressure through the vault.

An important limit in the chain of transverse stiffening is f formed by the vaults of the middle aisle, to which falls the task to transfer the varying thrusts, and it is advisable to briefly consider the vault on the ground of its function. That the vault may transfer a greater thrust, or the same thing, c can receive a greater sidewise stress, there must be formed w within it flatter lines of pressure than usual, for otherwise under the greater pressure the crown would be raised and it would eventually be destroyed, and for this reason a light tunnel vault would be little suitable for transverse stiffening, while an equally light cross vault might be able to undertake this by the peculiarity of its form.

The vault can generally maintain itself in equilibrium only if the external forces, that act on it sidewise are exactly as great as the thrusts exerted by the vault outside, since in general a condition of rest can only be conceived, when force and counter force everywhere neutralize each other. If the stressing force is too great, the vault is forced upward, but if it is too small, then would the vault press downward. Usually the thrust of the vault is neutralized by the opposed counter pressure of the abutment, which is to be regarded as the stressing force for the vault. If the wind pressure is added at one side, this joins the opposing pressure of the abutment to form a greater counter pressure, that must be opposed directly by a greater thrust of the vault, if equilibrium is to be preserved. A vault can with a permanent loading exert a greater thrust only if flatter lines of pressure be formed within it. But this greater thrust not only acts at the windward side, but also at the leeward side, where it must be opposed by the counter pressure of the abutment mass, and indeed mostly by a flying buttress.

In unfavorable conditions the thrust received by this flying buttress on the leeward side is increased to the sum of the ordinary vault thrust, and the wind pressure against the clearstory wall with the entire wind pressure against the surface of the roof. But it will usually be stressed less, since the middle pier takes a part of the stress on itself.

As previously stated, a tunnel vault is little suited to transfer the wind thrust, since in it the lines of pressure have little play, and then the vault must be very thick, have a ba-

backing, and moreover being so heavy and having such great thrust, that the wind pressure is relatively small in comparison to the vault thrust itself. Quite otherwise is it for the cross vault, even if its compartments are very thin, and the cross arch must have a deeper cross section in which flatter lines of pressure are possible; but this is not the only way, the cross section of a cross vault is horizontal on the middle axis or is always rather flat in a raised vault, and in this upper part of the cross vault can be formed flatter lines of pressure, here is a transverse stress on transverse shoring is possible, if one prefers, as indicated by lines in the cross cross section of Fig. 411 and in plan in Fig. 412. What the cross arch with its backing cannot undertake must the ridge of the vault take on itself. The wind pressure transferred by the ridge of the vault acts at c d on the opposite side on the wall and seeks to bend it outward, which must be opposed by by its stiffness, it acts like a horizontal arch in plan and transfers the pressure to the supporting points e and f. Here must be attached the flying buttress to receive this pressure. Therefore the problem is solved, wherefore very many flying buttresses stop close beneath the eaves of the roof. Naturally must a flying buttress be applied far higher and not be too h heavy, so that it may not ordinarily press the wall too strongly inward. Since it is also concerned to receive the vault thrust acting lower, the flying buttress must join it with a high vertical surface (Figs. 403, 405). If this surface be too high and the arch be undesirably heavy, it is better to employ two in its place, a higher one that chiefly serves to receive the warying wind thrust at the ridge of the vault according to Fig. 412, that strongly tends to bend the leeward wall e f, and therefore this is not to be too thin over the windows; in old examples it has been skilfully made thicker by corbelled arches outside and inside, and made more resistant by heavy gables.

Stiffening by cross arches.

This stressing of the wall may be entirely or partially avoided, if the cross arch is made sufficiently stiff and is suitable to thereby transfer the wind thrust instead of the ridge of the vault, as the section in Fig. 413 indicates at the right and left in two places. It must now transfer the pressure from the wall struck by

the wall struck by the wind to the points a and b, which is h here easily possible, since this transfer is distributed over the entire height and besides the pressure here opposed in direction to the vault thrust is less injurious. Thereby the variations would be kebt far from the middle part of the vault and further would receive in the plane of the cross arch a strong closed buttress system, the one great arch as if extending from the ground at the left through buttress, flying buttress and stiff cross arch over to the right even to the base of the buttress. Whether stiffening the cross arch in the perforated manner of Fig. 413 as executed in the historical examples is unknown at this time, a good backing of the spandrels and of the lower springing of the cross arch is also executed in the same little shafts. --- One can frequently observe, that the ancients had arranged at proper places loadings of cross arch, which modern times have removed from ignorance.

It should also be mentioned, that the roof beams could take an essential part in the transfer of the wind thrust and in the entire transverse stiffening, and in many cases did in fact. At least they are suited to transfer to the leeward wall and the flying buttress there the entire wind thrust of the roof as soon as the windward wall inclines only the least; this is then possible even when they are not firmly fastened to the wall but merely placed against it (By the friction on the pearing surface).

Bending of clearstory wall.

In closing, another effect of the wind on the clearstory wall is to be emphasized. A high wall will tend to bend in the part between the side aisle and the vault of the middle aisle in an entirely similar way as a board standing vertically bends under a side pressure. Fig. 414 I represents the scheme of the d distribution of the stresses in the cross section of the wall pier.

The magnitude of the stresses is computed by seeking the bending moment in the same way as for a loaded beam with a single difference, that the latter lies horizontally while the wall pier stands vertically. In the same case the bottom end of the pier is to be regarded as one support (regarded as fixed) and the upper end of the wall at the vault as the other support. The maximum bending moment will be expected at a certain height

above the roof of the side aisle, but nothing will be added c concerning the simple mode of seeking it, as this reference to this effect of the wind and the final conclusions to be deduced from them may suffice.

To the tensile and compressile stresses of the cross section represented in Fig. 414 I are joined those of the wall and pier pressure D produced by their upper loads (Fig. 414 II). If D passes straight through the centre of gravity of the cross section, it produced uniformly distributed compression. The stresses in I and II are added together algebraically, so that the total stresses in the cross section are shown by Fig. 414 III. At the outer edge are added the compressions of I and II, and at the inner edge is the difference of tension and compression. If the tension is here greatest, an excess of tension would remain as shown in the Fig. The last is avoided and the entire distribution of the stresses is more uniform, when the compression D is not applied at the middle but nearer the inner edge as illustrated by the corresponding representation of the stresses in Fig. 415.

Thereby the wind pressure against high clearstory walls is made desirable, to keep the line of pressure in the upper half of the wall nearer the inside, but for the lower part of the pier it is better for similar reasons to keep the compression far from the inside; therefore the course of the compression as in Fig. 410 is designated as favorable. But according to t the statements on p. 161 this can be attained by a flying buttress attached not too low and not pressing too strongly. Two flying buttresses above each other again act better and will further hold the top of the wall more safely, so that it assumes more the properties of the end of a firmly fixed beam.

Likewise this bending effect of the wind on the clearstory wall is not to be underestimated, for it is so important in t the great cathedrals, that the cross section of the wall or t the wall pier is so correctly dimensioned as to receive it with sufficient safety. That the powerful wind pressure against the great surfaces of the windows likewise demands great consideration, and is also found in the mode of construction, is only incidentally mentioned here.

If the restricted space did not forbid, we should gladly follow the effect of the wind on the buttress system farther, the

more because within our knowledge nowhere else is sufficient consideration given to it. In any case this consideration only serves to enhance the high esteem of the old masters; the more one penetrates into the details of their construction, the more he learns to wonder. --- Our modern timer have brought forth nothing in the domain of masonry construction, which in boldness of thought ayd in grandeur of structural conception can be approximately compared to those works of the ancients.

1. Construction of Vases.

III. PIERS, COLUMNS AND CORBELLINGS.

1. Construction of Piers.

The function of the pier consists in transferring the load of the arches and vaults to the foundation.

This foundation is formally expressed in a twofold way, and accordingly is determined the difference between the compound pier and the single column.

The pier transfers the multiplicity of the springings of vaults to the simple square of the foundation, when it gathers the different more or less richly moulded arches by means of the capitals into certain groups forming the members of the pier, the latter then expressing a later simplification and extension by the base, and thus resting on the foundation generally appearing in its original or modified form.

On the contrary the column is the simple support without tendency. Its shaft forms an insertion between the foundation and capital, it carries in a certain sense the foundation up to t the weight of the springings of the vaults, where it repeats these in a simple or modified form in the abacus of the capital.

Both modes of treatment find their origins in antique art, from which come the Romanesque, developed them according to t their needs and presented them to the Gothic.

If the support had to serve a relatively simple purpose, if it had to bear a beam ceiling, an arcade or even the similar vaults of a room like a hall, then its entire development was made quite simple and regular. On the contrary if combined requirements existed, were they vaults of adjacent aisles of different spans and separated by broad dividing arches, perhaps also even of different heights and supported by the pier, then was there opportunity for inexhaustibly varied solutions, which the middle ages places before one's eyes in ever novel transformations from the Romanesque period downward.

The form of the compound pier already in the later time of Romanesque art and in the transition style to a superior richness of development, which the Gothic art of the 13 th century found something to add, only because the vaulting system and therefore also the plan of the springings of the vaults had assumed a more complex form.

The subdivided angular pier.

Projections from the pier.

The Romanesque pier first consists of a square nucleus, that has to support the dividing arches between the aisles, and then the various projections that stand beneath the cross and diagonal arches of the vaults of the middle and side aisles. (Figs. 416, 417). When the depth of the dividing arches became so great that they had to be composed of two concentric rings, and therefore their cross sections assumed the form shown in Fig. 418, the plan of the nucleus must also have a more complicated shape, where it either follows the profile of the dividing arch, or receives the addition of a projecting half column, which by its capital passed into the square form a b c d of F Fig. 419. The projections toward the aisles could then best h have the same plan, while the diagonal rested on the square angles and the cross arches came on the half columns, so that the plan of the pier assumed the form of a Greek cross with p projecting half columns before the ends of the arms (Fig. 419, left). The next addition resulted as soon as instead of the groin arches actually projecting diagonal arches were admitted, that on the one hand had no proper bearing on the angles of t the arms of the cross, and on the other hand the cross arches seemed to demand corresponding projecting columns. Accordingly there resulted the form shown in the right half of Fig. 419. If the projecting half columns were originally semicircular, then since consistency thus required that under the diagonal arches there must be quadrants, which could form no sufficient bearing. Forst must the latter have a larger portion of the circle, and then also those projections from the surfaces must exceed the semicircle. The separation from the nucleus thus obtained, the independent effect of these parts removed them from the character of half columns, and therefore from now onward even this name became unsuitable.

Bonded and attached shafts.

In the language of the mediaeval stonemasons these little c columns received the name of bowtell (dienst) and this expression is recommended by just its definiteness and exclusiveness. The bowtells or shafts are either fully bonded with the nucleus of the pier, so that the separate courses of cut stone extend through the entire form of the plan, or they are attached to the nucleus of the pier, so that they are connected to it only by the capital and base, or sometimes of greater height are

also joined to it by the so-called bands.

The independent shafts consist of tall cut stones set on end and not on bed, which surround the nucleus either free, so that spaces remain, or are in contact. The bands form concentric m mouldings, that are bonded in the nucleus, and thus retain the attached pieces in their positions as in Fig. 420. But sometimes they are not expressed at all and then form only a lower bonded course. The latter arrangement is chiefly peculiar to the developed works of Gothic art, while the former is common in the works of the transition style, yet also reappears in certain early Gothic works.

First is the material on which depends the choice between the two methods of construction, of the banded or independent shafts; for the stone must be of the most uniform and fine grain to permit its being set on edge. But even the same material in different works in the same city and only separated by a brief time, as for example the churches of Notre Dame and of S. Benigne in Dijon, we find in the first independent and in the last bonded shafts, and both modes of construction have been retained for almost six centuries. In certain German churches, as in Wetter, are found both methods of construction connected together; while elsewhere in Germany the bonding of the shafts is the general arrangement. Experience also seems equally favorable to both.

Which of the two solutions stands higher esthetically is indeed only to be decided in each case, and it is to be added, that in some remaining examples the pier was colored and then over the nucleus extended white beds, but the shafts were left free and thus represented monoliths in a certain way, as for example in the former painting of S. Elisabeth's church in Marburg and in the church in Wetter. The most extended and spirited treatment of the construction of attached shafts is found in the Article Construction by Viollet-le-Duc.

Multiplication of shafts.

The form shown in the right half of Fig. 419 is found almost unchanged in certain Gothic works, as on the crossing piers of the church at Haina and of S. Blasien in Mühlhausen, where the plan of the nucleus becomes that of a regular Greek cross, so that the projecting shafts suggest the dividing arches, and t arms of the cross with the the shafts standing in the angles of the crossbear the diagonal

arches (Fig. 421,left). Accordingly there still remains visible of the nucleus of the pier the angles under the upper ring of the dividing arches. But men soon began to extend the adopted principle also to these angles of the pier, i.e., to open these by a rectangular cut and to place in the latter a shaft in the same way, as a similar one was already found in the angles between the arms of the cross, and thus they passed to t the form represented in the right half of Fig. 421. In a certain respect the last is also found already in those Romanesque forms of piers, where the rectangular angles were divided by inserted shafts. But while these little columns beneath the c capital of the pier and above the base of the pier passed into the right angle, this transition is omitted on Gothic piers a and each separate shaft has its own capital, its separate base, and both parts entirely enclose the nucleus, yet its edges are still visible between the shafts.

An ordinary aisle pier treated according to this principle would therefore consist of 12 shafts, one of which supported each cross rib, as well as three supported each dividing arch. Since in early Gothic works the cross arch is lower than the diagonal rib, and also the lower ring of the dividing arches freely projects and is likewise larger than the portion of t the upper ring visible over it, these shafts support also the arches mentioned, and thus the shafts in the axes of the plan of the piermust be larger than the others. Now to make the entire form entirely regular, the larger and smaller rounds could be assumed equal, although such regularity is not based on the nature of the thing. According to this system are treated the main piers in the collegiate church in Mantes.

Treatment varied in both directions.

The number of shafts always increased according to the number of the arches. Thus in the nave of the cathedral at Rouen, the division arches consist of three rings, so that a b in Fig. 422 represents half of a cross section of a dividing arch, which therefore requires five shafts on each side, and the entire pier is composed of 16 shafts. While they stand beneath the dividing arches in the ordinary way just as in Fig. 420, they are connected with the nucleus and appear to indicate an entirely regular form of the whole, the arrangement is according to that of the aisles, yet the shafts bearing the vault ribs become ey-

entirely different. The reason may be sought in that it was unsuited for the springing of the vault to project the cross rib so far. To avoid this projection lay nearest the right angle of the nucleus, that in a certain sense determined the position of the shaft, was omitted and then was given a prominence to the shafts suited to the need. Since by this was already g given up the regular form of the plan of the pier, there was no further reason in regard to the diameter of the shaft for requiring a uniformity not based on the nature of the matter. Thus in Fig. 422 the shafts supporteng the lower ring of the dividing arch, that also came to lie on the longitudinal axis of the piers, is greater than that lying on the transverse axis and supporting the cross rib, so that the entire plan of t the pier received an indeed symmetrical, tough no longer concentric but extended form in the direction of the length. More decidedly prominent appears the proportion of length in the piers of the Cross church in Breslau.

That in general the nature of the matter did not afford concentric symmetry is most clearly evident by the cases in which, where by retaining the regular plan a special arrangement of the forms of arches was allowed, which then led to an irregular section of the dividing arches. As examples we mention the nave cross of the minster at Strasburg aod at Freiberg -i-B; Fig. 423 shows the plan of the former. Men started from the prinpiple that the dimensions of the vault ribs depended in the span of the vault, so therefore the cross arches and diagonal ribs of the middle aisle must be larger than those of the vaults of the side aisles. The piers themselves consist of eight large shafts standing in the axes of the diagonals of the plan of the pier and eight smaller shafts between them. While now the cross ribs of the vaults of the side aisles rest on one of the larger shafts, according to the preceding principle those of the middle aisle require three, thus a larger and two smaller ones. According to this while the cross ribs of the side aisle stands on a smaller shaft, those of the middle aisle are separated by those larger ones standing in the diagonal, and could also again have the proper proportion of dimensions. According to this there also remain four shafts for each dividing arch, which therefore prescribes for them the form seen in Fig. 423, so that their cross section toward the side aisle consists of

three, and toward the middle aisle of two rectangular corners. The same proportion is found in the minster of Freiburg.

In the enclosure of the middle square of the cross and further in the piers supporting the inner angles of the towers is found the converse proportion, while here the equality of four dividing arches resting on the pier and four diagonal ribs also leads to a regular plan of the pier.

Treatment of the nucleus between the shafts.

The form of Fig. 422 also shows many partly very characteristic peculiarities. Thus the shafts are partly joined to the nucleus of the shaft by a neck, i.e., by a part bounded by straight lines and recessed at the back. This arrangement has t the advantage that the effect of the shaft becomes bolder anf by the freer position results a large space for the development of the capitals. A richer form is further given thereby, that between the shafts the still visible surfaces have moulded chamfers, which die beneath the capitals of the shafts and at the height of the base of the shaft pass into the square by a stop chamfer. However in this form are also expressed the stepped form of the nucleus by the square angles. But how the latter were left in regard to the shafts next the aisles has already been shown above. They stop entirely to be perceptible to any one, as soon as the right angle is rounded at d, and thus the shafts are connected together only by hollows, as for example those opposite the same piers and forminm the entrance to the shapels arranged between the buttresses exhibit shafts. In the example mentioned the two forms are indeed separated by a century. But they stand still abruptly and at intervals of perhaps scarcely a decade opposite each other in the aisle piers od Strasburg and Freiburg. While as evident from Fig. 423, the first exactly follows the old system, in the latter the otherwisi similar in number and style rounds are directly connected by hollows as given in the upper quarter in Fig. 423 by dotted lines. Men appreciated little the nature of the matter for such rich piers, and only from a distant similarity introduced clustered piers with an expression brought from a foreign object.

The development of the middle and later periods belongs to freer forms of piers, of which Figs. 437 to 440 afford examples, and we shall describe them later.

The round pier and its members.

Round pier without little shafts.

We have sought above to designate the nature of the single round pier, that reproduces the form of the foundation, which appears in the abacus, rises to the height of the springings of the arches and in a certain way must form an intermediate between the two. Whether the entire arrangement of the Gothic round pier originated from the Romanesque column, or the growing Gothic art gathered again the many members of the Romanesque pier into a unity, to attempt ti divide it in a new way, is a question unimportant to our purpose. On early Gothic works in Germany is found but seldom the simple round pier, but therefore the more common is it in France, and there occurs from the end of the 12 th to the middle of the 13 th centuries both exclusively in entire rows, for example in Notre Dame at Paris and at Dijon, as alternating with compound piers in the arrangement of the hexapartite vaults,so that the compound piers receive the cross and diagonal ribs, the round piers bearing the bisecting rips.

According to what was said above, the square form of the abacus of the capital is nearest, at the same time also best corresponding to the original shape of the cut stone. In such wise terminate the round piers of the cathedrals of Paris and Laon in square and slightly rounded capitals. The great projection obtained by such capitals, especially when seen diagonally, c contributes no little to their stately and solemn appearance, affording opportunity for the arrangement of rich and bold ornament. To the much subdivided form of the springing of the arlhes least corresponds the simple square form. Therefore it must yield to other basal forms, first to the regular octagon, when the principal of the compound pier passed to the round pier, and the form of the springing of the arches determined that of the capital of the pier. Yet any regular form of the abacus introduced everywhere difficulties and restrictions, w where the direction of the dividing arches deviated from the simple straight line or those intersecting at right angles, as for example is the case for avery polygonal choir enclosed by an aisle (Fig. 424). The French works indeed now show frequently solutions of these difficulties only effected by a modification of the basal form of the abacus, to which we shall later

return, yet the simplest and nearest means might be found in this, that the pier being connected with one or more shafts according to the plan of the Springing of the arches, abandons the regular ground form and assumes one corresponding to its function.

Round pier with little shafts.

An example of this kind with greater consistency is shown by the round pier in the polygonal choir of the cathedral at Meaux. (Fig. 425). It supports on an irregular capital of octagonal form adapted to the decagonal shape of the choir the dividing arches, the cross and diagonal ribs of the vaults of the choir aisle and the round a, on which rests the side arch of the choir vault. On the contrary the diagonal ribs of this vault are borne by the rounds b, that extend from the ground and are connected with the cylindrical pier by flat hollows in which intersect the capital beneath the dividing arch, cutting as shown by the perspective view of Fig. 425 a.

But also for the piers separating the aisles, the peculiarities of the arch system have led to similar forms of piers entirely differing from the square basal form. Such an example is offered by the collegiate church at Mantes, whose vaults a are hexapartite. Fig. 426 exhibits the basal forms of the weaker pier from which spring the bisecting ribs. Here the cylindrical nucleus bears only the dividing arches, yet at the side toward the aisle is connected with three rounds, so that the side next the side aisle bears its cross and diagonal ribs, b but that toward the middle aisle supports the bisecting rib a and the side arch. A like arrangement is shown by the piers of the almost still Romanesque cathedral at Besancon, thoroughly changed by later alterations. Here also extend over the middle aisle the usual oblong cross vaults, so that the rounds toward the middle aisle have the same function as those of the side aisle. The ground form differs from that of the pier at Mantes only in that instead of the rounds being connected by flats, they are joined to the cylindrical nucleus by hollows. Undeniable are the clearness and consistency of such an arrangement. This could be transformed in a very suitable manner, according to the arrangement found in the minster at Strasburg of larger ribs in the vault of the middle aisle, if to the side of the nearly cylindrical nucleus toward the middle aisle were added

three rounds, with only one on that toward the side aisle, from the latter spring the cross and diagonal ribs of the side aisle, while the larger ribs of the middle aisle would each be borne by a separate round, or if toward the side aisle are found th- three with five rounds next the middle aisle.

But just as well as for the nave piers is the plan of the s springings of the vaults harmonized with a regular placing of the rounds in the most varied manner.

Round pier with four similar rounds.

Thus is formed a pier continuing through the earliest and middle periods of Gothic art, occurring just on the noblest w works that the combinations of the cylindrical nucleus with four rounds set on the axes of the ground form. However simple is this form in itself, the most trifling modifications in regard to the proportions of the diameters of the rounds and that of the cylindrical nucleus, producing so varied an effect. On the older works, as in the Hessian churches at Marburg, Haina and Wetter, then in the French cathedrals of Rheims, Amiens a and Dijon, the rounds are relatively larger, their centres project more strongly, their bodies are usually separated from t the nucleus by a flat band, and the effect is correspondingly vivid and bold. In the churches at Friedberg and Frankenberg, in the nave of the church at Wetzlar, their diameters already diminish and the centres are nearer the circumference of the pier; still greater is the reduction in the church of S. Stephen at Mayence, whereby in spite of the great diameters of the rounds is produced a feeble and less tasteful effect. This defect is even increased by the slight projection of the base, the small importance of the abacus of the capital, the small scale of the foliage on the same, in brief the difference of the latter from those mentioned before is very striking in spite of the similar basal form. Much varied is further the relation in which a pier of this form stands to the springing of the arches.

According to the older system the rounds toward the aisle bear only the cross arches, thosestanding on the longitudinal axis support the lower rings of the dividing arches and the nucleus carries the upper rings of the latter as well as the diagonal ribs. This arrangement is first found in connection with the arrangement of aisles of equal height, in all the

Hessian churches mentioned and the older Westphalian. In modern times this arrangement has been frequently and incorrectly termed as still undeveloped.

Generally such designations are almost as common for the treatment of the forms of early Gothic works, as those of the degeneration, rottenness and drynes for the works of the late period. Both kinds of treatment of form are not to be judged so superficially. Both form only the expression of the structural system of the works concerned. But the latter require in each separate case very thorough investigations in order to be understood, researches to which one must bring not only good will, but also favorable opportunities and some experience in construction. But in the cases it should be assumed, that such an investigation must lead to the avoidance of the before mentioned striking expressions.

This arrangement of the springings of the arches mentioned above on the round pier connected with four shafts is already based first on this, that section of the pier itself in order to resist the excess of the vault of the middle aisle against that of the narrower side aisle, requires one of a diameter exceeding the breadth of the dividing arches, thereby forming an excess of bearing area, which of itself leads to the receiving of the diagonal rib (see the right half of Fig. 427). Besides this reason taken from simple practice, also others derived f from the nature of the matter lead to this arrangement. The entire vault is divided into bays by the cross and dividing a arches. To form these divisions first suffice the lowerirings of the dividing arches. Therefore the latter are parallel to the cross arches (?), and in the older works generally have the same profile as those but project more, and therefore were borne by rounds set on the axis of the pier. But the cross ribs most clearly express the character of the Gothic vault, and according to the location of the beds in the compartments the g greatest part of the thrust of the vault is transferred to the pier, thus representing the proper force of thrust. But strengthening by the upper ring of the dividing arch is necessary to represent the wall resting on the dividing arch, whose load together with the roof construction resting thereon increases the resisting force of the pier. Hence justly the nucleus of the latter bears the cross ribs that denote the separating force,

and those increased dimensions of the dividing arches that on the contrary increase the resistance, while the rounds again support the dividing, and the pier the connecting and stiffening arches.

By the partial increase of the springings of the arches certain arrangements further become possible, that again have their reasons in certain peculiarities of the plan. Thus in the church at Wetter the crossing piers are no larger than the nave p piers and are just large enough to receive four dividing arches meeting thereon, so that the diagonal ribs project between the latter and increase with two of those in their base. This location of the diagonal ribs then produces the same stresses, also to retain on the nave piers, so that these with half the p profile increase with the diagonal arches, and between them and the cross ribs are visible a part a b od the face of the compartment in the left half of Fig. 427.

If we so far had in view only the design of piers of this g ground form in churches with aisles of equal height, there is likewise found the same condition in the arrangement of the arches in the design of a raised middle aisle, like the cathedrals of Rheims, Amiens, Chartres, Dijon and as shown on many German churches. In the side aisles the arrangement remains unchanged (see right half of Fig. 427), but in the middle aisle the round rises above the capital to bear the cross arch above. As a characteristic example is given in Fig. 428 the clear development of a pier from S. Jacob's church at Einbeck (Note) belonging to the 13 th century. The round of the middle aisle is surrounded by a band at the height of the capital, which c continues above the nucleus of the pier and then passes into the abacus of the capital of the side shaft. Above this moulding at each side of the round of the middle aisle, small mouldings on shafts rest on the nucleus of the pier, intended to support above the ribs the side arches of the vaults of the middle aisle. These little shaft members do not find their start on the parts of the nucleus of the pier not used by the dividing arches, since they would otherwise rest on the available border of the capital of a round pier without little shafts.

For the older round piers with four rounds, stated to bear usually only the cross arches, the ribs rested on the nucleus. Later the cross arch and ribs usually developed together from

shaft, and then the nucleus and both side shafts remain for t the use of the dividing arches. As examples may be mentioned S. Stephen at Mayence and the Minorites church at Höxter, to the latter belonging the simple development of the pier represented in Fig. 429.

A peculiar arrangement is found in S. Blasien in Mühlhausen. The nave piers as in Wetter are as large as the crossing piers, whose size on their part is determined by the combined four d dividing arches. While here the diagonal ribs start between t the dividing arches, this arrangement is changed for the nave piers, so that between diagonal ribs and dividing arches is inserted a piece of the surface of the compartment, as it appears in Wetter (Fig. 427 left, a b).

In this place is to be mentioned also an alteration entirely favorable in effect in the cathedral at Chartres: for here alternate cylindrical piers with octagonal, and the round ones with four having seven sides of the octagon, the polygonal ones with as many round shafts. (See both plans in Fig. 430).

Note. From the drawing by C. W. Hase at Hanover.

The Crossing Piers.

As already mentioned are found on the crossing piers at the crossing and further at the internal angles of the towers, four arches together having the dimensions of the dividing arches, between which the diagonal ribs must find their bearing. This condition first leads to an enlargement of the pier mentioned (Note), obtained in the most varied ways. By the arrangement of compound nave piers, the crossing piers were subdivided in the same way and only received an increase on the number of l little shafts and the angles of the nucleus, those consisting of about 16 rounds, while the nave piers had but 12. A crossing pier of this kind is shown by the right half of Fig. 421. But even with the arrangement of single cylindrical piers or those connected with rounds in the aisle, the crossing piers are generally subdivided according to the arches resting on t them, thus in Notre Dame at Dijon, and those at the southwest angle of the middle square of the cathedral there, also in Chartres, etc. Simpler forms of this kind are shown by the already mentioned crossing piers of the churches at Haina and Mühlhausen.

Note. Why this enlargement disappeared in the churches mentioned at Wetter, Mühlhausen and elsewhere, will be investigated later.

On the contrary in other works, as in the church of S. Elisabeth at Marburg, the churches at Colmar, at Altenburg, etc., it is found that the relation to the round piers with four rounds in the aisles is there continued in a higher degree, that also the crossing piers consist of a cylindrical pier with 8 instead of 4 shafts, namely 4 larger and 4 smaller rounds connected with the nucleus, whose diameter is increased according to the measure of the plan of the springing of the arches. The tower piers of the church at Colmar are formed according to t the same principle except with an octagonal nucleus.

The most consistent arrangement would then consist in this, that from the plan of the nave pier should be cut out in the lowest part, and from the combination of four such shafts, by the rounds beneath the diagonal ribs, all different parts formed the crossing pier. In this wise with a certain freedom in treatment is constructed the northern crossing pier of the cathedral of Dijon, Fig. 431. Accordingly this combines all elements of the other piers in itself, the circular shape of the nave pier, the rectangular recessions of the opposite crossing pier and a number of rounds corresponding to its function. Sometimes as in the nave of the church at Friedberg, the nave pier still always formed with a round nucleus is combined with 8 rounds, so that a round stands beneath each arch, while the crossing pier is but slightly older and according to its formation is combined with only 4 rounds. In spite of the slight difference in style, chiefly expressed in the treatment of t the foliage, must the increase in the number of the rounds, at least with the retaining of their regular position, must be regarded as contrary to the proper nature of the construction, since the shafts set beneath the diagonal ribs are exactly according to the octagonal division of the circle and restrict the width of the dividing arches, unless the diameter of the pier receives a corresponding increase. Accordingly the increase of the rounds is moderate in the way, that to the plan of the piers of Mantes and Besancon represented in Fig. 426 were added two rounds in the longitudinal axis, or that around the circular pier were set 12 rounds, three each standing under

the dividing arches, whereby the latter received a moderate b breadth without increasing the size of the pier.

This consideration of the size of the dividing arches certainly stops, when the arches supporting the wall are first turned above the compartments and then all formal treatment can be omitted, beneath the compartment but in the same direction is turned a simple rib entirely like the others, so that from each pier starts 8 entirely similar ribs, between which are v visible the surfaces of the compartments in larger or smaller widths according to the size of the pier. It becomes possible by such a plan to place a separate round under each rib, and to attain in general a really ideal regularity, though it still ends in a denial of the nature of the thing, produces the effect of a tiring monotony at least in wide interiors, and is recommended only by its economy, in so far as the arches turned above the compartments and not in connection with them, could be erected with rubble or bricks with horizontal balancing and to support the framework of the roof. This arrangement is found in a relatively still happy form in the nave of the cathedral of Frankfort (Fig. 432).

The exactly similar forms of the ribs, or at least those over the sides and diagonals of the bays, on the contrary is everywhere required by the nature of the matter, where the function of the dividing arches is lost, as for example in vaulted halls, and then in churches composed of two aisles separated by a middle row of piers, which are chiefly found on the Rhine as in Namedy and Bornhofen, but further in the plans of five-aisled churches, for which the rows of piers separating the s side aisles, in brief in all cases, where the ribs extendind in the direction of the dividing arches stand in an exclusive relation to the vaults.

The compound piers of the middle and later times.

Transformation of the members of the dividing arches.

Before we pass to the freer plans of the piers belonging to the middle period, we must examine the corresponding transformations of the profile of the dividing arches. The original and on earlier works almost typical profile of these was directly formed from the rectangle of the cut stone, that is represented in Figs. 423 to 427, already received about the middle of the 13 th century all sorts of additions and a partial transf-

transformation. Thus in S. Blasien in Mühlhausen the rectangle of the dividing arch consisting of but one ring received coves in the corners. A more compound form of this kind is then shown by the dividing arches of the cathedral of Dijon 'Fig. 431), but in both cases in retained the characteristic of the older form, the horizontal underside. Yet the latter disappears when a round is placed on it as at c d in Fig. 433, and the main form of the profile approaches a square set diagonally. The l latter is more clearly expressed if a flat be added to the round as in the cathedral of Rheims. But also the form of the a arch with two rings soon fails to be perceptible, and the joint then separates the lower hollow from the round of the upper ring. Fig. 433 represents these changes. There a b c d is the older and already more richly treated profile, which by the addition of the ogee round in the form a b c f d and by the hollow between the rounds passes into the form characteristic of the middle period.

In the addition of the round or rather of the ogee member of the lower flat and the basal form of the diagonal square thereby obtained for the entire profile, it has been desired to recogniqe after the example of Kugler the form characteristic f for the best period of Gothic architecture, and to include that earlier one with horizontal underside with the still undeveloped transition forms, to which would accordingly belong the greater number of the standard French cathedrals, also the cathedrals of Strasburg and Freiburg in Germany as well as the church of S. Elisabeth in Marburg .nd numberless others. A certain narrowmindedness seems to us to lie in this. So pleasing is the effect of a dividing arch treated in this later form, that it manifestly elevates the difference between the character of a strong dividing arch supporting a wall and the diagonal rib only bearing the light masonry of the compartment. For the profile of the latter is first important the height, while the dividing arch must first have the breadth to receive the wall. The change of the underside into an angle ends directly in the denial of the breadth and also produces in reality the corresponding effect. Conversely we might give the preference to the older form and also prefer a correspondingly simplified shape for the cross arches, as soon as the latter generally b became larger than the diagonal ribs (see the right half of

Fig. 427). The profile a g c f d of Fig. 433 further emphasizes the separation of the two rings of the arch, and therefore is first only in place where the limited dimensions make possible the form of the divided arch with one ring.

But the prominent eeffect of the arch profile ending beneath in an angle asserts in general its right, that it was retained for all richer designs in the late time, and only suffered in the details of the profile certain changes corresponding to t the general character of the period. Thus care was taken first to enhance its character to greater richness by the increase of members, by fillets or roundings inserted between the rounds and hollows, till men began to border the rounds by more straight lines, by members corresponding to that simpler shape of vault ribs or replaced by a simple intersection the coves in the angles.

The dividing arches of the church in Friedberg, of which Fig. 434 represents the older older ones found first in the transverse aisle, and Fig. 435 the slightly later ones of the western bay, makes apparent this last transition. A further example of the latter only consists of hollow members, then are shown by the dividing arches in the southern side aisle added to the collegiate church in Fritzlar about the end of the 14 th century (Fig. 436).

Proportion of the dividing arches to the rounds.

At the same time that this change in the form of the profile of the dividing arches was modified its proportion to the rounds of the pier. While according to the older system each round entirely bore a separate arch, we already see on the nave piers of the cathedral of Dijon the rounds standing beneath only the farthest projecting parts of the dividing arches. But men soon went farther and brought the shafts into separate relations to the different rounds of that profile, whereby also the diameter of the former must not exceed that of the latter, and then also carried the hollows down the pier, so that the nucleus of the latter entirely disappeared. Yet here at first is found a difference between pier and arch members, so that the rounds of the latter are larger than the shafts and vary from them by the added curves, so that they are at least generally more complex, and the capitals fulfil a real function. Such an example taken from the church of S. Katherine in Oppenheim is shown

by Fig. 437. But usually these differences entirely disappear, as shown by the pier reproduced in Fig. 438 from the church of S. Mary in Mühlhausen bwilt about the middle of the 14 th century; the profile of the dividing arch there is the same as that of the pier, and the capitals are exceptionally necessary to the shafts standing beneath the cross and diagonal ribs, c carried around the entire pier until men succeeced in entirely omitting them, forming the pier only by vertically continuiong the members of the arch. Figs. 440 and 439 show plainly two s such piers, the first being from the Wiesen church in Soest of the second half of the 14 th century, and the last from S. Maclou in Rouen of the last time of the 14 th century. On the former the dividing and cross arches are alike and the cross ribs intersect at the angles between both. Conversely in S. Maclou the cross ribs partly increase with the diagonal ribs and the extreme members of the dividing arch and are extended in this form from the base of the pier.

Pier of the basal form of square set diagonally.

The ground form of all these members, both of the dividing arches as well as of the pier, is that of the square set diagonally, or yet restricted in the manner that only certain parts project beyond it. In it is met at the same time both earlier systems, for the cylindrical pier according to the size and the projection of the shafts passes into this so far sometimes, t that it is entirely inscribed, and in the form of the fier divided in rectangular steps is this directly contained, when the separate sides of these steps are alike.

Very instructive in this respect is the treatment of the piers in the minster of Freiburg, where the stepped surfaces are entirely omitted, as shown in the upper half of Fig. 423, while conversely the members of the dividing arches are developed w with entire distinctness from the structural motive of the different concentric arches. The joints in the intrados of the d different arches therefore lead to retaining the steps, while the end joints of the piers could lie in the direction of the sides of the diagonal square.

The entire transformation thus consists in this, that first the direction of the diagonals in all details replaces that of the sides of the square, and further that the hollows ever become wider or more numerous, the rounds and shafts being dim-

diminished in the same proportion.

The development of the separate members from the diagonal square is indicated by aiding lines in Figs. 434 to 440. Although we cannot guarantee exact accuracy of this construction employed in the drawings, yet will they serve for for the approximate determination of the proportions of the details of the members to each other.

The undivided pier of the late time.

Besides the richer forms mentioned, to which the ground form of the single pier passed by combined combination with the shafts and by fusion with the formation of the membered pier, t there also extend through the periods of Gothic art numerous examples of retaining all this simplicity. It will even become more common about the end of the same, so that thereby almost a transition may seem prepared to the antique-like columns of the Renaissance, if not the latest Gothic forms of piers had not in principle opposed more strongly these columns, than the simple round piers of the early Gothic period. The course of the transformation of the early round pier into the late Gothic forms is the same, that we have just proved in the compound pier, and is expressed in the increasing harmony of the basal form of the prier with that of the dividing arch. But this harmony will be obtained conversely as in the compound pier, for while the latter vertical continuation of the profile of the arch forms the plan of the pier, the dividing arch is here formed according to the plan of the pier, and thus in both the c capital is superfluous.

Thus the dividing arch first retains a form differing from that of the pier and more or less richly membered, only its p projection becomes less and accordingly the sections of the v vault ribs are more compressed, so that the entire mass of the arch members finds room on the moderately projecting abacus of the capital (Fig. 441). There the pier can have a round or polygonal form. But the hollows of the profile of the arch still cut into the basal form of the pier. To avoid this loss of size, the arrangement is sometimes found so modified, that the plan of the pier continues above the capital and partly combines with the members. Then to avoid the projection of certain members beyond the continued nucleus of the pier, the nucleus is either above beyond the pier, so that it circulscribes the

springing of the arches is shown by a b c d in Fig. 441, or t the mass of the springing is contracted so that the plan of the pier can be inscribed withon it, as shown in the right haff of Fig. 441.

In the first case the capital still fulfils an essential function, but in the latter it serves a more ornamental purpose, at most still indicating the beginning of the arches and therefore is finally omitted. Between the two arrangements lie those, according to which either the nucleus assumes a basal form differing from that of the pier with the same magnitude, and hence the capital must effect the transition from the round pier to the octagonal nucleus, or where the lowest member of the section of the dividing arch projects beyond the pier and the same condition appears with reference to the vault ribs, then the pier without a capital has four corbels, on which rest the projecting members mentioned, while the rest of the members of t the dividing arches grow out of the pier. (Figs. 442, 442a).

Also sometimes these corbels are arranged only for the vault ribs, while the members of the dividing arches are inscribed within the plan of the pier, thus entirely growing out of it, as indicated in the right half of Fig. 442. Likewise can be arranged shafts instead of the corbels, indeed either four or e even but two beneath the springings of the arches. Then may t piers and shafts be without or with capitals, or merely the shafts may have them.

Polygonal piers.

Instead of the round plans of piers in Fig. 442 may also be introduced any polygon. The polygonal pier is found in the economically built churches of the mendicant orders already from the beginning of the 4 th century onward, as in the Dominican church at Erfurt in the way, that the octagonal ground form of the pier continues in the dividing arch, whose springing is indicated by a slightly projecting capital. There separate corbels are arranged for the springings of the ribs, which either project above the edge of the capital as shown in plan in Fig. 444 and in elevation in Fig. 444a, or they start directly from the mass of the capital. Thus the moulding of the abacus can extend around the projection of the corbels, or the latter receives the finer treatment, or finally the projection may assume a freer form. A most happy form of the last kind is shown

by the little church of the village of Gottesbüren in Hesse, built about the end of the 14 th or the beginning of the 15 th centuries, of which a perspective view is given in Fig. 445.

In consequence of the slight elevation of the vault of the middle aisle amounting to a few feet above those of the side aisles, the differently shaped projections here bear short shafts, on whose capitals rest the cross and diagonal ribs, while for the side arches are arranged separate corbels projecting from the edge of the capital. With the same ground lines of t the vaults of the three aisles, the springings of the ribs rest directly on the corbels projecting from the capital, the side arches on the projection of the abacus.

The exaggeration of the thoughtful arrangement mentioned is shown by the capital from the princes' hall of the city hall in Breslau, where these corbels in small size project from all eight sides of the edge of the capital, arrarently being there only to bear the extreme members of the different vault ribs, that might well have found room on the edge of the capital itself.

The use of these corbellings above or in connection with the capitals is however nowise a peculiarity of the middle and late periods, but is adready found in principle in the works of the transition style, as in the aisle of the church of S. Sebald in Nuremburg and in many early Gothic works in France and England, only according to the arrangement of the raised middle aisle, so that these corbels bear the shafts that receive the ribs of the middle aisle. (An extremely beautiful example of a corbelling from the capital of the church in Semur in Burgundy is found in the Dictionary of Viollet-le-Duc, II, p. 514). The peculiarity of the later treatmentthus lies only in the greater hardness or economy of the treatment, thereby making itself particularly instructive. On the very slender round piers of the hall church of S. Croix at Liege all ribs are set on corbels. There for equalizing the heights of the ridges the corbels for the arches of the middle aisle of wide span are set lower than the others.

Also the older arrangement occurring in the cathedral of Paris, whereby the rounds belonging to the vaults of the middle aisle rest on the margin of the capital of the pier beneath the diagonal arch, are found in reduced size in the middle and

later periods. An example of this kind from the church at Immenhausen built about the beginning of the 15 th century is shown by the right half of Fig. 443 in plan. The octagonal pier continues in the dividing arch,and the springing of the ribs and the dividing arch rests on the edge of the capital. The f further reduction indicating the last period then consists in this, that the useless capital for the dividing arch is omitted and the springings of the arches either rest on corbels or grow out of the sides of the octagon toward the aisles. But conversely would easily result a function for the capital, whose omission produces the effect of a certain dryness, if the profile of the dividing arch has a form varying from that of the plan of the pier, as given in the left half of Fig. 443, and further in the left lower quarter of Fig. 444.

The polygonal ground form of the pier is sometimes modified in the last period by a concave form of the sides, yet so that the edges of the pier formed by the intersection of these segmental arcs always remain rectangular. Here is manifestly an endeavor for varied effect of shadow, that has caused this use of the nagain renewed polygonal plan. In the cathedral of Erfurt according to the same system the octagonal pier is carried into a richer system, particularly so that the angles are formed by rounds and the sides of hollows, that are separated from the former by fillets. However such a form is entirely different from the compound pier mentioned above, so far as any relation between the members of the pier and the different arches fails. It is far more nearly allied to those with the penetrations mentioned at the springings of the vault (Fig. 291), and varies from that only in the members of the pier including the capital.

Form of shafts.

Ground form of shafts.

Shafts with polygonal plan are rare, examples with octagonal shafts may be mentioned in the cathedral of Chartres and the side aisle of the collegiate church in Fritzlar; in the church at Wolfhagen near Cassel, which belongs to the early Gothic school of Westphalia, are found on each of the heavy round piers four shafts of square section. Otherwise round shafts p prevail, whose cross sections represent a larger or smaller portion of a circle. But from the 14 th century onward is found

the round plan sometimes transferred by a front projection into the sharp or pear shaped ogee shaft.

The origin of this not very fortunate form must be sought in this, that men sought to show the direction of the upper arch already in the form of the shaft itself. But if in Fig. 446 the lines a b etc. give the direction of the ribs, they determine at the same time that of the projection. The latter occurs in various relations to the bases and capitals.

In the simplest cases the base remains round and the projection rests on its upper member, that it intersects according to the shape of the base moulding, and then extends down to t the projection beneath, or the base moulding encloses the projection as indicated at g h g in Fig. 446. In the same way the projection either runs under the lowest member, stops at the astragal of the capital or intersects at and joins the greater projection of the mass of the capital or the foliage, or it is enclosed by the astragal, or finally by the entire mass of the capital.

Groups of shafts.

As already stated, in the simplest cases a shaft bears all vault ribs, as conversely in the richest design as when for e each rib is arranged a separate shaft. Such a group of shafts then forms a part of a compound pier, whose entirety originates by the combination of the required number of shafts. But sometimes are found in the works of the middle ages very peculiar forms of such groups of shafts. A happy example of this kind is shown by the castle chapel in Marburg represented in plan in Fig. 447. Here actual shafts are only arranged for the diagonal ribs, which are connected together by a hollow. The latter is then continued above the capital of the shaft in the cross arch, while the outer members of the latter, the members a b and c d grow out of the side surfaces of the diagonal ribs.

Rounds above the capital of the pier.

The rounds are as already mentioned if they are connected w with a pier, either if set on the ground or in aisles of unequal heights being placed on the projection of the pier capital under the dividing archor corpelled out over the latter. Placing them on the pier capital is found in certain French works of the early time, as in Notre Dame in Chalons, S. Remy of Rheims in the modified way, that on the capital of a larger round

standing on the ground, stand three smaller ones, when the capital is either at the height of the pier or above it. The last arrangement is found in Chalons. The position on the capital of the pier is found in an especially happy way in Notre Dame in Dijon, where the round stands without bonding with the upper wall, and the members of the dividing arch extend behind the round bearing the diagonal rib, so that it again becomes visible between the latter and that supporting the cross rib. We have represented the plan of this form in Fig. 286, which Fig. 448 shows in elevation.

Rounds corbelled out from the fier.

Frequently only a part of the shaft extends down on the pier, while another is corbelled out higher on it. This already occurs on Romanesque and early Gothic works, that the shafts for the ribs first begin higher, and among other things there may have led to this a later decision for projecting groin arches. The cathedral at Riga was at first begun as a hall church at the beginning of the 13 th century and shows this solution in the stepped and entirely Romanesque piers of the aisle and the wall (Fig. 450). Likewise also the round piers in many Westphaliay churches, like the cathedral at Minden, the church at Volkmarsen, etc., have four corbelled shafts for the ribs, while the four shafts for the cross and dividing arches start next the ground, so that the arrangement is entirely regular.

For round piers with four projections the shaft next the middle aisle is sometimes placed higher, as in the church at Haina, where it begins about 12 ft. above the ground. In the church of S. Christopher at Mayence, also dating from the 13 th century, the shafts for the clearstory even first begin above the capital. Io the market church at Hanover (15 th century) conversely the shafts under the dividing arches are corbelled out of the mass of the cylindrical pier, and thuse next the aisle start from the ground.

To extend the view under the free space may have first led to this abrupt corbelling, that is very justifiable statically. Thus in the predominance of the thrust of the middle aisle it would naturally result to save partly the shafts of the middle aisle no longer stressed, but conversely would be quite proper to extend down the most strongle stressed shafts in the side aisle and set then on a widened foundation.

the walls.

Shafts on the walls.

Shafts are attached to the walls as well as to the piers, but some special arrangements occur there. For sometimes the shafts only start from the moulding extending under the window sills, or it is found in the arrangement of three or five rounds that this plan is only employed on the outside ones, while the middle ones axtend down to the ground.

Another arrangement already peculiar to the transition style is likewise found in the before mentioned Westphalian churches in which in particular a segment of a pier is first placed at the surface of the wall, and the shafts are corbelled from this. In Volkmarsen the execution of this form is very simple as shown by Fig. 449, as this was already brought in by the spanning of the side aisles with vaults without ribs. On the contrary in Minden is it executed with the greatest richness. Above a figure standing on a corbel a semicircular canopy forms the basis of the corbelling, that by a bold hollow covered by leaves obtains a larger surface and forms the cornice of the canopy, on which rests a short portion of a pier bonded with the wall, to which are attached five rounds, namely a larger one for the cross rib and four smaller ones, two of which bear the side arch and two the diagonal rib. The smaller rounds stand on the projecting cornice edge of the lower corbelling, but for the larger is arranged a corbel projecting from this edge. All rounds have capitals, whose upper moulding extends around the c corbelled nucleus of the pier.

Such treatments are of use, since they leave the lower part of the wall flat and those seats in churches and benches or o other furniture in secular buildings may be set close against the wall without being restricted by the shafts extending downward, thereby producing a rich and massive effect; they further strengthen the abutment since they lessen the spans of ribs.

Ratio of size between pier anf springing of arch.

As for what relates to the ratio of the supported to the supporting part, the plan of the ribs to that of the shafts, that of the entire springing of the arches to the pier, the general rule is valid, that the area of the supported part at laeast equals that of the support, but is mostly larger than the latter. The basis of this almost regularly occurring appearance is to be sought in this, that the plan of the springing of a

vault for architectural and practical reasons only allows a reduction within certain limits, and that on the contrary the plan of the pier when no thrust is in question can be very strongly reduced. If only the resistance of the material to compression is considered, then as a rule the dimensions of the pier can be made very slender.

A prismatic pier of sandstone or limestone (specific gravity = 2.5) can be built 80 m high before producing a compression of 20 kil per q cm at bottom. If 40 kil per q cm be allowed t the pier might even be 160 m high. If the beds were sufficiently hard, the crushing of the material need not be feared with a much greater height)..

Taking a pier that must support a vaulted area of 50 q m (about 7 × 7 m) with a weight of 25,000 kil, with an allowed compression of 20 kil per q cm, $\frac{25000}{20}$ = 1250 q cm area, or about 35 × 35 cm would be required. But to reduce the springing of a vault od 7 m span to this little area would scarcely be considered.

Usually the proper area of the springing of arches and of pier are fixed separately, so that one is often led to have the arches project as far as the projection of the capital permits. Thereby is obtained a smaller span of the arches, a less compression of the members and usually an easier construction.

If the springing of the arch and the plan of the pier are to be exactly equal in area, then usually a greater diameter is given to the springing of the arch, since this is weakened by the projecting angles of the members, but the pier has an entire outline. It is evident that the projection of the mass diminishes when the plan of the pier is simpler than that of the springing of the vault, and conversely that the projection will be less, the more nearly similar the pier and springing become, until finally with complete harmony of both all projections d disappears. The later time of the middle ages has particularly sought a direct transition from pier to springing of the vault without intermediate capital (Figs. 288, 291).

A certain justification cannot be refused to endeavors like these, since after combining the forms of the vault in the springing of the arches the material suffers the same compression as in the part of the pier beneath. If the pier and springing can be made of the same stone with equally careful joints, it

is admissible to give them equal areas in plan. It has just b been stated that other reasons again oppose this equality.

With smaller dimensions as may occur in secular buildings, halls etc., the reduction in size of piers has limits set by conditions of execution and of resistance to accidental injuries; since then in such cases the otherwise small stresses can afford no such advantage opposed to an increase in the dimensions of the springing of the arch, but on the contrary it may produce a heavy effect, and then its projection beyond the face of the pier must be omitted. An example of this kind is offered by one of the halls of the monastery of Haina, the so-called bitter chamber, whose springings of the arches formed in accordance with the principle of Fig. 287 scarcely project beyond the line of the cylindrical pier supporting them. Further much depends here on the nature of the material. Thus on the extremely slender granite piers belonging to the so-called Brief chapel in the church of S. Maria at Lübeck, as well as that found in the refectory of the castle of Marienberg, the springings of the ribs project but slightly beyond the face of the pier, trusting the excellent quality of the bricks composing it, while for the equally slender piers of the refectory of S. Martin des Champs in Paris, the diameter of the springing of the ribs is apparently thrice the upper diameter of the column.

The bold piers at Lübeck and Marienburg seem comparatively even bolder in comparison to the yet bolder springings of the vaults, which in consequence of the recesses have even a smaller area than the piers, in spite of the springings being of brick and the piers of the much harder granite. One might conclude from this that the piers should have been far more slender; but it is not so. --- With very slender columns there is not only the resistance to compression, but the danger of bending or breaking is to be considered. Further in very small piers is increased the possibility of splitting in consequence of hidden defects in the material, entirely aside from the fact that the pier is more exposed than the springing of the vault to accidental blows or injuries.

However just at the springing in brick is required a certain greater projection for construction and artistic effect, although in a given case to the unusual and aspiring effect of the vaulting cannot be denied an almost fascinating charm.

2. The Capitals.

Form of capital with round shaft and square abacus.

General form of the capital.

The capital usually has to fulfil two purposes, it first has to create by its projection a greater area to receive the supported members, and second to transfer the cross section of the support into another suitable form of plan. With especial frequency it concerns the change from a round shaft to a square abacus; both for beams as also for simple arched members d does the square abacus offer a bearing area suited to the purpose, and further it gives the most natural form of a cut stone.

In Romanesque art the fulfilment of both requirements was combined in one part, when the single body of the capital produced projection as well as transition, where yet the projection was by a boldly moulded slab, frequently enlarged by the addition of a separate cut stone. Figs. 451 to 453 represent three ornamental additions.

The Gothic takes care to separate the two functions, and like the Corinthian capital it effects the projection by the bell of the capital, but the transition is by the circular form of the abacus laid thereon as shown by Fig. 454 in contrast to Fig. 453. The abacus may project more or less beyond or be inscribed within it. (See plans in Figs. 454a to 454c).

Supporting leaves at the angles.

The overhanging corners of the abacus (Figs. 454a, b) were supported by a corner leaf, a bunch of leaves or a supporting body formed in another way. Even if the angles do not project, as in the plan of Fig. 454c and on the capital from Volkmarsen represented in Figs. 455 to 455b on Plate 37, still is recommended a strengthening of the angles by supporting leaves; for as shown by the diagonal section in Fig. 455b, there is less of the bell beneath it than at the side of the slab, and it therefore is next to strengthen the edge of the capital beneath the capital by a support, so that the line a b c becomes the line a d c.

Fig. 456 exhibits the elevation of such a simple form of capital, that has angle supports of full square cross section at top, flattened to the shaft below and finally passing into it. Accordingly the form of this support approximates the form of the fleshy leaf or stem cut off at top, Fig. 457 representing

its elevation at a largeR scale. More animated than these cut off "dead"members are the leaves developed entirely to the point, that are represented in simplest form in Fig. 458.

Since the point e in the plan of Fig. 458a denotes the extreme corner of the cut block, then may the angle leaf project so far beyond the edge of the bell as the block permits, so t that its end as given by the right half of the plan and elevation shows, with leaves cut off is moved back to h i, or with pointed leaves nearly to e. Further to separate these leaf supports more sharply from the surface of the nucleus of the capital remaining between them, they are supdivided below and indeed in the simplest cases by two hollows g sunk in the right half of Fig. 457, which also flatten below and lie against the nucleus of the capital, hence assuming the form here given in plan in Fig. 457a or by a richer profiling, visible in the left half of the same Fig. More animated is this treatment, even if the lower also cracks, and both its parts separate in the mass as they near the nucleus as in Fig. 457 at x.

The form of this support as here represented, for example as found on the corbels of the church at Haina, is not the oldest, but on the contrary is to be regarded as one of the earlier and most richly derived, but we hold it advisable for its clarity and simplicity, made easy by the geometrical treatment in such a high degree, to allow it to precede the richer forms and to regard it as the root of the latter.

Two rows of leaves above each other. Development of the angle leaves.

For a greater height of the capital the need of architectural animation and the utilization of the mass of the cut stone leads to the form similar to the leaf-like supports in half the height or somewhat higher, and to its repetition, so that the leaves curl forward free from the nucleus, yet the angles indicate a diagonal square within the square of the cut block. Then the upper leaves spring from between two lower ones, so that both rows have the same position as the acanthus leaves on the Corinthian capital. Accordingly there results the form represented in Figs. 459 and 459.. These are more graceful when the plant character of these supports more strongly appears, and also if instead of teeth the ends are formed by points of leaves coiled upward or downward. Such very simple capitals are

formed on the round piers of the high choir of the collegiate church at Mantes. Fig. 400. (Note). It is generally first the varied mode of treatment of these supports and their ends, sometimes carried to the greatest richness, which characterizes the various capitals of this kind. We can here indicate this endless diversity in only a few ways. That triangle e a e projecting over the edge of the bell in Fig. 455a gives the mass of these endings, from which develop knobs, buds or leaf forms, that lie before the edge of the bell, transform this into the square ground form, and produce the same effect as the volutes of the Corinthian capital, in a very happy way as we think. F Figs. 461 to 461d, 462 to 466, 469, 473 to 480, show different examples of gradual progressive development of these forms, w whice includes a transformation like that of the development of the bud into the leaf.

Figs 461 and 461b show the buds still entirely closed in the simplest form like knobs. Characteristic for this simple form is the almost typical arrangement of two knobs into whimh the support divides. From this develops the more unrolled leaf divided below in a form like Fig. 464, which reappears with special frequency in Germany and France, and by its easy recognition produces a peculiarly tasteful effect. Fig. 463 then shows a simple leaf closed as in the bud, while Figs. 465, 469, 469a and 480 exhibit recher but still closed forms of buds. Fig. 4 462 shows a fully developed leaf, Fig. 466 has a formal bunch of leaves, and Fig. 474 is a later and more conventional shape. Simpler forms are the leafy endings replaced by heads, as in the choir of the cathedral at Wetzlar (Fig. 471), or the entire support has become a great animal's head, of which Fig. 470 is an example from the same place, and Fig. 472 represents a second from the cathedral of Besancon.

Note. A capital from the same row of piers is found in Viollet-le-Duc's Dictionary, II, p. 512, in which the ends have two leaves above, with two curved downward.

With the existence of a second row of leaves the upper one can be of the same form, but the lower row usually has a different shape, and consists only of more or less strongly conventionalized leaves added to the nucleus of the capital. Examples of these are given in Figs. 461, 462, 480. It is essential to the effect if the entire capital, that all parts attached to

the capital, the supports at the angles as well as the leaves in the lower row, should curve outward in a line similar to t the profile of the bell and thus enhance the effect thereof. Therefore the line of the profile visible in the Fig. above is particularly characteristic.

With a stronger projection of the capital,the middle of the edge of the bell may also be strengthened by supports in the same manner as the angles of the abacus. In this wise are then formed the supports of the capitals in the aisle of the cathedral of Rouen (Fig. 463).

The body of the support, that we have previously seen animated by a simple subdivision, is sometimes ornamented by underlaid leaves, and these leaves are either simple as in Fig. 463 or are arranged in greater number,and in the last case lie in the most varied arrangements from the middle lines at both supports toward both sides. A simple example recalling Romanesque ornament of this kind is shown by Fig. 465 from the rood screen of the church in Friedberg.

Transition from the bell to the abacus.

We have placed the lower square of the abacus within the circle of the edge of the bell represented in Fig. 456.. But thereby becomes necessarn, particularly with a greater diameter of the column, a larger projection of the margin of the bell beyond the sides of the abacus and wide projections of the members of the latter, if it is not to remain behind the edge of the bell, and further the edge of the bell itself must already have a considerable projection, if the abacus is generally to project still from the line of the column, and so finally these strong projections and ressessions also require a height for the entire capital, which must lead to a heavy disproportion. But this tendency will be avoided when the lower square of the abacus extends its corners beyond the edge of the bell, while the latter still retains the propections of the middle of its sides of the abacus. But by a similar arrangement will the need of the angle supports be increased, that now those p projecting angles of the slab lie upon, so that the edge of the bell either stops against the sides of the abacus or the endings of the supports, or at the latter.

The oldest form of this kind is that, where the projecting angles retain their horizontal undersides, that remain visible

between the supports and the edge of the bell, as shown in Figs. 462 and 461, the first from the collegiate church in Mantes and the latter from a portal located on the eastern side of the north transept of the cathedral in Mayence. In the last Fig. the ground planmakes clear the proportion by which the triangle a b c represents that horizontal surface over the support. On the first Fig. we call attention to the edge of the bell placed on four arches, an arrangement that is also found in a similar way in the choir of the church at Gelnhausen and produces an extremely animated effect.

But men soon sought to avoid these horizontal under surfaces and thus passed to the forms represented in Figs. 466 and 464. Fig. 466 represents a second capital from the Friedberg rood screen, and the body of the abacus rests directly on the bunch of leaves that forms the end of the support of a triangle a b c that has a corresponding size, while the edge of the bell cuts into the thickness of these leaves, and there also rises from it a wash, that rests against the side of the abacus. Another shape results when the abacus rests with a cove on the top of the bell, but this cove follows down on the angles above the edge of the bell and intersects the wash forming the back of the support (Fig. 464). A more complex but entirely happy solution is shown by a capital from the sedilia in S. Blasien in Mühlhausen (Fig. 467), which in a way joins together the peculiarities of both forms last mentioned. Here a wash rises from the edge of the bell that joins on the lower edge of the abacus but goes down over the edge of the bell around the vertical continuation of the body of the slab resting on the leaves of the capital, at which the edge of the bell also stops. The projection of the corners of the abacus beyond the edge of the bell is further reduced or entirely prevented by cutting off the angmes, so that now the ground form of the abacus is an octagon with four large and four small sides. An example of the last kind, which at the same time forms the transition to the capital with polygonal abacus is shown by Fig. 460.

Form of the bell.

In the Figs. so far represented are given the most diverse forms of the three parts of the capital, namely the abacus, b bell and astragal. The profile of the bell, formed as an extension of the shaft of the column by a cavetto, yet in a way that

the latter still extends above the astragal, only passing into the cavetto at about midheight of the bell or higher, which is nearly typical and varies only in regard to the projection of the edge of the bell and the height at which this cavetto begins. Seldom fails the continuation of the shaft above the astragal, as in one from the hall of the former Dominican monastery in Erfurt, and the bell is then shaped in a freer curve. The thickness of the foliage projecting far from the ground of the bell sometimes extends at right angles to it on the older works. But already in the first half of the 13 th century the leaves are also undercut, so that their edges form an acute angle with the nucleus. Such an example from the nave of the minster at Strasburg is shown by Fig. 514. But sometimes the nucleus of the capital assumes a section line swelled to near the main profile of the foliage as shown in Fig. 467, so that thereby the projection of the leaves is reduced. This form is shown by the capital from the church at Frankenberg dating from the second half of the 14 th century, and it made the execution much easier, but also causes a far weaker effect of shadow. The edge of the bell is formed by a fillet in the simplest way, as may be seen in Fig. 510a at b. This fillet sometimes receives a wash above or is rounded off, either only above or also below; in the same way is also broken the lower edge by a chamfer as in Fig. 461 or by a cove. More rarely does the edge of the bell assume a form differing from the circle. A very peculiar example of this kind is shown by Fig. 468 from the vestibule of the cathedral at Dijon, where the ground form of the edge of the bell still strikingly recalls the Corinthian capital. Another example is given in Fig. 462a.

Height of the members of the abacus.

Of special importance for the good effect of the entire capital is a certain height of the abacus, at least for the previously described and mort structural forms of capital, in which the abacus generally assumes a predominant individuality. This height is in a certain proportion to the projection of the capital or rather to the size of its form of plan. It cannot indeed be formulated since in general Gothic architecture rejects all timid restraint by proportions. Already the endless diversity of its forms would make any such regulation impossible. Usually the ratio of the height of the abacus to the side of

the square lies between 1/4 and 1/2, being usually high in the early time and lower in the late time. The profile of the abacus almost always shows a vertical side, which becomes an almost essential part, if the angles of the abacus project beyond the edge of the bell. The upper edge of it is surrounded by a moulding, whose simplest form is a coved enlargement (Fig. 481). Instead of the upper fillet this cove is sometimes terminated by a round above, and then is also undercut as in Fig. 482, or a round is formed with fillet above as in Fig. 483. The moulding is richer when the cove is separated below by a recession or a little round from the vertical side as in Figs. 484 and 485. A particularly effective moulding is that shown in Figs. 486 and 487 (also see Figs. 461, 465 and 469). One projecting farther and strongly undercut in form is then shown by Figs. 468 and 469 (also 480), and Fig. 468 is one more antique. That lower side of the abacus denotes as a rule the extreme projection of the arch or rib resting on the capital, especially if the upper edge has a strong projection with small height as in Figs. 488 and 489, while with a steeper profile as in Figs. 481 to 485 can also the projection of the edge at least partly serve the rib as a support.

If the uppermost member of this edge is a fillet, then it e ends either in a rectangular edge or a chamfer as in Fig. 486. This chamfer is replaced by rounding as in Fig. 489 or even by an ogee, two examples of which are given in Figs. 490 and 491, the first from the lower blind arch in the cathedral of Chalons, the latter being from the corresponding part of the cathedral of Rouen. Further the chamfer can be formed as a wash, that r rises from the extreme edge to the deepest part between the intersecting members of the arches. Especially in the open air this arrangement has a certain practical utility, so far as it removes rainwater from the joint; itstarts from the member of the arch and must then be left on the capital, so that its form is that shown in the perspective view of Fig. 492.

Form of the astragal.

The astragal is most simply formed as a round as in Figs. 493 and 494, whose profile however is seldom an actual circular arc, but generally shows a free movement as in Figs. 495 and 496. But more decided is the effect if it has a lenticular shape (Fig.

497), which either consists of two symmetrical or unsymmetrical curves, the lower having a flat recurvature and in undercutting (Fig. 498), the latter being sometimes more sharply expressed by an added fillet as in Fig. 499. The form is richer if a hollow is cut in the lower half as in Figs. 500 and 501. The last shape then usually suffers a modification by the upper s surface formed as a curve being changed to a wash (Fig. 501a).

On capitals of very small projection is sometimes found a t transformation of the profiles described in the manner, that these increase in height in a certain way in the same proportion as they lose in projection, so that the members of the abacus are only formed out of a vertical surface. Such an example from the chapter hall of the monastery of Haina is shown by Fig. 536.

Capitals with polygonal and round abacuses.

Octagonal abacus.

We have already described above an abacus of square plan with angles cut off. By a corresponding increase of this cutting the square plan passes to the regular octagon. The advantages of the polygonal slab may be comprised in this, that the total m mass of the arch mouldings resting on the capital as a rule differs much from the square and therefore approaches a polygon, it being consequently that the angles of the square remain without loading, and a form of capital with an ugly projection diagonally would be required. The general acceptance of the polygon during all periods of Gothic art, the first of the octagonal form is mainly connected with the endeavor to express the vertical direction in an enhanced manner, and to assign to the horizontal an always subordinate place. The effect of the vertical direction, which at least in the interior is first of all expressed in the system of the pier and rounds and in the connection of the last with the lines of the arch, but at least when seen diagonally is substantially injured by the great projection of the rectangular angles. Men then already in the transition style had believed that they found an inconvenience in this, and usually placed on the square capital the lowest block of the arch in square form, but in the latter formed the transition from the right angle to the arch mouldings in a far richer way, than could be directly placing it on the capital.

To this is added, that the grounds deduced above from the o original form of the cut block for the square shape of the capital fail, as soon as it concerns the forming the capital for a group of shafts or for a compound pier. But also structural advantages are connected with the adoption of the polygonal form of capital. First this form more easily permits a departure from the shape regular on all sides that the square, but then the arrangement of the washes represented in Fig. 492 occupies much less height than for the square.

Thus now the octagon generally comes nearest to the springing of the vault consisting of several arches and ribs, hence for a single pier or for all rounds supporting several ribs it is the most suitable form of capital, so that the hexagonal capital set diagonally frequently corresponds better to the plan of a certain rib, and the predominant relative height of the profile of the rib may lead to making the angle placed in the direction of the rib more acute, nearly equal to a right angle and so pass over the regular polygon. An example of such a hexagonal capital is seen in Fig. 511.

Likewise in certain cases the peculiarity of the plan of the arch leads to the change of the regular into an irregular octagon. Such cases occur first on the pier arcades of polygonal choirs furnished with a choir aisle (Fig. 425), but still may also be caused in nave piers by special arrangements like those of the pier from Notre Dame in Dijon shown in elevation in Fig. 448 and in plan in Fig. 286. But yet more commonly is found to be caused such a departure from the regular form, because that on the capital of the round bearing the diagonal rib must set also the side rib or a round supporting it, so that then the abacus demands an enlargement of the surface enclosed by the regular polygon.

Since in all these cases the plan of the springing of the arch dominates that of the abacus, thus on certain shafts in the south side aisle od the Minorites' church in Cologne, the placing of cross arch, two diagonal arches and two side ribs on one capital led to a star shaped blan of the abacus.

Elevation of the capital with polygonal abacus.

The form of the elevation occurs with the just mentioned form of plan of the abacus in the same way as for the square. The body of the abacus borderes by the vertical side surfaces

are set on edge of the bell or projects over it, indeed only by the angles or so that the circle of the bell is inscribed within the octagon. Sometimes then the area of the margin of the bell projects by a chamfer formed on the lower edge of the abacus (Fig. 480). But in a certain way is modified the proportions of the leafy support. If as on square capitals this support of the projection of the corners, under each of the eight angles is one, therefore being eight on the entire capital, or sixteen if two rows of them are formed. Such an example is shown by Fig. 448. - similar form is sometimes already found on square capitals with cut off corners, as on certain rounds of the nave piers of the collegiate church in Mantes. This supporting of the angles of capitals is particularly necessary where this entirely or partially projects over the edge of the bell, as for the irregular form in Fig. 448. But where the last is not the case, where the capitals rest on the edge of the bell, this support of the angle necessarily disappears, and supports occur exclusively in relation to the edge of the bell, and in a certain sense form a strengthening of it in a way, similar to a cornice slab supported by corbels, and accordingly may also have a different number and position, when they are placed beneath the middles of the octagonal sides, are arranged in f fours on the bell, and either rise from it in the direction of the side as in Fig. 480, or in the diagonal of the square.

On the capitals of the rounds and columns of the 14 th and 15 th centuries, the transition from the round to the polygonal form of the top is made by a rounding of the surfaces and cutting off the angles from above downward. The profile of the capital then corresponds to the one developed from the round, i.e., a projection of the edge of the bell becomes superfluous, and the supports of the angles are omitted, or rather are replaced by the gradually formed angles, to which they stand in a similar relation as the ribs to the groins in the vault. Thus there also lies in the apearance of these angles the means for obtaining any irregular form of polygon for the top. On capitals with foliage the angles will be entirely or partly concealed by the foliage, and they appear only at the edge. The entire condition will be clearly illustrated by comparison of Figs. 502 and 503, the first of these representing a capital without leaves formed in the older manner from the cathedral of Dijon,

the latter being one shaped as stated above.

Circular abacus.

Allied to the polygonal is the round form of abacus, that is the rule on nave piers in England, occurs commonly on the earliest works in Westphalia and Hesse in Germany, and here first passed into the polygonal in the 14 th century. The advantages of it over the square are substantially the same as for the r regular polygon. On the contrary it lacks flexibility, which is peculiar to the latter by the transition to the irregular polygon.

Since the round capital remains in the form of the plan of the shaft or column, it merely has only to form a projection. However the arrangement still remains the same in at least the older examples, that was developed from the functions of the square and polygonal capitals. The bell is terminated by a fillet on which lies the round abacus, and even the supports retain their places. Fig. 510 shows such a pier capital from the middle aisle of the church in Wetter in perspective, and 510a is the corresponding profile. Fig. 505 then exhibits the capital of a round of the church at Haina, in which the original shape of the support is retained in more decided form, and Fig. 506 is a capital from a window pier of the church of S. Elisabeth in Marburg. But it cannot be denied, that this arrangement becomes purely conventional, that strictly taken the abacus is merely the moulded band of the bell, the leaves form less a support than an ornament of the bell, and that it therefore was only to bring a changed proportion into expression, like that on the capital of the church in Volksmarsen (Fig. 507), but even more decidedly in the capitals of the rounds in the cloister at Wimpfen im Thale (Figs.508 and 509). Retaining that older mode of treatment accordingly merely finds its full justification by the unsurpassed clarity of its effect.

Capitals of polygonal piers.

The capitals of square piers have in common with the capitals of round piers, that no transition to one form occurs in the other. Accordingly the projecting edge of the bell is omitted, the supports under their angles become necessary only by their greater projection and are repeated as a rule on the greater breadth of the capital or again at the sides.

Very beautiful examples of this kind are found in the choir

of the cathedral at Wetzlar, two examples of which we bring in Figs. 469 and 470. The arrangement of Fig. 470 is more consistent, as the greater projection of the angle here has also found a bolder support. On Fig. 469 is the extremely thoughtful arrangement of the leaves placed under and between the supports, the upper ones leaning toward the support and thereby producing an extremely living effect, as well as the beautiful a and powerful treatment of the foliage is to be considered, of which our Fig. indeed can only give but an imperfect conception. Fig. 469a then shows the ending of another support of the same capital.

The capitals of polygonal piers either continue in the basal form of the piers, or change into the square. In the first case their development in elevation becomes that of the round or octagonal, but in the latter corresponds to that of the capital of the square column, and the support where attached to the nucleus of the pier has a profile corresponding to the angle of the pier. It is further to be noted, that the transition of t the square is easier from the diagonally placed octagon, since then the corners of the abacus come to stand on those of the edge of the bell, but are best when the octagon of the edge of the bell is inscribed in the square of the abacus. However the border of the capital could also take the round form, and then in the body of the bell itself is to be made a transition from the octagon to the circle. This occurs thereby with angles over the astragal yet corresponding to the angles of the polygon and connected by the sides of the polygon ever become more flattened, and are entirely lost under the capital, and that in the same the connecting surfaces, originally plane, gradually increase and finally pass into arches corresponding to the rounded octagon.

Foliage capitals of the middle and later time.

Branches of leaves.

The forms of foliage capitals of the middle and later periods are rooted in the different arrangements of the early Gothic. Thus we have already stated above how at first the closed leaves expand and they conceal the body of the support, which accordingly serves merely to allow the projection of the bunch of leaves before the body of the support, and particularly when this original function of support in the changes mentioned re-

recedes more to the form of the capital. Therefore it receives an ever more subordinate form, and soon is expressed only in the stems becoming visible beneath the bunch of leaves, while the connection of the leaves with the capital is made by the angular or oblique direction on the surface of the parts last worked, which already appear in Fig. 466. Therefore the entire form consists of bunches of leaves with stems attached to the nucleus of the capital in either oblique or angular direction, and in the last case may be concealed by the leaves themselves as in Fig. 526. Fig. 507 exhibits an example of the kind from the church in Volkmarsen. These bunches begin with two or three leaves and are generally so arranged that the middle leaf forms a wrapper or a bold projection from the others. Fig. 511 shows an example taken from the northern stairs of the rood screen of Mayence cathedral, where by a similar arrangement, by the contrast of the strongly projecting middle with the flat side leaves, the quiet and clear effect of the support finds a happy addition. These bunches of leaves are repeated either around the capital in one or two rows, or sometimess the entire ornament of the capital consists of three leaves expanding from t the adjacent stems, as on a capital in the choir of S. Blasien at Mühlhausen (Fig. 513). In the nave of the same church is also found the less happy arrangement, where the leaves have their points turned down and attached to the edge of the bell.

Just as on the older capitals that support the attached leaves forming the lower series (Fig. 480) are sometimes replaced by a lower row of supports projecting out free from the nucleus, there is also found the converse. Then is shown by an example in Fig. 514 taken from the nave of Strasburg cathedral, the supports are replaced by a repetition of that row of leaves. The strong and powerful line of the latter allows them to appear not unsuited to the changed purpose.

In Fig. 467 we took an early Gothic example in which two rows of entirely regular leaves are attached to the nucleus of the capital. But usually this vertical position of the leaves is replaced by a more flexible bending sidewise. They either all bend in the same direction or the points of each pair separate; either parallel or divergent in two rows. They either lie open or are partly concealed. But almost always their arrangement is characteristic, such that it impresses itself as a happy

melody and affords proof, that the stonecutter who executed it worked not merely for decorative effect, but that he actually invented the use of a modern impression, in brief the use of a motive therein. A very simple and graceful capital of this kind is shown in Fig. 512 from the southern stairs of the rood screen of Mayence.

Attached branch of leaves.

Already in the works of the transition style are sometimes found capitals, whose ornament consists of a branch with growing leaves, flowers and fruits twining around it in apparently naturalistic form. (Note). Likewise sometimes occur on the early Gothic and round capitals branches attached between the supports, whose leaves expand in an entirely unsymmetrical way; thus on the capitals of the vestibule of the foundation church in Fritzlar, dating from the first half of the 13 th century. In the so-called Jews' bath at Friedberg are then found square capitals, on which by the systematic arrangement of these branches with the leaves growing on them and partly bending around them are replaced the supports of the corners, or rather compose a formless mass entirely covered by these branches; Fig. 515 shows one of these capitals. But soon and already at the end of the 13 th century, men began to replace attached branches of leaves by smaller branches with few leaves applied to t the body of the capital. Thereby was a means given for obtaining greater variety, when the buds, flowers, berries and fruits growing on those branches were drawn into the circle of ornamentation. Meanwhile such examples are already found on early Gothic work, although occasional and in severer treatment, yet for example here the bunches of grapes are yet enclosed by the leaves with the flowers seldom expanded. In Fig. 516 we give an example of such branches of the middle of the 14 th century from the capitals of the portal of the south transept of S. M Maria's church in Mühlhausen, and in Fig. 517 is a similar one from a pier capital in the interior of the same. Besides this arrangement allied to the before mentioned branches of leaves, also sometimes the entire capital is surrounded by such a branch, that either stands obliquely or lies like a garland on it, so that the leaves grow of themselveu on all sides and are interwoven by flowers and fruits.

Note.

Note. A beautiful example of this kind from the cathedral of Karlsburg in Siebenbürgen is found in the Jahrbuch of the K.K. Centralcommission. Vol. III. p. 168.

Treatment of the foliage.

Such a pretext then requires also a free technical treatment, and thus the thickness of the leaves is undercut in a direction forming a very sharp angle with their surfaces, whereby the acute angle at the corner is avoided by a chamfer or a rounding. In the same manner are also undercut the fruits, flowers and stems, so that freed parts are already found on certain strongly Romanesque capitals at Gelnhausen.

If the forms described now advance in the path of a progressive imitation of nature, then also occur beside them other motives likewise derived from early Gothic works, whose different mode of treatment led to the opposed results, and even in late Gothic works supplanted those naturalistic forms, to replace them by sketched outlines of foliage. Thus are sometimes found obscured in the attached bunches of leaves the division of the separate leaves by a freer mode of treatment, so that the three leaves appear comprised in a single larger and compound one. Such an example was already shown in Fig. 468. But beside it are also found capitals, which indicate the previously considered treatment of such rich leaf forms, indeed already from the first half of the 13 th centurw, as in the eastern side of the cloister of the cathedral in Erfurt (Fig. 518) in an extremely delicate manner of treatment, almost like miniature. And later and still more graceful forms of the same kind as then shown by the capital (Fig. 520) taken from the lower blind arch in the interior of the cathedral of Chalons.

Where the distance of such leaves from the eye was greater, on account of recognition they must naturally be in larger lines, as shown by some of the upper capitals of rounds in the cathedral of Rheims (Fig. 519). But generally the size of the surface occupied by such a leaf requires an increased modeling, of sharper accenting of the outlines. Especially worthy of imitation in just this respect is the mode of treatment peculiar to certain forms of capitals of this kind from the second half of the 13 th century, an example of which we give in Fig. 520. The characteristic of these particularly consists in this, that

by the arrangement and location of th

by the arrangement and location of the separate parts of the leaves are formed certain parts, and thus the clarity and quiet of the earlier capitals with supports are attained. Thus Fig. 520 repeats the geometrical principle of Fig. 480 in an entirely altered form. The supports are formed by the upper ends of the four principal leaves lying under the edge of the bell, w whose lower sides bend obliquely over the smaller intermediate leaves, covering the latter in a certain way., and in connection with them fomming a substitute for the projection of the lower row of leaves. Similar forms are then found on the capitals of the columns of the blind arches of Strasburg cathedral and in a more naturalistic way also in Freiburg. In the latter forms of capitals these thoughtful arrangements disappear, and from the 15 th century onward it was sought to animate these great surfaces of leaves by excessive movement of certain leaves, by increased bendings and swollen excrescences. Meanwhile are found yet in the last period beside this overloaded shape even more simply treated works distinguished by execution with a certain economy. We have given in Fig. 521 an example of the last kind which belongs to an alteration made in the church in Volksmarsen executed in the 14 th century.

As for what concerns the proper treatment of the foliage, we can only indicate this in merely a few great lines, since in general little has been done here in words and even in drawings made at small scale, and rich information can only be obtained by the study of the monuments. In the first Gothic works appeared the strongly conventionalized Romanesque leaf, but it soon disappeared. The Foliage of all periods of Gothic art finds its models in nature. Scarcely can a tree or a plant be mentioned, which was not drawn into the circle of ornamental forms. This diversity is sometimes so great in even the simplest and smaller works, that as Kreuser already stated, the endeavor to give all that lies on the earth its place in the church cannot be denied. But preferably are the maple, oak, ash and beech, white bryony, celandine, vine, ivy and hop, rose, aconite, turnip, clover and cabbage, have served as motives. The expression od motives is so far incorrect, since in the best periods an actual imitation of the natural vegetation was not intinded, but its forms were transformed by the style into the various orna-

ornamental forms.

Foliage of the earlier and middle periods.

In the early Gothic period the different leaves aid in producing the capital, they fulfil in a certain sense a structural purpose, and their removal would classify the body of the capital as a deformity. Accordingly it is the profile of the bell or the rounded ending of the support which prescribes the characteristic line of the leaf. The characteristics of the natural leaf must therefore be simplified according to these predominating simple curves and be reproduced in larger lines. Thus are found everywhere sharply accented contours, on which all little points and bends are omitted, broad surfaces either entirely without ribs or divided by merely angles and deeply cut channels, the modeling is kept simple, so that in the leaf itself are formed broad and soft tones of shade. But where the profile line of the leaf makes a short bend, this effect is sometimes strengthened by rounded swellings, whose bold shadows contrast with those softer ones, and so aid the entirety to a more vivid effect. But in the measure that the foliage becomes an ornament attached to the bell, men strove to multiply these effects, also to bring the swellings where they were indicated by the principal lines of the leaf, until also the latter gave the form a more wavy movement. Likewise here the natural leaf presents the motive, indeed in its full development at high noon, when the rays of the sun act thereon, and compel the same to certain bendings, which enhance the manifold effect of the shadows. There is found when several leaves are arranged in groups or bunches, an alternation in regard to the sides of the leaves turned outward. If this be not carefully executed, so that one leaf should have the form in which the other could be cast, but the peculiarities of the different sides be reproduced in the arrangement of the ribs as well as in the relief of the s surfaces. Soon the endeavor makes itself felt to even express mor shapely the basal form of the leaf, i.e., the geometrical figure forming its ground. The shape of the nucleus of the capital, thus the bell, is then an impelling influence for the f forms of the foliage only so far as the leaves lie thereon in their separate parts and bend below the edge.

Foliage of the later time.

Increased movement in the modeling as in the contours then

characterizes the foliage of the late Gothic capitals, so that the natural model is only recognizable in the character of the different outlines and also by the fruits connected therewith. The surfaces often bend very abruptly from each other or even appear imitated from diseased plants, show by their withering and recurved edges and points, and first of all show those rounds and swellings and depressions, in whose excess men long saw the proper nature of Gothic ornaments. Such a capital of the supporteng column is still moderately treated, from the f pulpit of S. Blasien in Mühlhausen is shown in Fig. 522. Another taken from the crossing piers of S. Maria's church there is that in Fig. 524, on which is still retained the arrangement of bunches of leaves, and only the leaves show this excessive modeling. There the incisions between the separate lobes of the leaves are evey deeper, as men began to seek generally the effect of shadows cast in these sinkings, and finally succeeded in giving the sinkings a power equal to that of the proper forms of the leaves, when they gave these definite forms like tracery. Finally even these forms were left and indeed to terminate them, the distinctness of the proper outlines of the leaves were neglected, when the points were allowed to combine from different leaves, and thus between them fish bladders or quatrefoil panels were obtained, but the proper character of the leaf was entirely obscured. Fig. 523 exhibits such a capital.

Different forms of capitals.

Capitals with little or no projection.

On the proper capitals of rounds in certain cases and for t the before mentioned reasons was the projection reduced, so t that the outward bending of the edge of the bell was lessened or entirely omitted, the body of the capital entirely corresponding to that of the column, and merely the astragal separating it from the latter. There the foliage can still be arranged in the same manner as in the actually projecting capitals, and consist of one or more rows of attached branches of leaves. Such capitals are found in the bitter chamber of the monaster: of Haina (Fig. 526). A different form is conversely shown in the capital from Mühlhausen reproduced in Fig. 524, where the plan of the capital continues above the astragal in the capital and ends beneath the strongly projecting octagonal abacus (Fig.

(Fig. 524a)., in whose hollow lies the branches of leaves of t the upper row. This conceals in such wise the transition tc t the octagon, and at the same time forms a support for the edge of the abacus. Conversely the lower ones only have their stems attached to the nucleus of the cylindrical capital.

The comparison of the two last forms permits that of Fig. 524 to appear more justifiable, since the upper bundles of leaves also fulfil a real purpose, which entirely disappears in Fig. 526. Yet the effect of the latter is better, since the projecting leaves of the upper row present a substitute for the lacking edge of the bell, although an apparent one, therefore the effect of the older form of capital comes closer.

The just mentioned capitals lacking a projection of their o own body are everywhere in place, where the plan of the springing of the members of the arches harmonizes with that of the column or pier, as for example usually occurs in regard to the window tracery and of the jambs. Strictly taken in these cases the apacus is also superfluous, and the delineation of the ground line of the arch that alone concerns it, is effected by the foliage attached to the column above the astragal. The effect of the latter thereby is nearer the usual form of capital, because seen as a whole it terminates at top in a horizontal line. Capitals of this kind are found in the windows of t the southern side aisle of the minster in Freiburg in different forms (Fig. 525)., further on the cathedrals of Chalons and Evreux. (Dictionnaire d'architecture, II, p. 533). More rarely is found a similar arrangement on the little columns at the sides of portals, as at S. Stephen of Mayence.

Capitals without astragal.

On some forms of capitals of the late period the astragal is omitted, and it is either replaced by the entwined stems as in Fig. 522, or the separate leaves are laid directly on the shaft of the column. Such forms will result from omitting the lower circle of leaves and the astragal in Figs. 524 and 526. Thereb: in the first Fig. the leaves would only form a decoration of the abacus, and this character becomes one more important if their stems remain in the hollow a in Fig. 524., so that the entire form is represented as a union of the otherwise assumed triple form of the capital, in which the hollow a forms the bell, an ogee member the abacus and the lower surface the astragal.

Capitals of the last kind are generally found in these simple churches of the 14 th and 15 th centuries, an which the octagonal basal form of the piers continue in the side arches, and in the simplest cases consist of a flat ended above by a fillet and below by a hollow (Fig. 445), but also usually take the form represented in Fig. 538, and in both cases may be covered by foliage or left plain. In a way the form represented in Fig. 527 is also to be counted with it.

The ornamentation of the members of such capitals is generally found in heads, indeed either naturalistic or combined with foliage (Note), and is also effected by entire figures, forms of animals, suspended shields of arms, inscriptions, etc. Forms of figures or heads are also usually found as actual supporters treated like caryatids; as on the beautiful fountain in Nuremburg, the heads in a sense are stuck on the shaft of the column and replace the capital. Just as the heads, the animal forms are usually interwoven in foliage, or their tails extend in form of foliage. Especially commonly reappear as in the keystone of Fig. 233 the representation of two animals with interlaced necks also on forms of capitals. If now an many cases the meaning of these forms to us is only possible by more or less risky hypotheses, it yet arises from the entire character of mediaeval art as well as the greater number of examples in which this is made entirely clear, that such a one was the basis in all cases, and that entirely removed from Gothic art was t the desire in so many works of modern art, to employ figure s shapes for themselves alone without any relation whatever, and even in frequent repetitions.

Note. Leaf heads according to Villard de Honnecourt.

As an example of a capital with figure ornament may serve Fig. 528 from the second half of the 14 th century, from the church at Gottsbüren near Cassel. In the upper are placed the heads with a purely ornamental purpose, while those figures of a praying nun projecting from the shaft of the round in a sense s support the projection of the edge of the moulding above, so that both motives mentioned are found combined.

Capitals without ornament.

The need of economy or more frequently the striving for a c certain simplicity has sometimes led to the complete absence of all ornament. Capitals of this kind are found in the cathe-

cathedrals of Dijon and of Narbonne, in the foundation churches of Colmar and of Troyes, in the church of the Minorites in Cologne and in many other churches, particularly of the mendicant orders. In the simplest cases they have the form entirely harmonizing with that previously mentioned, so that the foliage merely seems omitted as shown in Figs. 502 and 529 from Dijon and Treysa. (The latter being a circular plan). The simple plain surface of the bell would then usually be animated by painting. Such an example is shown by the little column of the jamb in the interior of the window of the choir of the Wiesen church in Soest, where on these surfaces is painted light green scroll work on dark green grounds. That also the relief ornament receives such animation by alternating colors will be more fully explained at the proper place.

But the nearest substitute for the lacking ornament is afforded by the enhanced richness of the members (Figs. 530to 534) in connection with that in Fig. 503 and in contrast with the transformation of the transition from the polygon to the circle shown in Fig. 502.

Penetrations.

Generally it is the arrangement of these transitions, which offered to the characteristic striving after artificial penetrations in the later period of Gothic art repeated opportunities have changed the forms of capitals. The ground motive of these forms is the penetration of the cylinder by a square or polygonal pier capital formed about like Fig. 527.

Already from the beginning of the 14 th century is found a capital of allied shape on the southern side of the cloister of the monastery of Haina before the no longer existing fountain chapel. The pier is round, but the springing of the arch is shaped like a square set diagonally, two sides ofit extending in the profiles of the arches on both sides. The capital represented in Fig. 532 therefore affects the transition from the abacus a likewise having the shape of the basal form of a square set diagonally to the cylinder through the four sides of the pyramid b penetrating the latter, with which again bineath the angles of the square intersect the four funnel shaped corbels c. But the side surfaces of the latter are not smooth as shown by the plan drawn at d, but are divided by flat hollows, and their lower points rest on branches of leaves e. The-

Therefore the entire form still shows a certain relation to t the older form of the square, in a sense an application of the principle of penetration of the same, and is often characterized preferably by its freer treatment than the forms belonging to the later period, in which indeed penetrations of very different forms prevail as a mystery.

A shape of the last kind is shown by Fig. 533, that represents the penetration of a capital belongong ao an octagonal into a cylindrical shaft, so that the lower octagon of the capital is inserted in the circle of the cylinder, and the little points make the transition. The latter become richer when the capital is formed below by combined members instead of a curve, for example as given in Fig. 534. The latter capital is shaped on a hexagonal plan, but the transition of the latter to the cylinder is placed under the astragal, so that the body of the bell affects only the increase of the projection. The plan is seen in Fig. 534a. The members in elevation thus make the transition from the hexagon inscribed in the circle of the cylinder to that circumscribed about it, so that between both hexagons lie the intersections, than can easily be constructed. In Fig. 535 is then the transition from the circle to the octagon is replaced by a simple slope instead of a moulding, and this extends around the circle of the shaft, hence forming a part of the surface of a cone. Thereby arises the intersection of the cone with the octagonal prism; Fig. 535a shows the form in perspective.

In the same manner would be formed the transition from a compound ground form to any simpler one, thus for example from the octagonal pier to a square capital, or from any ground form to one set diagonally, and also finally and generally in an irregular way are all transitions from one to the other.

The further execution of these forms is suited in a higher degree to the working of wood than that of stone, and therefore also finds its chief employment in the treatment of wooden stalls and in furniture.

Prismatic capitals and the like.

In Fig. 535, which represents the capital of a round belonging to the church at Immenhausen, is omitted all reference to the original bell form, and the body of the bell is replaced by a short octagonal prism. The surfacer of the prism then give

opportunity for richer treatment. The simplest case would then be to form rectangular panels recessed by a moulding, whse grunnd is then occupied by a rosette or other foliage; or these panels might receive a form very like tracery, as by cusps inserted in the sides of the rectangle, as on the middle piers of the church at Bornhofen, and further by any more complex pattern. Such richer forms are more rarely found on actual rounds than on those modern columns near the eye, which are placed as supports of statues or any more ornamentel arrangement, like a shrine, finial, or as a starting point of a corbelling as under pulpits, bay windows, etc. Here is then found sometimes the almost overloaded artificial arrangement, that the tracery is perforated and within the surfaces thus formed as in a cage swings a body of capital shape from the upper abacus of an actual capital beneath it, so that the object ornamented by tracery represente in a sense the pedestal of the figure or an intermediate, as Fig. 537 shows in section.

With the forms of this kind are to be counted those extremely rich capitals of the piers of the cathedral in Milan, that are likewise formed by the insertion of prismatic bodies, whose side surfaces represent niches arranged beside each other, separated by finials and crowned by gables, but thereby the proper laws of the formation of capitals are just concealed by their magnificence.

Arrangement of the plans of capitals on compound piers.

Capitals on compound piers of the early time.

On compound piers each round receives its separate capital with independently expressed ground form of the abacus.(Note). The junction of these different capitals is thus arranged according to the plan of the pier and that of the different abacuses. According to those oldest ground forms of piers with rectangular projections and with rounds in the angles reproduced here, therefore is also the general form of the capitals of t these rectangular recessions. For oblong bays of the cross vault, seldom for square bays, sometimes appear modifications, that the square capitals under the diagonal ribs stand in the same direction as those set diagonally. So that generally there is soon with a freer treatment of the pier or group of shafts in the direction of the axes of the square or polygon of the abacus concerned. In the direction of the arch may be placed

either a side or an angle of the folygon. For the good effect of the arrangement it is essential, that the sidesof the polygon does not meet in two acute angles, and that reference thereto the choice and location of the polygon requires above all a good general form as more important than a rich ornamentation.

Note. Exceptions to this, as for example they appear on the columns of the jambs of portals, will find their explanation at the proper places.

Already for stepped forms of piers cannot usually be expressed all little members in the total form of the capital, and the same condition is also found in the rounds connected by hollows in the earlier period, and the hollows and properly beneath the abacus of the capital. In the same manner like the abacus, also the angle leaves of the earlier form of capital grow together, indeed so that their ends entirely disappear as shown in plan in Fig. 539, or two such supports join in the s same leaf knob (Fig. 540).

Only the astragal sometimes continues around on the hollows, which then extend above it and stop beneath the capitals of t the rounds, as shown in the same Fig.

Capitals on round piers with little shafts.

On round piers joined with four rounds the nucleus of the p pier receives a capital, which intersects those of the rounds and either has the same projection as they, as on most German works, a larger one on the older French cathedrals, or a smaller one if the nucleus of the pier is only surrounded by the abacus, as on certain Westphalian churches (also see S. Jacobi at Einbeck, Fig. 428).

As a rule in Germany, the pier capital remains in the round form. But by the animated projections of the shaft capitals, even if the latter remain round, the rather stumpy appearance of this ground form is modified, and passes into the more distinct effect of the square set diagonally (see piers at Wetter in Figs. 427 and 510).

Yet the effect will also be more tasteful here, if the capitals of the rounds take a polygonal form, since usually the contrast of the short sides of the polygonal and circular parts is very happy. This arrangement is already found in the nave of the church at Haina, dating from the second half of the 13 th century, that is about the same at the old cathedral of Dijon

of the church of the Minorites at Höxter (Fig. 427 right and Fig. 429).

But likewise the pier capital may assume a polygonal form and be combined in the same way with polygonal capitals of the rounds. Very happy in this respect is the arrangement of the pier capitals in the cathedral at Rheims, where the nucleus of the pier bears a capital in the form of a diagonal square, whose angles combine with the four octagonal capitals of the rounds.

Capitals on compound piers of the later time.

On the compound piers of the middle and late periods as a r rule is found only a very small and sometimes no projection of the plan of the arch, and accordingly only a small projection of the capital. Since now the proportion of the widths of the hollows to the diameters of the rounds has also increased, then also the extension of the hollows under the capitals of the r rounds is no longer possible. Accordingly the capital follows the entire plan of the pier, when also the line of the hollows is accompanied by concentric arches. But there it may be necessary to replace the latter arches by straight lines to establish continuity. An example of this kind is shown by Fig. 438. Since where the hollows as members of the pier continue as slightly modified or unchanged forms in the members of the arch, each greater projection of the capital in them will be superfluous, and accordingly the capital extending around first assumes a simpler form, while the projection of the bell as w well as the foliage thereon is omitted.

That complete agreement of the hollows of the pier with those of the arch, the capital in the former becomes superfluous, and it results that the arrangement common from the middle of the 14 th century onward, whereby only the rounds have capitals, which stop horizontally in the hollows continuing in the arches, conversely to the hollows and angles of the members in the older style, those at parts representing proper angles of the nucleus of the pier stop vertically under the capitals of the rounds.

The form mentioned then makes the transition to the piers w without capitals.

Elevation of capitals of compound piers.

Different heights of the capitals.

In general is the rule correct, that the upper edge of the

capitals denotes the base line of the arch. Following this therefore leads to a complication of the rounds or little columns everywhere, and especially to an unequal height of the capitals of the rounds, where the base lines of the arches lie at different heights. Such examples occur in window tracery, when the base line of the tracery falls below that of the arch. But it more necessarily results on vault bays with strongly differing lengths of sides, first already on the vaults of every polygonal choir, where a separate round is arranged for each rib. When here the side arches and the diagonal ribs must commence on o one base line, then as previously stated, their crowns must either lie far below the crown of the vault, or their forms must become extremely pointed. Both will be avoided by raising the base line of the arch of smaller span, and therefore according to the preceding rule the capitals of the rounds under them.

By these unequal heights of the capitals will be increased the animated effect of the whole, as well as if the capitals of the different piers filling the window in the space between the rounds of the side arches again fall at different heights as required by the forms of the tracery.

Separate stilted arches.

But for all rounds the same height of the capitals can be obtained by a corresponding stilting of the arches of lesser span. But diagonal ribs and side arches rest on a common round, then also stilting would be avoided by the arrangement of smaller rounds set on the capitals for the side arches, as for example in the choir of the church of the Minorites in Duisburg.

Similar conditions result on the capitals of the detached p piers with unequal lengths of sides of bays. Thus for a round pier connected with four rounds and separating two aisles of equal height, the capital of the round supporteng the diagonal rib, whose height is fixed by the base line of the rib, in the simplest case extends around the entire pier, and the narrower dividing arch would be stilted, indeed this arrangement becomes a necessity, if as in Fig. 427 the diagonal rib, whose span exceeds that of the cross rib, rests on the nucleus of the pier, therefore determining the height of the capital on the latter. On the contrary, if the nucleus of the pier only supports the dividing arches, its capital and the rounds belonging to it a also in the base line of the dividing arch, can therefore be

placed higher than the capitals of the rounds bearing the cross and diagonal ribs. Then the higher capital of the pier will cut into the continuation of the vertical surface of the compartment lying in the profile of the rib. Still more easily would be found an arrangement of the last kind in connection with t the plan of pier represented in Fig. 426.

An adjustment of both systems is found on the round piers with four rounds of S. Blasius in Mühlhausen. Here at the height of the base line s of the diagonal and cross ribs, the capitals decorated by foliage extend around the entire pier and on them rest the dividing arches with a simply profiled plan in a manner, so that the lower end of the profile stands on the round, as Fig. 451 shows in perspective. The dividing arches are stilted high, but the weight of their actual base line is indicated by a slightly projecting moulding a intersecting the hollow of the profile of the dividing arch, so that the proper capital nf the pier rests on a second lower pier, which bears the dividing arch.

Separated capitals of the rounds.

The arrangement of the capitals at unequal heights occurs e even more in those piers of the later period chiefly composed of hollows, on which only the rounds have capitals, as they a are found in extremely rich forms in the choir of S. Ouen in Rouen. Here rises each of the rounds, which form the body of the pier in their construction by means of hollows, independently of the nearest one, so far as the base line of the arch requires and then bears its separate capital, denying all relation to the entirety of the pier, therefore being only in a certain relation to the diameter of the round. We have here n not only unequal heights, but even unequal heights of the separate capitals, and consequently a loss of all independence of the pier, and the transition to the form without a capital, c continuing in the members of the arch.

From the rule given above, that the base line of the arch determines the location of the capital, there is also found the exception formed by the stilted arches, there is still a second that consists in this, that the capitals in a sense are taken from the height of the arch, when they range in the base line with the under edge of the astragal. This condition is most

distinctly expressed in the smaller arcades of Notre Dame at Dijon (Fig. 542) or the windows, triforiums, etc., where the arch lines above the clear opening are full pointed arches, while the concentric plan of the shaft continued through them into the arch represents a broken arch resting on the capital. The entire form harmonizes its nature with the change of the springing of the arch shown in Fig. 273, as of use, that first the intersection of the round is avoided and second the little column receives a more slender form. An application of it to a pier composed of several columns would lead to an unequal a arrangement of the heights of the capitals on them.

Height of capitals and of ashlars.

On the proportion of the entire height of the capital, to the diameter of the column or of the projection as well as of the other parts to each other, no determination is possible. An eye trained by the study of old works in connection with the most accurate understanding of the special conditions can alone find the right one in a given case. If already in general a larger column requires a higher capital than a slender round, a direct proportion of the height of the capital to the diameter of the column does not exist. In ordinary conditions the capital is cut in a course either with or without the abacus, therefore the nature and size of the material already give certain starting points and limits. Execution in a single block is therefore first required for a compound pier having the same height of capitals for the rounds and intermediate members without regard to their different sizes, so that also for a round pier with four little shafts their capitals with the same heights continue around the nucleus of the pier as shown in Fig. 510.

When extreme conditions of the size of the whole allow the height of capital possibly obtained by one course not to appear sufficient, and permit the execution of the capital in two courses lying above each other, no construction is yet known to us in Germany, where this construction is expressed in the form of the capital. On the contrary forms affected thereby are found in certain French works. For when the necessity of a greater height of the capital concerned only the nucleus of the pier, the lower course was only utilized in the capital for the latter, but for the shafts and the continuation of the shaft occu-

occupied it up to the capital formed in the upper course, as in the cathedral of Amiens. On the contrary there is found in Rheims beneath the capital of the rounds made in the upper course, a second slightly projecting one, whose decoration remains without any conncetion with the upper one, and it lies beneath the astragal of the latter, which continues in the foliage of the proper capital of the pier, while the astragal belonging to the lower capital extends around beneath the capital of the pier. But since the astragal belonging to the upper capital of the round has only a small projection, the entire form only differs in principle and not in effect from that common in Germany.

In contrast to the form of the capital of one block or of two lying on each other, in any case must we return to the peculiar arrangement of the older Westphalian works, whereby capitals were only arranged for the rounds, but whose abacus extends concentrically around the round pier. This arrangement had its origin in an execution of the pier in smaller materials, which were then bound together by capitals formed of longer cut stones, so that the piers lack combination by an united capital; accordingly it forms the transition to the form of the pier in brick construction, to which we shall return later below.

Bases of columns and of piers.

Relation between capitals and bases.

The base has the purpose of continuing the pier to the foundation, hence providing an enlargement of the section and a t transition from the more complicated and smaller area of the former to the square and larger form of the latter. Accordingly the base has in common with the capital the projections and transitions. Statically a projection of the capital and base satisfies similar conditions. Since in a quiet body all forces occur in pairs, at every cross section of the support the weight of the upper part is opposed by an equal reaction of the lower part. If a projection exists, i.e., if the compression is transferred to a larger area, then is it the same under the effect of these two forces, whether the projection is above ((capital) or belon (base).

Since when capital and base are under like conditions, i.e., where an entirely definite compression is transferred to a projection of distinct form and size, it can accordingly be j

justified to form them exactly similar, which sometimes occurred in the Romanesque period and more frequently in late Gothic.

But as a rule the conditions for capital and base are not exactly similar, rather their functions differ in more than one point, so that a different architectural treatment is required for both.

First in massive piers the compression at the top of the base is considerably greater than above the capital, since on the capital is only the overload, while on the base further rests the weight of the pier itself.

Then on the capital are set much divided members, while on the base it attains only a larger simple area.

Freely projecting ornamental members that are in place on t the capital must be avoided on the base chiefly for reasons of suitability.

The capital is seen by the observer more obliquely than the base.

Finally an independent pier already has much an effect as a separate form, that its upper termination must express not only support, but also a crowning.

All these requirements were taken in the most varied ways by the middle ages according to the conditions; capitals and bases accurately show in a high degree the refined stylistic feeling, that permeates all the better works of the middle ages. The Gothic knows no ready made column, that like the turned wooden pieces in a child's box of building blocks can be employed here or there at pleasure; it rather creates for each place a support belonging there.

Members of the base.

Consider first the form of the base of the simple column on round, then here as on the bell of the capital, the nearest purpose is an enlargement of the basal form, which by the base mouldings extending concentric with the circle of the column is formed its proper base. But in regard to the shape of the latter the ground form of the mass of the base at first remains without influence, for it is similar for the square, circular or any polygonal form of the latter.

At first the base in many older works is still found entirely corresponding to the Attic type; thus on the round bases in the choir of the church at Volkmassen (Fig. 543). But to the

idea of spreading the pressure from above underneath as opposed by the form of the torus in a semicircle, that expressed a similar function of this member above and below. Since already the Greeke partially departed from the pure circular line, and partly expressed these conditions by the section of the torus accordind to b in Fig. 543a instead of like that at a, it found in Gothic art a still more emphatic accenting by the flattened form entirely differing from the circular line. On steep bases the torus takes the form of Fig. 544, or from low ones those of Figs. 545 and 546 (from the monastery at Walkenried) or of Fig. 547 (round pier from Notre Dame in Dijon). But the shape of this line is of endless variety, and it is about between the limits shown in Figs. 547 and 548.

The upper torus, which is still separated from the shaft of the column by a fillet, then rests directly on it, indeed with a continuation by a curved line from the distance a b in Fig. 547, whereby results a sinking, from which again sometimes a splay rises to the shaft, yet it also generally takes the form of a flattened curve, or of one pressed in at the middle, as shown by the profile of the base in the church at Mantes represented in Fig. 548. The latter line is capable of the most varied shape, according as the convex or the concave principle predominates. Both toruses are separated by a deeply cut scotia as in the Roman Attic bases, that either is combined with little angular members as in Figs. 546 to 548, or as in Fig. 551 directly cuts into this. The lower margin a of it remains either as in Fig. 548 within the vertical drawn through the extreme outer point of the upper torus or projects beyond it. Likewise the lowest point of the scotia lies at the height c (Fig. 547) or may sink below it. Thus generally the line of t the scotia itself is very variable, and its purpose is chiefly to be sought as producing a very deep shadow, thus animating the effect of the members by the contrast with all soft shades of the flat profile of the torus.

A definite construction of such a profile is impossible as in all similar cases. As for what concerns the proportion of the height to the projection, both are sometimes equal, sometimes the first slightly exceeds and sometimes the latter. Bases lying above the eye of the observer already in the Romanesque time were steeper (Figs. 548 and 544) than those below it.

For the proportions of the different members to each other it must be characteristic, that the lower torus occupies at least half the height of the entire member. In Figs. 549 and 550 we attempt by aiding lines to give only some general starting points for indicated constructions.

There a simplification of the form of profile just represented was then developed, peculiar to the middle and later periods, when men came from that to omit the fillet over the scotia, as having no structural importance, and indeed when either the upper torus was omitted or the hollow, whose size was also reduced and had lost its distinctness by the loss of the fillet. By the first way was reached there Fig. 550 through Figs. 551 and 552, on the latter after widening the hollow and reducing the lower torus through Figs. 553 to 554 and 556. The former shape is already found in the boss form on the little colums on the jambs of a portal in the south transept of the cathedral at Mayence, dating from the middle of the 13 th century, then fully developed at the end of the same century in the nave piers of the church at Naina, and lonsequently has a better effect than the second, that from the beginning of the 14 th century is found in the church of the Minorites in Soest, and then by reduction and omission of the torus disowns all characteristics of the base as such, passes from Fig. 356 for example in the cathedral of Frankfort. But the form of the last is in exact relation to the general arrangement of the later forms of bases, particularly of the transition from the circle to the square or polygon of the mass of the base.

The base with the part of the plinth lying beneath it is always wrought from one block. The low printh of the earlier Romanesque column soon becomes a higher prismatic body.

Subdivision of the height and members of the base.

The proportions of the height of the base are just as flexible as its projection. The total height is from 30 to 60 cm, or sometimes amounts to more. The usual height requires two courses according to the ordinary dimensions of the cut blocks, and this division in two parts is expressed in a projection of the lower part, made by a wash, cove or by a compound member. This division into two parts is then almost typical for the r richer forms of bases, even where the entire base consists of a single block, as on the little columns of window jambs and

With the flat base forming the margin of the upper stone, t the members of the lower block usually contrast by their steep direction that give a decided stamp to the outline of the entire base. It consists mostly of a scotia, that forms both divisions of the base by fillets, chamfers or roundings, more rarely is entirely simple; since it is chiefly seen in elevation, its effect must be obtained by a sharper separation of the upper torus from the body of the base, a sinking below the horizontal extension, as at a′ in Fig. 548. Figs. 548, 558 and 564 give different examples of it.

In the later time of Gothic art men began to make these opposed directions of both members less prominent, making them p parallel or reversing the proportions and forming the lower one flatter than the upper.

Of the two divisions of the base separated by the lower member, as a rule the upper is higher, yet even this difference also alternates, sometimes disappears or becomes opposite. The last condition leads to an always lesser height of the upper division and finally to a combination of both members. For French works of the 14 th and 15 th centuries is characteristic the form in Fig. 557, resulting from the simplification and c combination of both members of the base, and it is repeated w with tiresome monotony.

Square bases.

For the proper body of the base as for the abacus of the capital, the square form is indicated for the same reasons. Indeed it is nearer than there, since it is already given by the form of the foundation. The upper surface of this square is then c concentric with the base accompanying the circle of the column, indeed first in the same manner as in Fig. 342, the edge of t the edge of the bell is placed beneath the abacus, so that the extreme circle of the base is circumscribed in that square. The remaining four triangles lying in the horizontal surface were already in Romanesque art covered by corner leaves formed with extreme diversity for static and esthetic reasons. The arrangement of the latter was continued during the 13 th century in France as well as in Germany, for example in S. Blasien in Mülhausen, in the aisle of Freiburg cathedral, and even on the columns of the rood screen in the cathedral at Lübeck (Fig. 562). At first are they found in that very rich variety peculiar tothe

Romanesque style (see the usually recurring form in Figs. 559 and 560, then they take a strongly recurved shape of leaf found almost everywhere (Fig. 561), so that even this monotony is characterized as vanishing. Fig. 558 shows the corner leaves on the bases of the piers of the cathedral of Rouen.

But since such a leaf could not cover the remaining area with mathematical accuracy, then according to the principle of Gothic art that allows the horizontal surface only as a bearing or floor surface, a chamfer of the upper edge of the square plinth of the base became necessary. Thisi chamfers denoted by f in Figs. 549 and 550 then extend beneath the torus of the b base, so that the latter projects at the middle of the side of the square beyond the chamfer. By the increase of this chamfer, which then usually changes into a hollow, was given a means of avoiding the horizontal triangular surface, or of reducing it to a minimum, and thus the covering leaves were omitted. Of t the contemporary bases in Figs. 545 and 546, the former has covering leaves and the latter has not.

By the increase of this chamfer mentioned, there increased in the same proportion the projection of the torus and thereby the possibility of its being injured. Therefore this member frequently did not project beyond the entire sides of the square, but only about one third of it, to e and f in Fig. 564 showing the way, so that between e and f the vertical surface remains and extends under the torus. The entire arrangement is preferably formed, if the circle of the torus projects a little beyond the square body of the plinth as assumed in Fig. 564.

An increase of this projection then leads to the above contrasted proportions in plan, so that the square of the plinth is inscribed in the outer circle of the base, whereby the edge of the torus forms a projection at each side of the plinth, w which makes necessary a special support at the middle. The latter is then obtained by a corbel remaining from the mass of the plinth, which as a rule is shaped like an irregular part of a polygon, producing a wider base, as shown by a b c d in the upper half of Fig. 565a. Thereby only the part a b c of the torus forms a free projection with a horizontal surface beneath.

But the size of this surface, as well as of the before mentioned corbel a b c d may be reduced, if the square of the plinth projects at the angles beyond the circle of the base, as at

f in the lower quarter of Fig. 565a. But these projecting angles of the plinth then first are broken beneath the base as at g h i or k l m n as in the right quarter of the same Fig.,and above the lower member of the plinth by a wash made in its original form, so that the plinth receives the form represented in elevation in Fig. 565. Likewise with the arrangement of the corbels mentioned with that represented in Fig. 564, the horizontal member breaking the angles of the plinth were connected. Such bases are found in the church at Colmar. The corbel mentioned under the edge of the base generally received a simple form. That given in Figs. 565 and 563 is almost typical for it. On the church of S. Thomas in Strasburg as well as on certain piers of the minster they are found replaced by leaves. Fig.565b. 564 A peculiarly rich form of plinth comprising most shapes mentioned is then found in the church at Gelnhausen (Fig. 563). Here the circle of the base projects a little beyond the middle of the sides of the square plinth. But the latter is divided by a recess, in which is visible a plinth in a quatrefoil plan set under the circle of the base. Above the lower plinth this sinking then extends into the square, while the moulding formed extends horizontally.

Thereby the sides of this recess cut beneath the base above form a projection from the circle of the base, which is covered by the corner leaves, just as the projection of the quatrefoil permits the arrangement of the before mentioned corbels. The effect of the entire form is extremely vivid because of t the varied shadows, that result from the rich alternation of the ground forms, and it is further of interest, that it properly is represented as an intersection, which results from the system of the square plinth transferred from Romanesque art with the round form predominating in the neighboring early Gothic works in Hesse and Westphalia.

By the projection of the base beyond the plinth given in Figs. 565 and 565a results both the origin of the twofold shape for those smaller bases, that are formed b: a single cut stone, from the necessary mass of the latter. It expresses this nowhere more clearly than on the bases of the chapel in Paris made od two layers and accordingly divided in two parts, on which the upper base with regard to that projection above the joint extends with a wash, so that the entire base is really triply

divided.

Instead of the recesses at the angles given in Fig. 565, these may be chamfered, by which the character of the square ground form given in the return at a in Fig. 565 always continues. Generally the skuare ground form of plinth recessed from all bases removed from human contact is most suitable, as on the little columns of window jambs, mullions, etc.

Polygonal bases.

But for all bases standing on the floor, is for all piers of vaults and shafts the square form has certain disadvantages in consequence, because by the wide projection of the angles they make a large area inaccessible, and are easily injured by repeated contact with the rectangular angles. Consequently it was next to chamfer also the plinth like the capitals and finally to adopt the polygonal ground form. First are the singme round or polygonal piers, in which also the forms of the transition to the square shown in Figs. 564 and 565 require a considerable magnitude and therefore demand an inconvenient height. In a f far less degree can the polygonal square form be obstructive on the bases of the rounds, and at the same time for piers composed of separate rounds with the combination of the separate square plinths, when those supporting the diagonal ribs are set diagonally, and a polygonal ground form easily results. T Therefore are sometimes found in the same work the round nave pier with the octagonal, the rounds and cluster of shafts witth square bases, thus in Notre Dame of Dijon.

Likewise also the separate square bases of the rounds can be combined on a polygonal support.

The adoption of the polygonal ground form for the plinth renders more easy for the considerable diameter of the column the formation of that transition, and makes it superfluous for a smaller diameter, since then the projection of the edge of the round base over the sides of the polygonal plinth is so small, even when the polygon is sincsiped in the circle, it can remain without support.

Only for detached piers can be produced a regular polygon, while on all whether a pier, a wall surface or below rounds joined together, the junction of the plinths sometimes presents the formation of a regular polygon, permitting only a square to originate from the corresponding division of the circular

segments remaining free. Thus in Fig. 566 the polygon of the plinth of the round resulting from the division of the curve into five parts, would be irregular in completion.

The different forms of the transition shown in Figs. 564 and 565 also sometimes are employed on the polygonal plinth for d decoration. Generally are found particularly the cuts taken from the angles of the polyponal plinth as in Fig. 565, and by the stronger accenting of the angles, they accent the vivid effect of the whole.

Round plinths etc.

The previously mentioned round plinths likewise in most cases consist of twofold projections characterized by members, and the forms of the members are indeed the same as for the square and polygonal plinths. Only the proportions change so far, that while on the latter the torus terminates the proper base a and the chamfer below belongs to the plinth, here a decided separation is absent. Hence the torus ending the base is combined with the chamfer, thus receiving a recurved shape as in Fig. 567, or is also omitted and the mouldings of the base end below with a hollow or chamfer as in Fig. 556.

What is said here of the round base is likewise true of all that have no transition from one ground form to another, or where this transition is formed by the moulding itself. This last arrangement that corresponds to the form of capital shown in Fig. 503 is shown in perspective in Fig. 568. The transition from the round to the polygon more easily occurs in the scotia of the base than in the bell of the capital, since the former becomes horizontal at a, so that as given in Fig. 568, only the insertion of the triangle a b c in the horizontal plane is necessary to effect the transition. In a similar way on small wall columns the plinth mouldings sometimes begin with a little horizontal plane (Fig. 568a) that makes the transition to the polygon.

A combination of the plan of the round with that of the polygonal plinth is indeed shown sometimes, chiefly in forms usually occurring in the 15 th century, where the plinth remains round, but the polygon has the flutes cut for a short distance below the base, that again by a wash pass into the circle. But sometimes this fluted body also passes into a polygon instead of the circle of peculiar form, such plinths are shown on rec-

recurved rounds as shown by Fig. 567.

Here the moulding of the base extends around the plan of the round, so that the little projection a intersecting the moulding has the width b c at the plinth. The arc b d of the plinth is then divided into three parts and each of these parts and also the breadth b c is hollowed in a flat circular arc, so t that the surface c b e f d becomes the form of the plan. These concave surfaces then stop below the base at a in Fig. 567, and at b pass by a wash into the corresponding polygon, which receives an entirely irregular form and by another wash broadens into the lower division of the base.

Different forms of plinths of the late period.

The lower plinth is then often entirely omitted on the smaller columns and rounds, while in the contrast between the fluted and smooth parts of the plinth is thought to be found a substitute. This contrast will then be increased, if the angles of the fluted portion instead of being vertical, rise spirally as in Fig. 569. Sometimes these spirals also extend from one point to both sides, thus crossing and forming little lozenges, within which the flutes appear, or from the same points also rise vertical angles, so that the number of intersections is increased and little triangles are formed. Or the edges are carried vertically for a short distance, then passing into the direction of the spirals and later back into verticals, or the surface of the cylindrical portion has scales; in brief the Gothic art of the 15 th and 16 centuries developed an inexhaustible diversity in the mode of treatment of just this part of the architecture, as proved by a rich choice of examples to be found in Kallenbach and Heideloff.

If even the modes of treatment described produces a puerile effect when applied to plinths of larger piers and rounds, yet it is still in place whenever very near the eye, on which it usually falls and for a longer time, as for example especially on the little columns and rounds on the window jambs of secular buildings.

An application of this ornamental motive to actual plinths of rounds is found on the piers of the church of S. Katherine in Eschwege (Fig. 570) dating from the second half of the 15 th century. Here beneath the round base is a fluted part, which passes again into the circle farther down. This transition is

not made by the usual wash, but by round arches falling at both sides to the circular line. The lower cylinder then passes through the lower moulding of the plinth into the octagon according to the way shown in Fig. 568. A similar form is shown by the plinth of a round in the choir of the church at Immenhausen in Fig. 571, on which the transition from the octagon with concave sides beneath the base into one larger and with straight sides is made by angular gables over the sides of the latter, while the proper twofold division of the plinth is expressed by a moulding extending around below those gables.

In all these arrangements, what is sought is not mistaken, they pass from the simple beauty of the older art, but they are still in a thoughtful way derived from the nature, with simple means producing an almost rich effect, and can serve as valuable starting points for the development of various forms of details, particularly on works of secular architecture.

Socle of the plinth.

Besides the already often mentioned two parts of the plinth, there is also sometimes found a socle or step on which rests the plinth, like the upper course of the foundation. The ground form of the simple square is however seldom shown, as a rule it is replaced by another form corresponding better to the entire plan of the plinth of the pier and representing a certain simplification. Accordingly is first of all the simple square set diagonally as in the church of the Minorites at Höxter (Fig. 572), or with clipped corners, also with four squares or octagons projecting from the angles, that gives the ground form. The additional area of this socle generally remains horizontal with simple sides and is rarely occupied by a wash, on which stand the plinths, like the rounds of the Liebfrauen church in Frankfort, Fig. 574.

Simpler is the form when the socle at the same time forms t the lower division of the plinth, or when the latter assumes a form differing from the plan of the upper plinth, simplifying the ground form of it. Such a form is shown in Fig. 573 from the Minorites church in Cologne. Here the plinth is a square set diagonally with clipped corners corresponding to the octagonal sides of the plinths of the rounds set on them, so that the surfaces f lie in the horizontal plane.

On the plinth of a round represented in Fig. 574 from the L Liebfrauen church the bases are round and then extend afterward in the plinth.

Use of the plinth on piers of compound plans.

Stepped piers.

As for what concerns the different arrangements of plans for piers or rounds of compound plans, then all that was said of capitals is applicable with few modifications. On the older stepped compound piers with rounds in the angles and on the outer sides, such as are found in Strasburg, Rouen, etc., the plinths of the rounds are square, are again found at right angles with each other, thus reproducing the ground form of the nucleus, which itself remains without plinth and rests in the angles on the plinths of the rounds as in Fig. 558 in Rouen. The members enclosing the angles of the nucleus of the pier pass again into the square form above the plinth of the round. This arrangement suffers the next modification, when the plinth squares of the rounds bearing the diagonal ribs are set in the direction of the latter as in Gelnhausen, Freiburg, Mantes, or when the plinths of certain rounds are polygonal instead of s square, or when all plinths are polygonal. The general form w was richer when the members of the base also continued the stepped nucleus, as on the piers in the transepts of the church of the monastery of Haina, arranged as in Fig. 421.

Round piers with rounds.

For round plans connected with four rounds, the same base extends around the round nucleus and the rounds. Then all plinths are either circular or those of the rounds take a square or polygonal form and unite with the round plinth of the pier. On certain works are also found the arrangement, that only the r round plinths of the rounds are divided into two parts, but the pier is only surrounded by the lower member of the plinth, and indeed this occurs particularly on the piers of those Westphalian works, whose capitals are formed according to the same principle before explained. Thus generally a certain harmony of the ground forms and the arrangements of the plinth and capital is usual, yet is not exactly to be regarded as a rule. Likewise also the plinth if the nucleus of the pier usually assumes a polygonal plan, and even the diagonally set square as in Rheims. The octagonal plan in Fig. 570 led to a peculiar

treatment. Here the greater differences between the circle and octagon caused by the greater diameter of the pier prescribed a magnitude for the plinth member forming the transition, unsuitable for the smaller plinths of the rounds, hence the different form of the latter, which then intersected the former. This inequality of the plinths of the pier and the rounds is found however in much earlier works. Generally the proportion between nucleus and rounds is so peculiar, that both arrangements as well as the similar base or plinth enclosing both as well as the difference between them can be justified.

The same combinations of the ground forms result on round piers with a group of rounds, of which we have given an example in Fig. 426. Just in the given case are the plinths of the rounds with their sides in the longitudinal direction, but the plinth of the round pier is an octagon, while on the pier of similar ground form the cathedral at Besancon the nucleus has a round, and the rounds have square plinths, yet so that the plinths of the rounds under the diagonal ribs are placed in t the direction of the ribs.

More easily than for the transition from the round pier with four rounds to the octagon of the plinth is made that for the round pier with eight rounds. This can either be placed so that the edges of the octagonal plinth are visible between the plinths of the rounds, as in the church of Colmar, or it is set d diagonally, when only the sides of the plinths of the rounds are visible.

On compound piers of the middle period, whose rounds are connected by hollows or by richer members, there are found different arrangements of the plinths.

Compound piers of the middle and later times.

First the members of the base surrounds the different parts of the plan of the pier parallel or concentric. With a smaller size of the hollows only the upper and less projecting member of the base can extend around, while the outer member of the bases of the rounds and also the plinth intersect, so that the hollows disappear in the plinth. But as a rule the hollow is also not expressed, where by its size would be possible, since the recesses produced thereby in the plinth itself would be too narrow. Then the upper member of the base already extends

before the hollow in the direction of its chord. What concerns the carrying of the moulding around an ogee member, this cannot be carried around the added edge in a concentric arc, but according to an approximate straight line or curve (Fig. 5-7), w where the middle part of the curve lies at c'.

Also as now on compound piers with a stepped nucleus only t the rounds have plinths, on which are set the angles of the nucleus (Fig. 558), the same system is also found used on these later plans of piers, on which the hollows and some intermediate members represent the nucleus of the pier. Only the form will here be the more striking, since the plinths of the rounds do not combine, but between them are allowed the hollows and fillets or members, then extending with them to a common plinth. Fig. 574 shows such an example from the Liebfrauen church in Frankfort, which is thereby still simply represented, so that the rounds yet have equal diameters, and therefore equal plinths. But since the separate plinths belong only to the rounds and not to the members, and their connections with them are o only represented by the common socle, by which could then be made the combination, if the plinths of the rounds were unlike, it is next to shape the plinths in proportion to the unequal sides of the rounds. Thus for example if we conceive the fillet a in the plan (Fig. 574a) to have a round, this would receive a smaller plinth similar to the larger ones, and thereby the diversity of the separate forms of plinths would be enhanced according to the scale of the members of the pier.

But almost inextricable will be the complication, if also the nucleus of the pier represented by the hollows receives i its separate low plinth, which is intersected by the plinths of the ogee members and prismatic bodies then terminating on a common socle.

Also as the latter form now resultsbecause also the hollows receive their separate plinths, so that one yields to them with the same right as to the rounds, then proceeding in the opposite direction, so that also for the rounds are omitted the plinths of all parts of the plan stop on a wash, which rises from the edge of a regular socle, generally shaped to correspond to the plan of the pier.

Thereby originated the forms of plinths occurring already in

the Wiesen church in Soest in the 14 th century and in the nave of Erfurt cathedral in the 15 th century. In the former the p plan of the pier forms a circumscribing square (Fig. 440), and in the latter the corresponding octagon is the ground form of the socle, from whose edge rises the wash. In both cases is then lacking the extension of the ground form, at least in the recognized outline of the whole, thus an essential requirement of the form of the plinth. Further the transition from the rich plan of the pier to the simple one of the socle by a wash is yet formed in a powerful way and finally the effect of the whole is dry in even a high degree, as that of the one previously described is complex, and entirely inferior to the quiet beauty of the lines of a plinth formed according to the earlier method.

Yet this affected form must not be too lightly esteemed. The entire necessity for its formation was given as soon as the rounds and the nucleus had been fused by members into a continuous entirety, instead of shaping each part according to a definite ground form, and their homogeneity was only expressed by their position. Thus in the early piers (Figs. 419 to 430) each of both parts can be conceived as existing for itself, or rather after removal of the rounds the nucleus still retains an independent ground form, while already in Fig. 438 such a separation by the removal of the rounds leaves the nucleus as a mutilated body. The lacking independence of the nucleus permits a widening of the hollows, an entirely capricious form, which finally scarcely deffers from any other profile of a moulding, and thus also permits the forms of plinth last mentioned.

Intersections.

And also the penetration of the more or less compound body of the pier by the cone or pyramid now further forms the theme, that the more ornamental forms of plinth in the late Gothic v varied with inexhaustible variety. As in Fig. 574 the transition to the cylindrical socle is formed thereby, so in general all transitions from one ground form to the other is thereby made possible, and also thus is affected the extension of the basal forms of plinths by these transitions.

In the right half of Fig. 575., let the inner rectangle with concave sides be the ground form of the upper division of the plinth of a column, whose plan is indicated by the inner circle,

and the circumscribed diagonal octagon by the ground form of the lower division of the plinth, then as shown in Fig. 475a, the transition by the pyramidal body b c d over the triangle abc, which makes the difference in the area of the two octagons, is found in Fig. 575, but in a richer way if a member formed according to the projection e f (Fig. 575b) rises from the edge of the lower octagon, whose angles pierce at the points g and h the concave surfaces of the upper mass of the plinth, so that instead of the pyramids mentioned, they form small moulded bodies. If then the lower octagon receives a greater projection, then would the moulding produced by it either join directly beneath this body, thus extending its moulding as in Fig. 575 r right, or by separated there by a short vertical piece.

It is not in the nature of such forms, that the ground forms placed on each other require each other, that further the plan so constructed also affects the dimensions of height in the elevation. Thus here the internal octagon first determines that circumscribed, and further the angle i of the square circumscribed about the internal octagon fixes the lower projection of the plinth. Thus in the elevation the height a b is fixed by the depth i k in the plan, the height c e by one side of the square described about the inner octagon, the height of the little pyramid or moulded body by the length a i in the plan, and in the right half the height f g is determined by a side of the inner octagon.

In the plan of Fig. 576a the proportion of the two inner octagons is then changed, so that the one set diagonally is described with the same length of sides as the first, instead of being circumscribed, so that the two octagons intersect each other. About both octagons are then described squares, yet so that the squares are set diagonally with the octagons and therefore intersect like the octagons and form a star plan by connecting the angles of these results the outer octagon giving the lowest ground form. Fig. 576 shows the corresponding development of the elevation. On account of clearness, the corresponding points in plan and elevation are indicated by the same letters.

Accordingly the intersection of the same ground forms of both octagons and of both squares, which has given the motive of the

form of plinth, and which in general affordan almost kaleidoscopic means of continually producing new forms. In Hoffstedt the term quadrature is assumed for the intersection of squares, and that of triangulature for that of triangles.

The development from the quadrature is expressed more decodedly, when both squares appear not at different heights as in Fig. 576, but in the same division in height. Accordingly is formed that of Figs. 577 and 578. The star ground form is expressed in the left half of the elevation by the prismatic body . b, c d, enclosing the octagonal nucleus, which at top forms the octagonal surface as at x in Fig. 575. These are set off by the plinth e f, so that also the members lying in the plane of this plinth project beyond its sides; by the lower member g is made the transition to the square. In a richer form the sides of the prismatic body can be still ornamented by blind tracery. In this manner is treated Fig. 578, which at the same t time differs from Fig. 577 in that from the star form the transition is not to the square but to the octagon, and indeed is not there only by a member, but is formed by a little prism inverted between the parts of the star.

Figs. 580 and 580a then exhibit a form of plinth from the t triangulature, which also has the peculiarity, that the members by which the triangular prism is attached to the hexagonal nucleus, not as before to the sides of the former, but here accompany the sides of the hexagon, thus intersecting the sides of each body at a b, etc. In the accessory Fig. 580b then the same treatment is applied to the lower offset.

The penetration of the octagon, which is given in plan in F Fig. 576a, then leads to an entirely peculiar development of the elevation, when especially as given in Fig. 582, one of t the octagons is placed below the upper, and the second is on the lower moulding of the plinth, when both intersect each o other for a short distance a b, and here produce the ground f form of the sixteen pointed star, where then the parts of star belonging to the lower octagon are attached by a wash to the surfaces of the upper one, and those of the latter by horizontal cuts to the surfales of the lower octagon. On the plinth of a round in the Liebfrauen church in Frankfort this motive is found to be expressed by the bands merely applied on the round nucleus of the plinth (Figs. 581, 581a).

To be able to compare the effect of this series of forms of plinths with the older simple forms, we also add in Fig. 579 a perspective view of such a later plinth. With all the animation peculiar to it by the alternation of light and shade produced by the entire arrangement, by its animated lines, it still remains far inferior to the quiet beauty of the older pronciple used as the basis for Figs. 558 to 565. It is even the lack of continuous horizontals or rather the loss by the animated forms of plan or by the weakened accenting that makes itself felt and that finally gives to the entire plinth the appearance of a crystal.

Therefore such forms can be given less to actual piers of v vaults, and then rather to those smaller posts for supporting a beam ceiling or to the bases of posts used for a corbelling. But first of all is the material wood, on which is indicated in a sense the impossibility of forming greater projections. However this treatment can only be obtained in the mass, where the principal parts of the old form of plinth at least are allowed to appear, when the different parts of the plan do not entirely abandon horizontals in the development of their elevation.

4. Vault Piers of Brickwork.

Their ground forms are the same as in stone construction, w with certain modifications due to the natura of the material, to the small influence of the separate pieces, and to the necessity of bonding. At first men yet adhered strictly to the forms of bricks, later was developed moulded bricks.

Thus it is first the sizes of bricks, which in the older works exceeded those now common (mostly 28 to 30 cm long by 8 to 10 cm high), which set a maximum for the shapes of the separate members of even the ground form of the rounds, and then the bond which indicated certain ground forms, allowing others only after overcoming certain difficulties.

Forms of plans.

From the direction of the end joints intersecting at right angles now first results the rectangular and the later cross or stepped forms of plan. The rectangular pier receives a richer treatment by constructing the corners with chamfered or moulded bricks, further by the projection of moulded parts from the sides, and also in a certain way by the arrangement of clusters of rounds. The bonding of these clustered rounds then

occurs in the same manner and also requires its treatment here. We have given this at the different sides of the square in different forms.

Fig. 584 then shows the plane of the pier in stepped form. The richness of such forms may be enhanced by complicated arrangement of the separate bricks. In the main plan it harmonizes w with the stepped plans occurring in stone construction, and is distinguished from that by the lack of detached rounds, for the yet small rounds can only serve as members at the angles, as such are also found on the piers of Rouen (Fig. 558), while the actual round is formed by the mass a b c d, which is therefore terminated by a rectangular capital, in which the separate members are collected together. The preference is peculiar to plans formed according to this priciple, that they directly result from the bonding, while all that cluster of rounds is in Fig. 583 left, which vary from the stepped form of plan, are only bonded by a departure from the ordinary arrangement of t the joints and require many different shapes of bricks.

Fig. 585 then shows only a bilateral symmetrical arrangement, when the parts beneath the dividing arches receive a form differing from those of the vaulting rounds. The last principle is most clearly expressed when in the entire plan the length exceeds the breadth or conversely, as in the Cross church in Breslau (Fig. 586), where the great width of the pier produced an extension of the pier in the direction of the length, and in the church at Bützow, where the converse condition occurs. (Fig. 587). Note.

Note. Essenwein. Brick architecture of north Germany.

A different shape can be obtained by setting the square pier diagonally (Fig. 588). The joints then run diagonally and the angles are chamfered, moulded or have projecting rounds. This form is to be mharacterized as especially happy, since it appears in suitable proportion to the members of the arches, but then brings out an elsewhere occurring practice of masons, whereby both in foundation masonry as well as in the treatment of projections a greater diversity is made in the entire direction of the joints by a diagonal position of the bricks. Here also the pier laid with diagonal joints stands beneath the cross arches and on the foundations, whose joints run in the directions of length and breadth. Bo an application of this sys-

system of the alternating courses then originates the octagonal plan. Fig. 589 shows the two courses lying on each other, each consisting of cross bond with the filling of the corners. Richer forms are also obtained for these ground forms by moulding the angles and rounds built in the surfaces. According to the same system of joints is then also constructed the round form. Yet here instead of the cross is also assumed the position of six rays, so that the diversity of the two courses in the same way, according as hexapartite division determines the direction of the rays starts from one or another point of the four sides. The round pier can also have attached rounds, but there is generally found the arrangement of four rounds.

Dividing arches.

The form of the dividing arches in their main forms are determined by the construction. According to their height they consist of two or more rowlock courses turned without bonding and concentric, their depth again decided by the width of a b brick, but must have a rectangular and more or less stepped cross section. The plan of a pier in Fig. 584 ma: therefore a also represent in the part a e f g h the profile of a dividing arch. Besides the separate forms of the bricks, this profile can also receive the desired diversities in form to obtain a different character, according to whether the lower rowlock c course has a width of more than one brick and as tc whether t the size of the step is increased. By such arrangements will then be obtained wide surfaces in the height and width of the arch which may be plastered and painted, for the painting of certain parts and generally every kind of surface demoration is particularly suited to the nature of the brick construction. Consequently the dividing arch may assume another character if a rib brick is set in the lower surface, so that the entire profile ends below in an edge. The profiling of the rib bricks and forming the springing of the rib is already explained above, so that the general mass of the springing of the rib can be c constructed.

Forms of capitals:

With a pier formed according to the principles of Figs. 584 and 585, therefore is the springing of the arch in almost complete harmony, and a capital would only be necessary on the parts

designated by a b c d, in order to obtain the difference of this ground form from the springing of the rib, but only on the other part of the pier if the profile of the bricks of the arch differs from those of the pier. But the ground form of the capital will best follow that of the steps, so that the separate members either pass beneath the capital into the square or stop against its lower surface, while the arrangement of the different members enclosing the capital would already be ineffective by their small size. But the latter is entirely in place when the middle projection a b c d is replaced by a more united form, by several adjoining cylindrical rounds, which are then concentrically enclosed by the projection of the capital. In Figs. 583 and 585 is given the first arrangement of the capital at k and the latter at k'.

In all these ground forms of piers, which are based entirely or partly on the diagonal direction of the joints, like those of Figs. 588 and 589, they by the form of the capital must be extended the upper surface, so that it is in condition to receive the stepped form of the arch. For example that seen in Fig. 588 so that either by a simple projection extending around the entire ground form as given in the plan there, or in the capiral itself by the arrangement of one or more corbelled courses is formed the transition from the diagonal direction to the stepped one, or in other words, that the change of direction of joints occurs in the capital itself instead of between the capital and the springing of the arch. Fig. 588a exhibits such a treatment in a perspective view.

On an octagonal or round pier the ground form of the capital also follows that of the pier and also extends around any projecting rounds. But in all cases the projection of the springing of the arch as well as of the capital from the mass of the pier is very small.

The construction of the capital occurs in the simplest way by projecting courses of the ordinary height or that of a course set on edge, that has the required form of profile. (Figs. 590, 591, 591a). In all forms of capitals composed of straight lines, the pieces that form the corners must be specially shaped to avoid joints and cutting the bricks. Conversely the rounded joint is in place over the projecting angles. The jointing of the corner pieces is avoided if the mouldings generally

at one side (Fig. 590a, or the round passes through pieces of special form into the square (Fig. 590b). For the circular plan the separate bricks naturally must be formed as circular arcs, whereby the joints come to lie above the bisecting lines of the reentrant angles. Yet also here the round form is usually avoided and the capital is not a square, but with several intersecting rounds is formed of adjacent squares (Fig. 590a).

For a capital formed like Fig. 590 or 591, the part corresponding to the bell serves no structural purpose, does not result from the magnitude of the materials, and therefore is to be regarded merely as an ornamental band, as the bearer of rich decoration. Its ornamentation may then be made by painting or by relief ornament. The latter can be made of terra cotta or of mortar. In the first case the capitals of the rounds must be made as solid pieces if possible, and therefore can only be of small size. Since already the nature of the material allows no great relief and forbids all imitation of stone capitals, then the ornament must be chiefly effective by its outlines (Figs. 592, 592a). Certain leaves with sharply expressed outlines, scrolls or plants with simple or overlying leaves, are in place here. Fig. 592a shows such an example from the church of the monastery of Chorin. The ornament must be quite modest as a mere surface decoration. Facing with slabs of terra cotta has the disadvantage, that the proper surface of the pier is thereby reduced by the thickness of the slabs. Even if the slab itself aids in supporting, the manufacture of the slabs leads to frequent repetition and removes all its character.

More frequently are found capitals like ornamental forms in general of mortar with sharp sand. Whether cast like ashlars, or it is spread wet on the surface of the wall or pier and the ornament is modeled therein, as men executed works in stucco in former centuries, we cannot decide. Probably both methods came into use according to the size of the surface, so that only the smaller capitals, corbels etc., were cast, but the ornament on larger surfaces was modeled in the applied mass of mortar. The still remaining parts of the former Dominican monastery (the castle) in Lübeck have a great part of their richness devoted to such works. But it must be noted, for example that if the corbels are actually cast, it was still disdained to use repetitions of the same model.

Stone capitals on piers of brick masonry could be executed in the same manner as on stone, and they differ alone by the smaller size of the members or rounds, which they crown as well as the hollows in which their members enter. Therefore here is also in place all overloaded forms.

Forms of plinths.

Even more than for the capitals results for the plinths the condition of a certain simplicity. The forms in Figs. 558 to 565 are derived from the ground forms of cut stone and are only possible by their size. Therefore if it was ever possible to burn pieces of like size in clay, the proper nature of brick construction would thereby be denied, entirely aside from the impression of insecurity, which such a clay ashlar must produce. In the first place it is the work of the mason that gives its stamp to brick construction, and which only at the cost of the genuine effect of the whole can be supplanted by that of the stucco work or brick maker.

Accordingly the plinth remains in the ground form of the pier or round and only forms an enlargement thereof, which is then made of projecting courses of moulded bricks (Fig. 593). The richer members of the pier cannot therefore be expressed in the plinth, and they recede over the plinth in one or more adjacent squares, that extend around the projection of the plinth, or they also remain without a base and rest on the common plinth of the pier (Figs. 593b, 593c), then extend so that the bricks forming them must be cut according to the sections of the vertical parts. Thus the rounds built in the angles or surfaces of the octagonal piers generally extend to the plinth of the octagon. But the cutting of bricks cannot be entirely avoided, where cutting becomes possible. It is a procedure required by the nature of brick construction, without which even with the most extreme use of moulded bricks cannot be executed a vault, and must then appear consistent, i.e., where applied to parts coming under the eye, if the mason's work is not done by the day and sinnks to mere slave labor. For even then the work of the mechanic is distinguished from that of the day laborers and manufacturers, because it will be seen and the stamp of its goodness or badness cannot be denied.

5. Ceiling Columns and Detached Posts.

Stone columns for ceilings.

Stone piers that remain in small number, which could serve for supporting wooden beams and girders, as under the organ galleries of small churches, beneath the roofs of vestibules (on so-called verandahs), or in halls of considerable width, differ substantially from piers of vaults.

Basal form of shaft.

What first concerns the ground form of the body proper or shaft is that such is chiefly simple as a square with chamfered angles, or formed as a polygon or circle. A form of pier requiring subdivision into actual rounds could indeed be indicated by certain ceiling constructions, yet according to the conditions of the affair might either become puerile or require an excessive expense in size and space.

On the contrary according to the fluting of Greek columns, there is formed a subdivision of that simple ground form in a way, that it returns to the ground form above the plinth and below the capital.

The ornamental character of such a treatment is also then expressed, in that these mouldings, particularly according to the later mode of treatment, are seldom er vertical than made in spirals, and also intersect spirals in the contrary direction or verticals, and thus approximate the character of a surface decoration.

A basal difference of such stone parts from vault piers is, that the former could not be conceived without a capital, since it brings the material of the shaft to a termination, therefore assuming an entirely detached position of the supported parts.

Ground form of the capital.

The basal form of the capital is square in the simplest cases. Variations from this will be determined by the construction of the beam ceiling in the same manner, as the piers of vaults by the plan of the springing of the arches.

According to the usual arrangement the post bears a girder on which lie the ceiling beams, just as the Greek column did the architrave on which rested the stone beams. Accordingly the lengthwise direction of the girder must be expressed in the form of the capital. This relation is not indicated on the capital of the Doric column, the ground form of the capital b being entirely concentric, so that the changed condition of the

angle column on which lies the architrave in a twofold direction is not expressed, or rather the form of the intermediate columns of every colonnade seem imitated from that of the corner column. A higher organism animates the Ionic capital. Here the longitudinal direction of the architrave is expressed in the movement of the volutes accompanying it, and a meeting of those coming frog both sides in the corner column permits its projection diagonally, so that its peculiar position and the character of the entire combination is repeesented in the most beautiful way.

It even harmonizes with this relation of the form of capital to the direction of the architrave, that according to a later transformation the Ionic capital received four such angle volutes projecting diagonally, if as generally occurred the stone beams at the height of the architrave had a size and form equal to that, thus finding their bearings on the column, and so each returned in the Greek arrangement to the conditions of the corner column.

Therefore in all this is seen indeed a refined and spirited indication of the structural conditions, but an aid produced by the construction, for the area of the capital remains in all cases an exact or approximate square, both affording the same bearing on the corner column with the meeting architraves as when it simply extended over them.

An improvement in the bearing, a transformation of the ground form of the capital corresponding thereto, on the contrary is found on ancient Indian forms of capitals combined with corbels. Gothic art that brings actual aid for every need, and only indicates what satisfies the condition, must also therefore in t this case pass to such a change of the ground form of the capital, that the free length of the girder is reduced thereby, so that the possibility of a wide spacing of the supports is afforded, and likewise a suitable bearing for girders crossing in the same plane, i.e., the ground form of the capital becomes a rectangle under a simple girder, or a cross beneath two that intersect, or meet by a T beneath two that abut. Variations therefore by retaining a simple concentric ground form indeed are frequently found. But then the post is either connected with other structural parts, like wooden caps, etc., or it ser-

serves exclusively to support a beam ceiling, or rather it first supports points especially loaded and near together, so t that the required closer spacing reduces the free length of the beams so much, that this aid becomes superfluous.

Elevation of the capital.

The means of obtaining the ground form mentioned, thus first for the transition from the concentric form of the shaft to t the rectangular, consists in the connection of the capital with two or more forms of corbels, by which both parts receive a more separated expression, or the body of the capital passes directly into the corbel form. The separation of both parts o occurs most distinctly in those Romanesque capitals recalling a similar purpose, whose tops received a corbelled shape. But such a form is less suited to the ground principles of Gothic art, which requires two pieces lying on each other, and therefore instead of placing both parts on each other, their junction occurs in one block.

A very beautiful example of a capital with side corbels is found in the Dictionary of Architecture by Viollet-le-Duc. Here the body of the column extends above the astragal, expands with a slight bending of the edge of the bell to receive an octagonal abacus for the ceiling members, and is carried around corbels at both sides, penetrating the body of the capital. The part of the bell remaining between the two corbels is then covered by a bunch of leaves laid thereon, and on the fronts of the corbels are placed shields of arms cut in the mass of the stone. The upper surface is therefore of the form indicated in Fig. 594. The purpose of the projections indicated by a b c d and e f g h will be explained below.

To obtain a bearing for a single beam sufficed a simple rectangular form, or for the corner two such intersecting at the angle of the corner, for example is shoun in Fig. 595a. The elevation can take the form given in Fig. 595, which may also be employed for the capital formed as a simple rectangle. the form could be simplified by omitting the foliage and the bent edge of the bell, whereby the member a could penetrate the cylindrical or prismatic body of the capital, and finally by a breadth of the corbel equal to that of the diameter of the column, whereby it projects from the middle of the column on both sides, and the body of the latter must penetrate beneath the projecting

moulding. An example of the latter is shown by Fig. 596.

In the same manner as Fig. 597 shows, the side projections would have leaf-like supports. Figs 597a and 597b show the plan and elevation of such a form.

The parts hatched in the plan indicate the origin of the corbels a projecting at both sides (Fig. 597b), whose upper edge extends beyond its lower surface and is supported by the part a b c of the beam indicated on the plan. Over this corbel and thus at b on the elevation then appears the vertical face indicated at d, which has penetrated the part of the bell developed on the part e f of the plan. The entire form lies on t the rectangle by the slab q projecting over the upper moulding.

Likewise with the projection at one side is also the form of capital according to Fig. 598 connected with two concentric c corbels, and thus is obtained the oblong surface, whose relative length is removed farther from that of the post.

Also by a junction with moulded corbels would be produceda proportional length of the upper slab, if the mouldings penetrate the body of the capital horizontally in the manner explained on p. 244 (Figs. 599, 599a).

The means for obtaining the different forms further result from joining the concentric capital of the column with heads or other figure forms, that support the desired projections.

Directly by the form of the capital is further obtained any form of the upper surface, whereby the projections of the body of the capital wary toward the different sides, and the surfaces enclosing them became partly oblique. Such an example is shown by Fig. 600.

Further the upper area obtained by a concentric projecting capital can be made oblong by cutting from the body of the capital. Thus in Fig. 601a from the octagon a b c d e of the projecting body of the capital is cut vertically the part designated by the triangle b c d, so that the surface a b d e remains, on which rests a rectangular moulded slab. By that cut are also produced on the mass of the capital at poth sides the vertical surface f in the elevation of Fig. 601 in the same manner as the semicircular surfaces on the Romanesque cushion capital by cuts from the sphere. Fig. 601b shows the corresponding side elevation.

The same procedure may then be applied to any other polygon,

to the octagon parallel to the direction of the beam, the hexagon, decagon, dodecagon, etc., and to those forms set diagonally. Fig. 602 shows the form of the duodecagon set diagonally, and Fig. 603 that from the diagonal square in perspective view. In the latter are retained the angles of the square falling under the beam, so that the transition from the moulding to the square beam must be formed thereby. But just as well could the upper ground form have been changed into the rectangle according to either Fig. 601 or Fig. 602.

Placing the beam.

The beam or girder lies on the abacus of the capital and is held in place by a pin. The latter can be omitted if a so-called groin is cut in the top of the abacus as shown by the cross section in Fig. 599, in which the beam lies. Instead of cutting the gain a b c d for the entire length of the slab, it can consist of two shorter gains, so that the capital has a cross shape and two cross arms extend the bearing of the beam, and two others serve as sides to support the beam, and thus the capital assumes the form given in Fig. 604. If such an arrangement is employed in the open air, it will be necessary to cut a little groove behind the projections, which will remove the rainwater entering the joint between the wood and stone, and the most varied shapes may be obtained by the form of these openings. These may be developed into formal strongly projecting parts, t that if permitted by the distance of the posts allow it, could even support gutters laid thereon, that have to receive the w water from an upper roof (Fig. 605).

The ground form of the cross is further indicated also for the upper surface, where the post stands under the crossing of two beams or girders. Such cases may be required by certain departures from the construction of ceilings now usual, and to be mentioned later. A simple example of a corresponding form is found in the court of the hospital of Beaune (Note), where the cross form of the capital had but three arms. The fourth was added when the end of the beam on the post had to bear the post of the upper story and thus the corbel supported it.

Note. Arch. civ. et dom. Viollet-le-Duc. Dict. Arch. p.548.

If the upper story is then to be composed likewise of stone piers, then must the nucleus of the lower pier continue while

the girder only rests on the projections of the capital, so t that there results the form of Fig. 594 in plan.

What now concerns the relation of the size of the post to t that of the girder, in the simplest case the latter may have the width of the capital, and the column only be less by the always slight projection of the capital. Yet to prevent the breaking of the upper horizontal edges of the capital by the load of the girder, this may be surrounded by a moulding projecting beyond the breadth of the girder as in Fig. 603, and thus at the same time receive a more complete ending. The moulding then terminates at top with a wash or a rounding. This increase in the width of the capital further becomes necessary by the use of a gain given in Figs. 599 and 604. The theckness of its sides can be obtained by either the general form of the capital or by the projection of the upper moulding, on which the wash is replaced by a vertical fillet.

To the forms of the plinths of such piers applies what has already been said above relating to the plinths of piers.

Wooden posts.

Far more commonly were wooden posts employed than stone piers. Actually certain advantages are peculiar to them by the ease of connecting with the beam or girder supported and the convenience of placing. Under existing conditions, where the wood must be wrought too green, this brings the disadvantage, that it cracks easily,and therefore makes necessary the most varied means of filling these cracks.

From the use of solid logs results the ground form of the octagon, and that of the square from the hewn timber. Since it is of importance to leave as large as possible the bearing surface as well as that by which the post rests on the foundation, then the ground form must be the top and bottom surfaces, and hence the posts variously reduced in their length must return to their full shape below the girder and above the plinth (Fig. 606). Since the entire height of the post is a single piece, therefore the reductions or the projections of the mouldings thereon cannot be very great.

Entirely nonsensical is aiding the lacking projections by nailing on fillets. Such expedients are comparable to those f false locks of hair still in the thirties forming a necessary part of female full dress, and indeed are so far beneath the

works of the hairdresser mentioned, since the latter were at least on a proper place, and not quite capriciously attached, as it sometimes happens with the mouldings concerned. Not the nailing is perverse, but the unfortunate intention to deceive. Thus the actual ornamental parts like shields of arms, etc., in which is innate no structural importance can be attached in other materials, but then so that they are recognized as accessories, standing free or placed over a joint.

Polygonal parts.

Thus bilateral projections on the capitals of stone piers cannot be formed in the body of the wooden post, but must be made of pieces otherwise attached, that accordingly aid in forming the capital of the post. Also the post generally stands on a stone plinth, that may then receive a projection of the required form and have its own shape (Fig. 606), which represents the simplest form of a post of polygonal shape. Then the oblique surface that makes the offsets at a and b may be replaced in a richer way by mouldings, that either extend in straight lines, or from the middle of the sides rise or fall in straight lines or curves (Fig. 607), and further by forms like capitals (Fig. 608), which can again be ornamented by foliage and offsets in the ground form, of which Fig. 609 gives a simple example, while richer ones may easily be developed according to what was said above.

In designing it is first to economize the projections and further to give the preference to those forms best suited to the material, which can be made by simple cuts, avoiding as far as possible cutting against the grain. Further the manual working of all such details that with the variously shaped chisels and gouges are done freehand are to be considered in the design, so far as they lead to the arrangement of curved surfaces, in contrast to that done in stone with the mallet and chisel, where the stroke first produces a plane surface. It is further to be considered that of all parts of the post the proper body or shaft must have the smallest diameter, also that no moulding may be cut in its surface as wrongly shown in Fig. 610, since otherwise is formed a weak place favoring breakage, that would be particularly defective at mid-height.

Richer forms are produced by the arrangement of certain capitals or rings remaining in the mass of the shafte For the

capital results a function by the arrangement of the head braces resting on a projection and fastening in the post by tenons, making unnecessary any further projection (Fig. 611). If as caused by ordinary conditions the head braces occur in only one direction, then between them the upper end of the capital may have a different treatment, in the simplest case ending above with a wash or moulding. Instead of this capital surrounding the entire shaft, there may also remain beneath the head braces only their supports in the mass of the shaft (Figs. 617,617a).

Further at the middle of the post or otherwise arranged distances may remain moulded or ornamented rings, whose importance is mostly ornamental, and only consists in this, that the shaft gives the projection, and thus the form of the post is represented as in one piece, see g in fig. 611.

Square posts.

The same forms are repeated on the carved wooden posts of s square shape, except that here a new motive of form arises from the need of chamfering. The chamfer may either be simple or moulded, or result by a change to the polygon. The change in the rectangular corner may either occur at the capital and plinth (Figs. 612, 612a), or be made below and above them. There is also sometimes found the mode of treatment given in Fig. 613 producing a very peculiar effect, whereby the chamfers instead of returning to the ground form below and above the mouldings of the capital, extend through both and only change beyond them.

Mode of treatment.

Richer forms result by combinations of mouldings and capitals with ornamental bands of tracery on the foliage, yet may also be more simply obtained by chamfer cuts (see g in Fig. 611), and further by decorating the surfaces remaining above the capitals. The ornament may either be sunk in those surfaces, or be made better by disks applied on them, shields of arms, bands of inscriptions, heads, leaves or tracery, or by those latter substitute forms, which consist of interlaced arches or figures in straight lines (Fig. 614).

Likewise also the side surfaces of the posts may be ornamented for their entire height by branches laid on them, by sunken panels enclosing tracery, or in the mode of the late Gothic by scrolls and by combinations of the latter forming richer

patterns. Retaining the diminution that resulted in very slight degree from the original form of the trunk is found only on late examples, thus among these are the raised galleries usually built in the older churches during the course of the 16 th century, whose posts retained their original character until the middle of the 17 th century. According to the same principles as for these posts are also formed those little columns of galleries, clocks, etc., for which modern art has introduced the turned baluster, which last forms the stair railings in ordinary structures, and were sold by the piec like hobnails.

As generally the peculiarities of work in wood indicates a more ornamental treatment of the various parts, thus finally certain originally structural forms of stone construction are used as decorative motives and are characterized like the wooden piece. Thus flat finials or gabled forms lie on one or more surfaces of the posts as in Fig. 615, which still utilize diminution in the treatment, and there can be made in such wise direct imitations of the richest stone architecture, when the forms of piers and of finials with their complete basal forms lie on the sides of posts and are carved in an excess of the size of the timber. These can then rest on the stone plinth or be corbelled out from the surfaces, but above be connected with certain cornice or gable forms likewise projecting before the plane of the girder or other connecting parts, thus forming an ornamental architecture overlying the proper construction, but producing thereby an extremely rich effect. Such certainly belong to the latest period in their detail forms in the decoration and are also found on wall parts, and indeed are especially common in the cities of Normandy.

But even the antique caryatids came into use in wooden construction. Thus are found free posts under a vestibule of the city hall in Treffurt, that represent human figures and are in elongated form with refeeence to the propertions of the post, and to avoid all excessive weakening are very compactly treated. The effect produced is certainly very original, more that of a good jest. Gothic art like life likes to mix such with seriousness. But nothing is more foreign to it than the cothurnus, which stumbles over every humorous circumstance. The humor of Gothic architecture is not opposed to an intended jest, which elsewhere springs from the predominantly tragic character of

the various modern Byzantine or Romanesque art figures.

The posts are either flush with the girder or project beyond it on both sides. In the first case they are tenoned into it, in the second the girder lies in a fork. With a polygonal shape (Fig. 611) this is formed by the parts a b c d in Fig. 611a. Yet as a rule the angles a and d are chamfered as given at f. If a cap is also placed beneath a girder the profile of this cap can be formed as in Fig. 611b, and the cap whose width projects beyond d e can be notched and then its reduced breadth also rests on the sides of the fork. In Fig. 611b b denotes the cap, whose full width is indicated by the dotted lines, and n is the girder.

Brackets and head braces.

Both the girder and the cap are supported by the post by means of brackets or top corbels, by which connected parts the proper projection of the post is produced. The brackets either have the width of the girder or less, and are either fixed in the girder and post by tenons and boxing, or are placed directly in the right angle between them and are spiked to the post, or are tenoned only into the girder. In the first case the triangle a b c in Fig. 616 is the basal form from which results a flat curve as the proper outline, at the middle of which is g generally inserted a moulding crosswise (Fig. 616), a form like the nose or a more or less decorated disk remains in order to avoid excessive weakening of the wood. From the wood to be removed in forming the curve is then sometimes formed a plant ornament or an animal form that lies on the background.

Likewise the timber at the junction of girder and post may be strengthened by a moulding (Fig. 617), which sometimes extends over the entire surfaceof the bracket (Fig. 618). Usually then certain parts are cut from the sides, either by completion of their ground form, when the enclosing circle is struck about the round (Fig. 616) or so that an entire part of the moulding in smaller width seems inserted in a hollow extending the entire width (Fig. 619). Rounds further are sometimes ornamented by flutes, waves or vertical incisions, the hollows by rosettes or disks. Also there are found frequently carved on the side surfaces rosettes, shields of arms, circles decorated by tracery.

When the brackets are spiked to the post, their profile must be such that the nail requires no excessive length, and it must

therefore agree with the post in distance and direction for a short space, while it extends below the nail (Figs. 620, 621). On brackets of this kind the direction of the fibres of the wood is generally vertical.

Head braces.

Head braces are boxed and tenoned into the girder and post, or indeed abut against them. Fig. 611 shows at b how the boxing is replaced by the arrangement of the capital. They are described in many places, thus on the butchers' stalls at Frankfort, in the monastery buildings at Haina and Eberbach, the arrangement is that the octagon exceeds the width of the head braces. If the latter are greater, the post must return to the original form to receive the braces.

The head braces will either be made of curved pieces or of straight wood. In the first the curves are prescribed for both sides and may receive a richer form either by a chamfer or cove on the angles. In the last case the backs retain the straight form, while to the lower line or true profile all applies that has been said of brackets. Yet the greater length of the arch causes that a decoration extending over the entire front as in Fig. 618 appears least suitable, and the form in a curve, whereby the middle of the brace has a projection as mentioned for brackets, (Note), or a compound arch line predominates. Likewise the brace may remain straight, and its underside be moulded with a return to the square at the junction with the girder and post. Generally those forms are most suitable by which the length of the brace are accented, while all mouldings cut across it require certain depth for the heaped arrangement to remain effective, and then they weaken the wood so much as to lend to the whole an unsafe appearance.

Note. In Verdier add in the Dictionary of architecture by Viollet-le-Duc are found examples, where these additions in t the form of dragons complete the proper body of the brace, so that the arch line almost touches the straight back.

Particularly unfortunately appear the forms so favored in modern wooden architecture, whose principal element consists of the curved modillion like the antique represented in Fig. 695, which is either used singly or doubled, or is separated by intermediate members. There are here criticized also other

common procedures in modern architecture, by which elements of the most different styles are employed in a sense entirely different from the original one, with the secret idea of contributing to the hoped for new style of architecture. Thus the effect of these consoles which so frequently form the brackets in the wooden buildings of the 16 th and 17 th centuries, as well as the ox eyes, pearl beads and other parts like the antique are especially rich, since they are used in their original or slightly modified forms of a means of decoration.

Wooden caps.

The head braces are usually connected with a wooden cap. Their combination is most clearly expressed by this, that their curves sometimes continue over the cap, and the boxing is radial (Fig. 622). By the treatment of the ends of the cap may t the simple curve pass into an ogee or compound curve. The latter sometimes becomes a second beam extending between the posts with a smaller width than the upper one and tenoned into both posts. In this case the curve of both braces can be continued over this second beam so that at the middle it joins a semicircle, pointed arch or a curve, and the crown of the curve is cut in the girder. The wooden parts to be wrought here, as for the corbels, may employ any moulding or other form, and thus reduce the weakening of the timber. (Fig. 623).

6. Corbels, Cantilevers and Corbellings.

General and statical.

No difference exists in the nature of consoles and corbels. By the former word is denoted the relation of the corresponding cut stone to the wall or pier with which it is bonded, and by the latter its purpose in general. The uncertainty thus arising appears to have occurred by the term "console" so favored in Germany.

Purpose of classification.

According to the form can be distinguished central and one-sided corbels, the first takink its development from a point and usually its upper plan forms a part of a polygon, the one-sided corbel on the contrary having a rectangular plan.

The problem assigned to the corbel may have a manifold nature, it can offer a bearing for a stone lintel or wooden beam, may receive members of vaults or rounds, and finally may be destined to bear statues, etc. Especially manifold is its use

for the springings of vaults. Already en the Romanesque period very frequently occur corbellings in churches and still more frequently in conasteries and secular buildings instead of rounds extending downward, and which either by a short round (Figs. 665, 666) or deriectly, receive the springings of the vaults (Figs. 654 to 658). That on the one hand is not to be employed statically the omission of the lower part of the round as separate from the wall shaft, was alreadn stated above (Abutment), p. 126 and Fig. 343). The thrust of the vault usually passes obliquely into the wall a considerable distance a above the ground line of the arch. The capital of a little shaft or projecting corbel is commonly not affected by the thrust of the vault, so that these members rather fulfil an architectural than a static function. On the contrary the stress in the corbel under beams, girders, etc., is sufficiently important to require a closer investigation.

Stress in the corbel.

Statically considered, there are three possibilities, the bonded corbel (Fig. 624) under the load may rotate about the lower point d, and can also be sheared on the surface a d, and finally it can me broken.

1. Safety against rotation. The load Q tends to rotate the stone about the edge d, and on the contrary it is prevented from rotation by the weight G of the masonry resting on the bonded portion a c e d. That the stone may be safe there must be G n greater than Q m. Accordingly a great weight of masonry and a longer lever arm a c of the stone are useful. From safety in calculation the point of rotation d is not taken at the face of the wall, but a few cm farther back toward d′. Good bedding of the stone at this place is of the greatest importance, and also the upper bed a c must have corresponding consideration. If the danger of rotation is great, then with good donding above the stone there must be placed as large a part of the masonry as possible to load it. If it can be assumed that the entire thickness of the upper wall acts as a united mass, then can be made the adjacent calculation, which presents a favorable result (Fig. 625).

The corbel tends to rotate the wall about the point f with the upward force K. To prevent this we must have G r greater than K u.

But now K t = Q m or K = $\frac{Q\ m}{t}$ and substituting this value; G r is greater than $\frac{Q\ m\ u}{t}$, or $\frac{G\ r\ t}{u}$ is greater than Q m.

2. Safety against shearing. Shearing or separation of the stone comes in question, if the load Q (Fig. 624) lies close to the wall. The shearing stress is very simply found by dividing the load Q (in kil) by the shearing area a d (in q cm). the stress thus obtained per q cm must not exceed the permissible limit that is very low for stone. Assuming the usual safety, there must be allowed only 1.5 to 4 kil per q cm according to the nature of the sandstone or bricks, 3 to 6 kil for limestone and 5 to 10 kil per q cm for granite.

Example. The cross section of a cantilever in the face of t the wall is to be determined in its required area, if it must support a load of 4500 kil, and it is made of good sandstone with a safe limit for shearing of 3 kil per q cm. According to the preceding the cross section is very simply calculated at $\frac{4500}{3}$ = 1500 q cm, so that the stone may be made 30 cm wide a and 50 cm deep.

If a corbel has a very slender frofile at is not to be forgotten, that the shearing may follow a shorter depth i k (Fig.626).

3. Safety against breaking. (By bending strass). Breaking by bending stress easily occurs, since the tensile resistance of most stones is less than their resistance to shear. If both are equal, then already by the lever arm of the load Q of more than 1/6 the depth of the stone, breaking will occur more easily t than shearing.

a calculation can be made by the well known formula, W = M s. In this M = bending moment (in Fig. 624 = Q m).

S = allowable tensile stress, that according to the nature of the material may be taken at 1 to 10 kil per k cm.

W = moment of resistance of the area of cross section bonded in the wall (for a rectangle = $\frac{b\ h^2}{6}$; for a triangle = $\frac{b\ h^2}{12}$).

Evidently no cracked stone, but only a good material resisting tension should be chosen for important corbels.

As a handy rule it may be assumed, that a corbel of moderately good material, whose projection does not exceed its height in the plane of the wall must have a cross section (at least) containing as many q cm as the load on it is in kil.

For a profile of the corbel shaped as in Fig. 626, there is naturally a fracture again to be feared at the shorter line i k.

If the stone is correctly shaped statically, then between the wall and the extreme end is to be found any section at which the stone can break more easily than at the wall. But according as a cententrated or uniform load exists, at least the triangular profile of Fig. 627 or the convex one of Fig. 628 is required. Instead of the triangle is rather recommended the trapezoid indicated by the dotted lines with regard to shocks at the end, defects in the stone and fracture surfaces extending obliquely. The basal form of Figs. 627 or 628 may be enlarged in any way, but cutting into these outlines is dangerous.

Proportion of heights.

On the ratio between the projection and the height no statement can be made according to the preceding; the greater the load the higher must be the corbel. That the springings of vaults are sometimes found supported by lower corbels comes f from the fact before stated, that the entire springing assumes the function of the corbel. For cross arches of wide span the corbelling certainly assumes this function and rekuires a correspondingly greater depth.

The ratio between projection and height mostly lies between $\frac{1}{1}$ and $\frac{1}{2}$, but even exceeds the latter. If a geometrical ratio is sought, then the height may correspond to that of the diagonal of a square constructed on the projection or to the diagonal of a cube constructed thereon.

Central corbels.

The simplest form of a central corbel is one half an inverted cone or pyramid (Fig. 629), which may pass by curvature of the sides into the forms of Figs. 630 and 631. By combining two s such forms is produced a compound corbel like that of Fig. 632.

Capital and corbel.

The most common function of the central corbel is to form a substitute for the round, also to support the springing of the arch, and accordingly its top corresponds to that of the capital of the round. This agreement that the function of the round is most clearly expressed, if the corbel takes the form of the capital of a round, whose lower circular surface is then occupied by an ornamental form, even to remove the character of a bearing surface as in Fig. 633 from the church of Haina and F Fig. 635 from the foundation church in Wetzlar, in the latter being made the transition to the square. The change of the lower

bearing area must be regarded thus as the motive distinguishing the form of the corbel from that of the capital.

Between the capital and the perfected corbel are to be mentioned various intermediate steps, one being represented in a slightly later nave of the church in Haina, on which the leaves placed in part on the edge of the lower capital extend in the sides of the capital, partly bending downward around it, concealing the astragal or replacing it by their own bodies, and so covering the lower horizontal surface. The plane area of t the latter is further lost in the mass, when the ornament receives a stronger modeling, but very decided in the example in Fig. 636 taken from the church in Volksmassen, where the astragal and the rosette covering the bottom of the capital have taken the inclined position.

If according to the form shown in Fig. 634 the difference b between the sides and underside at least disappears in the treatment, it is still more the case if the astragal is replaced by a branch extending around the lower edge or a plait, from which the leaves ascend at both sides. But all that agreement with the form of the capital vanishes when from the lower end the foliage extends up over the entire corbel. The nucleus of the corbel covered by foliage must then also experience the corresponding change, and first its lower edge is rounded as Fig. 637 showsin an example taken from the Preaching church in Erfurt. On simpler forms is omitted also then the outward bending of the upper edge, and thus all recalling of the bell shape of the capital (Fig. 639). The form of the nucleus is either concealed by the leaves lying thereon as in Fig. 638, or in very visible distinction between them is in Fig. 639 from the late Gothic cloister of the foundation church in Fritzlar.

Simply subdivided corbels.

An entirely similar transition is made from the capital form by a reduction of the lower bearing surface, thus by a change in the line of projection of the capital as shown by Fig. 640. According to the same principle are shaped Figs. 641 and 642, the former from the vestibule of the foundation church in Fritzlar and the latter from the nave of the church in Haina. Thus is followed a stage from the capital to the simple corbel. The latter can also exhibit the profiles of Figs. 643 aod 644 besides the forms given in Figs. 629 to 631. By chamfering the

angles results the simple but corresponding form of Fig. 645. By varied profiling can forms originate like those of Fig.646 to 653 in section or elevation.

Compound corbels.

Corbel forms of greater height are generally obtained by a combination of two or more plainly distinct parts as indicated in Fig. 632.

The simplest case is a body like a capital in the upper part, supported by a simple form of corbel below, as in Fig. 657 from the cloister of the cathedral at Riga.

The character of the whole changes according as the capital or the lower corbel predominates. In the first case the latter becomes a mere ending without essentially increasing the strength, and forms a continuation of the moulding of the astragal as in Figs. 658 and 635; in the latter the proper corbel generally is a projection at one side for obtaining a more rectangular surface, from which the capital thereon makes it central. Such examples are shown by Fig. 656 from the rood scraan of t the foundation church at Oberwessel and Figs. 654 and 655 from the north side aisle of the monastery church in Haina.

The latter bear the rounds starting above the sills of the upper row of windows, and are of special interest for the beauty of their treatment, of which the small scale of our Fig. o only allows an imperfect conception. To it by its location in the height of the mouldings extending beneath the window sill is added an originality, to which we cannot refuse our attention. Instead of carrying this cornice around the corbel as a terminating moulding, as would have been done in the sense of modern architecture, thus serving different purposes in the same form, it extends into the capital of the projection, whose leaves bend over the edge of the moulding in an extremely graceful way, while the abacus thereby recedes above the cornice in question and intersects its wash, so that the base of the round rests on it entirely free (Fig. 655).

The body inclosed in leaves in Fig. 654 can serve just as w well as a cornice extending around as a capital. Generally the form of such a cornice decorated by foliage is so nearly allied to that of the body of the capital, that the former may be regarded as a low capital, as shown by comparison with Fig. 658 to Fig. 659.

Development of the corbel.

Forms of figures like animals, heads, etc., come into use in various ways. As in Fig. 654 they may sit before the body of the projection, or as in Fig. 656 be employed at smaller scale, may form only the termination of the corbel. Figs. 660 aod 661 show examples of the last kind from the church in Frankenberg. Further and especially heads may also furnish the corbel without a capital resting thereon, then being surrounded by either an edge moulding, a flower in front, etc., those not terminated above or even lacking such a margin, end horizontally and bear the rib on its upper surface, or as a head with foliage growing from it form the corbel. Likewise may it rest on the body as a mere ornament, either in connection with foliage or without it.

That an importance must lie at the ground of such forms has already been noted above. In corbels serving to support statue this is given by its relation to the figure. In the same sense may also be employed bends with inscriptions, shields of arms (Fig. 653), where the location and outline of the shield are given by dotted lines.

The possible arrangement of bands with inscriptions and legends on the corbel indicating the importance of the figure well aids understanding them in the old works. In modern times the characteristics of the figure itself makes this explanation unnecessary, but in cases not too rare are given a riddle.

The arrangement of statues on corbels is sometimes found connected with the springing of the rib in such wise that the latter rests on the canopy over the figure, as in the church of the castle of Marienberg and at the cathedral of Minden. Thereby the springing of the rib continues vertically before the eye and replaces the effect of the round.

Also to support in Fig. 658 the surfaces of the lower moulded part below the astragal a b against the wall, this arrangement for connecting the form of the corbel with different ground forms combined, so that the projecting parts of the upper body are supported by the sides of the lower, thus imitated as on late Gothic plinths of piers.

The lower ends of the corbels are sometimes concealed by bunches of leaves or branches from which then rises the bell shaed body, or such branches lie directly below the springing of the

rib and replace the corbel. Such an example is shown by Fig. 632 from the cloister of the collegiate church in Fritzlar.

We have already shown above by Fig. 278 how by a limited area of the capital under the springing of the arch, the corbelling of separate arches or certain parts of them could be arranged. Such a very graceful example is shown by Fig. 663 from the choir of the church in Volksmarsen. Fig. 663a represents the profile of the rib, of which only the part a b c d finds a bearing on the capital. From the surface b c then first projects the member b e c, so that the rib takes the form a b e c d, from which by the upper corbelling shown in elevation it passes into its actual cross section a f g h d.

In Fig. 282 we have given the springing of a rib resting on three corbels combined, taken from the Erfurt cloister. But sometimes certain irregularities in arrangement and the need for recovering lost planes on greater and more effective forms of this kind lead to a combination of different kinds of corbelling. For example as shown in Fig. 664 by the springing of an arch in the northwest corner of the church at Wetter.

Corbel with a short round.

As in Figs. 654 to 657 a greater size of corbel is obtained by placing a capital on the corbel proper, so that the magnitude of the whole is also increased if a portion of the round or wall pier replaced by the insertion of the corbel.

Fig. 665 shows the corbelled wall round in the choir of the Minorites' church at Höxter, while Fig. 666 represents the support of the cross arch recurring in nearly all churches of Reval, and it shows there like an architectural member a dry simplicity compelled by the hardness of the stone employed.

Richer forms of this kind are found on the western wall of S. S. Maria's church at Mühlhausen below the springing of the dividing arches. These start from the upper octagon, that corresponds about to the size of the springing of the arch, each being formed of five sides of the octagon, crowned my a part of the pier with a foliage capital, so that the projection from one to the other and from the upper to the octagon including the basal form of the springing of the arch are effected by capitals, below which on the bodies of the two upper piers are found on each side surface two slightly projecting round arches with cusps, while the side surfaces of the lower part of the

pier are decorated by tracery. The capitals thereby receive a greater projection in front than at the sides, so that the ground form of the part of the pier remains everywhere bounded by the five sides of the octagon.

By the use of what has been described can be developed an endless variety of forms. Fig. 667 may serve as a further example, that contains a motive reappearing in all periods of Gothic art.

Strongly projecting corbels.

Only when the corbel has the form of a semicircle or half p polygon will its lower plan be geometrically similar to the upper one; if the upper area exceedr half the ground form, then will the lower do this in a still higher degree, and finally be the entire form, which is free before the wall or the surface of the pier. Thus the upper area in Fig. 654 is formed by a part of a circle comprising about 225°, and thereby the lower part is nearly the entire circle, that must be entirely separated from the face of the wall by a considerable projection of the bell as in Fig. 641.

Still more decidedly appears this condition in corbels projecting from a point as shown in Figs. 668 and 667. As shown in the first Fig. so the side view, the corbel would terminate in a point hanging free before the face of the wall, and thus its lower portion is undercut (also see Fig. 665). If such projecting knobs already produce a specially bold effect, then is still thereby lost a part of the height if the ashlar for the s support in a useless way. But these free hanging points may be avoided, if only in the front half the projection appears concentric from the middle line, and conversely as in Fig. 668 t the near undercut is omitted, so that the moulding penetrates the face of the wall in a direction parallel to the projection. Thereby the point below is changed into a horizontal edge or a rounding.

Another means of transition results from the oblique development of the corbel when the lower part is placed in the face of the wall, while the middle of the upper part projects from it. When this solution is applied to a simple corbel which accordingly projects from a point lying in the face of the wall to an upper surface formed of five sides of the octagon or four of the hexagon, then the sides become oblique. Such forms are

then employed for both simple corbels as well as those covered by foliage, but in the last case they would be concealed by the leaves lying before them. Examples of this kind are shown by Fig. 655 from the church at Haina and Fig. 669. Here also belongs the arrangement particularly common in the 15 th century, whereby the corbel imitates a little column with a capital, w whose shaft forms the actual projection instead of remaining vertical, also with an arch that sometimes leaves the face of the wall horizontally and bends to the vertical in the axis of the capital,(Fig. 679) from the rood screen of the collegiate church in Oberwesel).

Figs. 669 and 670, strictly taken, are to be already counted with the corbels projecting at one side, to which the latter also belongs according to their function.

Corbels projecting sidewise.

The use of corbels projecting sidewise has the purpose of forming a bearing for wall strips, beams, lintels of doors and windows, and further for all those for filling the tympanums of arches and slabs to fill above openings for doors and resting on edge, then the slabs lying flat that form the floors of balconies and bays or bottoms of gutters for water. They further serve to support springings of arches in all directions, so that in the last sense for the previously described corbels of concentric form, they appear or are combined with them as in Fig. 654.

According to its shape and function, it consists of one or more cut stones with the beds lying on each other or of one slab set on edge.

As a good form for all shapes, or rather as the necessary c cross section for them we must also assume here the triangular or swelled profile in Figs.672,and 671. The ratio of height to projection must then vary according to the load received.

There Fig. 671 first follows the form of the quadrant, which also in smaller dimensions or in simpler treatment is employed without any further addition. Partly to strengthen the upper edge, but partly to meet the fact, that every form loses a portion of its expression in execution, and hence the quadrant w will appear as a smaller segment, this generally receives an addition by a rectangle a b c f, that without increasing the height causes the centre c to move on the line ac nearer to c.

In the same manner is formed the corbel with the simple slope (Fig. 672), either by the addition of the rectangle a b e f, or just as for the quadrant by the reduced slope parallel to d b, whence results a quarter of an octagon, and the corbel projects from the face of the wall at right angles.

A lighter expression is found for the corbel by a concave a arched form (Fig. 673), either shaped according to a b, c d or b c. These shapes would then be somewhat varied by rounding the courses. The rounding becomes necessary if the centre of the curve lies so high that sharp corners result at c or d, and t then they lead to an ogee form, such as the cross section of Fig. 703, yet may indeed be replaced by a slope.

Richer forms then result from rounding, slopes and coves, by combination of similar ones as shown in Figs. 674 to 677, or from different ones, either with rectangular fillets between (Fig. 678), or without these and hence by tangency without them. Further with a more animated treatment, for example like Figs. 679 and 680 in two little examples serving as supports of the former art clock in the southern transept of Strasburg cathedral. Here as on the forms of corbels explained above the triangle forms the necessary cross section. For the good effect of the line it is also useful to enclose it in any regular ground form, even if circumscribed with a certain freedom, thus like the form of profiling between two lines, for example b d and the corresponding curved line (Fig. 681). Within the area so limited may then occur even cuttings, as in Figs. 679 and 680, although these fulfil no useful purpose.

Front edge with projecting rib.

Other forms result by a combination of different shapes of corbels in thickness, for example when in the mass of the ashlar is cut a concave form of corbel accompanied by a thinner part left in a line projecting further, which then stands on the surface of the hollow of the rib, and then serves to strengthen the whole (Fig. 682).

Instead of a lesser thickness may this rib also be formed as a square set diagonally in the entire width of the entire corbel, so that as shown by Fig. 684 it is gradually extended from the front surface. But also may the mass between the lines a b and b c project horizontally from the front of the corbel, as

indicated by dotted lines a c and b c in the same Fig. Further the entire profile of the corbel may also be wrought in the e entire thickness in the direction of the lines a b and b c, instead or horizontally.

The entire proportion of this rib to the mass of the corbel is most clearly expressed, if the latter is angular and bordered by the horizontal and vertical directions so that the rib supports the horizontal surface from the vertical (Fig. 685). Now in the same manner also the horizontal underside a b, against which is placed the rib for its depth, would be supported by the sides of the latter, and thus results the connection of both parts a chamfer, a member, in brief a continuous corbel.

Particularly often that projecting rib receiveu the form of a cusp, at least in the later periods of Gothic art, and generally so that the cusp is inserted within an arch. Also in the last case as in Fig. 686, it man have a rectangular section, so that its sides may form a tangent continuiong the line of the cavetto, or also in a similar manner it may be subdivided as in actual tracery, and then as there have the same connection with the mass of the corbel.

Chamfering and moulding the angles.

But also the angles of simple corbels as represented in Figs. 671 to 678 may be chamfered or moulded (Figs. 687 to 689). But this moulding may be wrought through the upper bearing area, only when it is continued in the supported part, arch or slab, and must in any case pass into the square beneath it. Just as little may it enter the mass of the wall, but must in the simplest cases stop on the surface a b c (Fig. 689) built in the face of the wall. Likewise results the simplest transmission from the moulding to the square at the upper end of the corbel, because the former is wrought on the front edge of the corbel, for which purpose a movement differing from the line of the corbel must be assumed, as given at d e in Fig. 689. But generally all kinds of transitions are applicable here, and by their use is already given the means to produce richer forms. But especially suitable is shown that in Fig. 688, whereby the moulding extends around the front surface and thus contributes to the projection of the corbel.

Corbels in several courses.

That form shown in Fig. 674 and composed of several quadrants placed over each other usually receives an addition of little intermediate members a in Fig. 690, whose width remains about the same as the fillet beneath the entire width. If the corbel consists of several stones laid on each other, then it is nearest to give the lower one less depth than the upper as in Fig. 691, and to use the excess of the latter for a moulding extending around it. The construction with several stones is then expressed more clearly, if the corbel is composed of several rectangular stone blocks extending beyond each other, whose lowest is enclosed by a moulding. By such forms at the same time is obtained a greater width for the upper surface of the corbel, whereby the upper portion may consist of two parts with an abutting joint. In its further extent these forms agree with the arrangement of concentric corbellings.

Development of the front surface.

Richer forms are further obtained by decoration of the mouldings, both those on the edges as well as those forming the f front. But further and in the most effective way according to the analogy of the forms of capitals, and thus by forming the mass of the corbel like a cavetto, whose upper edge is then supported by a leafy bearing. According to the scale of the p proportions of the front must then the supporting leaves be more extended than for the capitals. The treatment of the supporting leaves may then correspond to the side projection of Fig. 5,7. (A particularly beautiful example of this kind is f found in Viollet-le-Duc, Vol. IV., p. 312).

Further in the same manner as on capitals, this bunch of leaves is usually found in a double row, and also some such supports bend out from the front of the corbel and animate the entire form, or are also replaced by applied leaves as on the c capitals. Likewise sometimes the supports are replaced by figures attached to the front, as under the upper roof gallery in the south transept of the colleciate church in Colmar, for then generally the mass of the stone lying outside the line b d is utilized for any ornamental form, that must be cut on the surface within this line if it retains its character of an actual support.

Likewise retaining a geometrically limited outline may be c connected the arrangement of richer leaf ornament, that lies

in the hollows formed between the rib and the mass of the corbel in the mode of Figs. 682 and 684, about as given in Fig. 683, or on a corbel with chamfered edges extends from the front over the chamfered surface and this finally covers the entire corbel as a single great leaf as in Fig. 669, or as complicated scrolls. In this case then the cross section of the corbel also takes a changed form projecting concentrically.

More rarely is found the arrangement recalling the antique by a great leaf only covering the front, as on the corbels beneath the cornice slab of Notre Dame of Dijon. On the contrary is first of all on corbels used as supports of lintels of doors is the use of little crouching figures or of monsters crouching thereon (Fig. 693 from the western doorway of the church in F Frankenberg).

Development of the sides.

Far less important in effect and without relation to the proper function of the corbel is an ornamentation of the sides, as by means of sunk panels, usual in modern architecture. The construction of the corbel-like tracery with a slab set on edge sometimes in late Gothic led to a treatment of the sides like tracery, even to the perforation of the entire corbel, so that it assumes just the appearance of a piece of tracery, as such was already introduced by the addition of the cusp. But the p perforations here are very injurious to the resistance, as justified by the function. Yet such forms on the old works generally contain a structural principle, that confers on them a c certain meaning, though superficially. Such an example is offered by the corbels under the gallery in hre south traysept of S. Severi in Erfurt (Figs. 694, 694a).

The same are formed as pointed arches with free piers projecting from a wall, yet so that the spandrel projecting from the wall passes through the crown of the pointed arch and entering at the upper end of the pier as a quadrant supports it. The s space between the upper part of the arch of the quadrant and the front part of the pointed arch is filled by open tracery, and the pointed arch has cusps. The principle of the entire shape, whose motive is required in Figs. 694 and 694a, is accordingly based on the hanging arch (p. 78), yet is more clearly expressed, since the proper support of the whole is visible in the mass formed above the quadrant. The piers thereby supported

serve at the same time as abutments of the arches in Fig.694a, representing the front view, extending in the longitudinal view and supporting the front edge of the slab and also with cusps, above which the spandrel is again perforated as tracery.

The inexhaustible wealth of forms of Gothic corbels, that we have endeavored to indicate in the preceding, we cannot refrain from comparing the manner in which the same forms are treated in modern antique art, and partly so. The ground form or generating curve is the line represented in Fig. 695, and the only freedom in its use consists in the position in which it is placed in the horizontal and vertical directions.

For either a b may be the vertical and b c the horizontal direction or conversely, and likewise d e may be vertical anf ef horizontal or conversely, and finally the same alternation may occur with the lines i g and g h. Besides these a diversity recalling a slight change is added when b c is vertical as a new volute dotted in our Fig. Further variations then consist in the number of turns of the volute, the form of the eye as well as of the anthemion or palm leaf, which fills the spandrels at the sides, and the leaves which lie on the front of one volute beneath the other. To Grecian and Roman and even Renaissance architecture such monotony.is foreign, for the former first develops the entire motive and indeed in a far more perfect way, but the two last animate it by the magnificence of its sculptures and manifold ways, but in our times the reversing indicated by Weinbrenner to Schinkel and with the latter lasts till the present time.

Connection of corbel with the supported parts.

The connection of the corbel with the parts supported occurs in the most varied ways according to the purpose. The upper s surface is dressed plane and the beam or cut stone is laid thereon. When at the same time any movement in a horizontal direction is prevented, as by the placing of the ridge plate of a shed roof will it be doweled, or there is left a raised margin rising from the top of the corbel, as a b c in Fig. 696.

Covering slab laid thereon.

The connection with a covering slab is by a direct bearing. The slab may be in line with the extreme projection of the corbel, the lower edge being chamfered is moulded between two cor-

corbels, the moulding either stopping square before the support, or turning vertically is continued along the corner of the corbel (Fig. 297). It may further project above that by a continuous moulding, and this projection either recedes by a wash to the line of the corbel, or continues on the vertical slab and further on the face of the upper wall or parapet.

While in antique architecture the forms of consoles, modillions, etc., allied to corbels are surrounded by a moulding at the upper edge, but which is actually wrought on the slab and from the beginning according to the depth of the undercutting, so that as shown by Fig. 699, the ogee a is broken around the console and between them is worked through the plane b, this weakening of the middle of the slab is found to be avoided in Gothic architecture. On the roof cornice of Notre Dame in Dijon this conversely receives a strengthening by the rosettes projecting from the underside between the corbels (Fig. 698). The slab requires no undercutting since the removal of water is done by the mouldings above.

Slab on edge between corbels.

Setting a slab on edge on the supporting corbels varies according as the slab stands in the direction of the corbels as on lintels of doors, tympanums of arches, etc., or makes an angle with them, most simply a right angle., as for example the arch a b in Fig. 694a, that may also be formed of one slab. T Transverse slabs sometimes have an entirely peculiar treatment, particularly on mantles belonging to the 15 th century, which represents the transition from the covering with a slab to that by vaulting. The jambs of such mantles are generally formed of slabs set on end and receive a corbelled form at their upper ends, by which the necessarily wide projection of the smoke m mantle at the ground is changed into a narrower one less restricting the area of the room, and bearing the slab set on edge and forming the front surface of the smoke mantle.

There for removing the projection as far as possible from the ground, it came to allow it to project to the lower edge of t the slab when possible. But then the slab could not be laid abruptly on the jambs, but as the perspective view in Fig. 600 shows inside and Fig. 700a shows outside, it is notched on the jambs as at a b c. This construction then leads to the usually repeated treatment represented in the same Figs., whereby the

outer half of the rib moulding forming the edge of the slab p, but whose inside is formed by a simple slope d e in Fig. 700a of the same height as the profile of the rib, that likewise c continues on the jamb at g in Fig. 700 and intersects the inside half of this moulding.

Arch and vault between corbels.

Now if the slab is to be replaced by an arch, thus first by a straight arch, the jambs must be so deeply inserted that the mass of the masonry between them cannot be crushed by the thrust of the arch acting as a lever, and have a sufficient strength not to be broken at the face of the wall by this force.

Likewise instead of the straight arch may be employed any other form of arch, and the abutment of the arch pe wrought on the corbel or on a stronger ashlar laid on it. The arch can be used to support the front edge nf a floor slab as in Fig. 694a.

Further the covering slab may be entirely omitted if the cross arch continues to the entire projection of the corbel from the face of the wall, thus to a tunnel vault turned between t them. Such an arrangement is found for every width of projection, but especially in that continuous corbelling, that either carries a projection of the upper above the lower face of the wall, or is to serve as the required support of wall beams. Sometimes as shown in Fig. 701 the lines of the arches continue above the corbelled support at its middle.

More favorably is received the thrust if the tunnel vault is replaced bn several concentric cross arches stepped slightly under each other, whose abutments are wrought either on the inner sides of the corbels, or better project from them, so that the sides are wrought with the projection of the inclined joints concerned. A very thoughtful arrangement of this kind, which corresponds in principle about to Fig. 702 is found beneath t the bay window of a subordinate building of the castle of Meissen, where by projecting continued and different cut stones results the support. The Fig. will make all explanation unnecessary.

There is sought the design of a cross vault between two corbels, since here the special advantage of it, obtaining height, is useless, but the effect of thrust remains about the same. Like the total thrust of the tunnel vault on a lever arm corr-

corresponding to half the projection of the corbel, so is the effect in the cross vault of half the thrust on the entire projection of the corbel as lever arm, while the other half acting against the face of the wall may be neglected. The only difference is that a part of the thrust acting in the direction of the projection of the corbels acts in an opposed direction, a and thus vanishes, it being always assumed that the abutment or springing of the rib is wrought in the corbel as shown in Fig. 70a. The tension in the upper part of the corbel is thereby increased. If these springings of arches are wrought on separate stones laid on the corbels and not built into the wall, such a construction is allied to that used for vaulting beneath a bay window, and thus generally the plan of windous in the sides of the bay window courses, with the greater part of the load acts on the ashlar under the angle pier, and thus ensures its position, and this makes a special point for receiving the thrust of the cross vault resistant.

Angular position of the corbel.

As a rule, corbels project at right angles to the face of the wall. But where the projection is to be continued around the corner, there generally for the better support of the slab, a corbel is set diagonally, conversely to the antique arrangement, according to which two were set at right angles at the corner, and the surfaces continued, but both came to be wrought from the same block, so that the corner of the slab remained without support. The diagonal corbel must then for its greater projection have the same proportion in height, strictly taken, or it may have the same height if the arrangement is found as given in Fig. 704, whereby the projection of all corbels remains the same. The arrangement of a diagonal corbel becomes an absolute necessity if the plate laid on it is replaced by arches.

If the line of the projections from the wall varies, then in the simplest cases, if by the projection a polygonal area is obtained, then must the corbel either be perpendicular to the face of the wall or to that of the projection. Figs. 705 and 705a show the first arrangement, according to which the arches a b turned beneath the oblique sides of the octagon on the face of the wall, also intersect obliquely the sides of the corbels, so that to the latter is added the abutting block suited to re-

receive this arch, and shown in perspective in Fig. 705b. An example of this kind is found under the bay window of the Princes' hall in the city hall of Breslau.

The position of the corbel perpendicular to the line of projection and oblique to the face of the wall is only possible when they leave sufficient space between them on the latter to be able to extend to a sufficient depth in the wall. But then the projection and accordingly the height of the separate corbels will be different.

Arched corbelling.

Here belong the supports of bay windows, pulpits, turrets etc., very frequently employed in the later periods of Gothic architecture, corbels formed as parts of arches as shown in Figs. 706 and 707.

Concentric treatment.

In such corbels the crown of the vault rises in the air, the keystone or ridge arch of an actual vault being replaced, either by a knob attached to the lower edge of the upper slab as in Fig. 707, or by an arch as in Fig. 706a extending horizontally and continuously beneath the same edge, as in Fig. 707a. It might appear as evidence of the caprice of late Gothic architecture, that two such different forms as a corbelling and a vault should be formed according to the same system. But a closer investigation shows, that the arrangement of such corbellings is based indeed on a labored, yet still always a structural principle. For any half of a vault may have its entire stability, when the counter thrust of the other half is replaced at the crown by the resistance of a wall or by anchoring. Thus in Fig. 796a the arch h c forming a half arch may be turned w with the least material, when the crown c is ensured against yielding horizontally by an iron anchor, and between two arches constructed in this way may even be turned compartments. But in Fig. 706a the iron anchor is more skilfully replaced by the slab f c a b d e built in at d e and ensured in this position, so that between the springings a b of the arches and the lower corbel b are likewise turned vault ribs, allowing compartments to be turned between the latter. If now as generally occurred, the entire form was built of horizontal layers, so that arches and surfaces of compartments as well as the springings of arch-

arches rest on the same blocks, then this arrangement only calculated for less material for the change to the more convenient mode of construction (as already on the Greek columnar orders the forms of certain parts must have been derived from wooden construction and from a different material), and the strongest objection to the entire form might be sought in this, that it does not plainly exhibit its construction, so far as it is impossible to see it externally, for example whether the middle block in Fig. 706a is ensured by a deep horizontal bed a b, or that it extends between the slabs placed above and below, and therefore has the shape given by the dotted lines.

The construction of the ribs in the same stone as the compartment, whereby the thickness to support the former must be wrought on the latter, then leads to the arrangement of a complex scheme of ribs and at the same time the height of the rib is limited, so that the front edge of the upper slab c must not be undercut too much. This undercutting may however be reduced, if the rips are joined with the mass of the compartment below the edge in the manner shown at the springing of the ribs as shown in Fig. 287a, so that the surface of the compartment extends under the edge according to the dotted line in Fig. 706a.

Continuous corbelling.

Such corbellings are to be formed by producing a concentric as well as a continuous surface. In the latter case (Fig. 707) the form of the net vault is the basis. A very artistic example of the last kind is found under the beams of the city hall in Cologne at the side next the Neumarkt.

The construction of such a corbelling and the arrangement of the joints are according to the nature of the material. If the ashlars are sufficiently large to place the highest bed so high, that the arch line is not too pointed, but intersects at an angle at least of 60° to 70°, then may it be laid horizontal, if above all the jointing of the stone allows an acute edge. In other cases with smaller or coarse-grained material, it is better to make the bed joints in front like the notching in wood construction at right angles through the arch lines of the ribs and the body of the compartment (see a b d), so that in the p plan of Fig. 706b, f f indicates the surface of the radial joint.

More complex will be the arrangement of a corbelling formed like Fig. 707, since here the beds of the ribs lie in two dir-

directions intersecting at an angle as shown in the perspective of Fig. 707a and the plan of Fig. 707b. The lower ashlar A enters the wall like the springing of a rib, and on the upper edge extends the radial bed surfacer f and f′ on three sides. On the bed surface f′ then lies the ashlar B, on which is wrought the intersection of the ribs. The sides of the latter (s in Fig. 707a) remain vertical and on the upper edges are wrought again the radial joints f″. On the joint surface f and between each two blocks B then lie the intermediate blocks C, on the upper edge of which continue the radial surfaces given by a e, so that the blocks of the upper slab D, that extend back into the wall and on which rest the uppermost parts of the ribs, are placed against the latter according to the extended surfaces d c b a e.

The stopping of the ribs under the upper edge of the corbelling is then found changed, so that the rib breaks around this edge and intersects the profile of the slab (Fig. 708). On the graceful pulpit of S. Blasien in Mühlhausen the profile of the rib projects beyond this edge of the wall of the parapet, and stops beneath the breast moulding of the pulpit, so that is p produced a moulded post at each angle of the polygon, and the surfaces between each two posts are ornamented by tracery.

Likewise the tunnel vault can form such corbelling, and then is formed a simple cavetto without ribs, or also with ribs the form in Fig. 709, indeed here the structural principle is clearer and the entire form is one fully justified. For in Fig. 709 the upper ashlar A enters the wall deeply and is anchored to it by the loading or by dovetail ends, so that here not only the lower blocks B of the ribs extend between them and the wall, but also the front block C of the upper edge holds its place by an oblique joint d a lying in the horizontal plane, and need not enter the face of the wall, and thus the slab proper may be omitted. But this block forms the ridge of the tunnel vault D between the ribs, which can therefore be executed in small pieces of material.

The use of the same construction in Fig. 707 also then leads here to fasten in the wall only the pieces on which the ribs intersect below the margin, extending between them the pieces of mouldings, so that here are avoided even the inclined joints seen in Fig. 709 at d e, replaced by the joints of the ribs ((Fig. 710). But in the last form the construction still has the

utility, that it allows placing the openings over each other, indeed is based thereon, so that if in Fig.710 are formed arched openings under the side arches, that start from the piers a, the upper piers stand over the crowns of these arched openings. Such constructions naturally cannot receive great loads.

Some purely ornamental forms are still to be mentioned.

The undercutting of the dege of the slab as seen at c in Fig. 706a, since the joint a b prescribes a greater height for the stone cut away in front, leads to the use of this height for arranging suspended arches extending around beneath the edge of the slab, that are preferably in place when the body of the projection is without ribs, but also generally occurs in connection with moulded corbels and at every scale. Likewise for the proportions of the stones for the arrangement of cusps on the edges (Fig. 711), and further for the arrangement of more or less undercut and even perforated tracery before the body of the projection as if enclosing it in a cage, an example of which is given by the pulpit of S. Leonard's church in Frankfort, as further all forms of ribs mentioned later in Figs. 95 to 102 are also employed on such corbellings.

Brick corbellings.

As generally the possibility of forming the greater and richer corbellings depends on the size of the materials, brick thus lends itself to it only with a certain difficulty.

Corbellings at a smaller scale as under the beginnings of r ribs are found made of terra cotta in a single piece, and only treated with less freedom than stone. Examples of this kind are shown by the remains of the Cistercian abbeys of Chorin a and Hude near Bremen, the castle of the order at Marienberg, etc. But obtaining greater projections is only possible by the continued projection of the upper courses beyond the lower ones, which may then occur in ways with or without the use of specially shaped bricks. Fig. 712 exhibits the different arrangements possible in a single example.

Simplest and nearest is the projection of the different courses over each other in ordinary bond as indicated from a to b. To secure a stepped ground surface with lesser projections sidewise may end in the manner shown between e and f. In the last way can a simple rectngular ground be produced by carrying higher the side projections up to the outer face. By adopting the

diagonal direction of the joints otherwise generally results on the whole the ground form of the diagonal square d, from w which then by a combination with certain bricks in the usual position, either as at d or d′, the ground form of the rectangle at c is again obtained. The position of the bricks given at d′, is employed diagonally above c, and forms one of the most useful means for forming a continuous corbelling as under cornices, that causes a different effect according to the number of courses lying on each other, as well as the distance by which the upper project beyond the lower, or indeed according to how such courses alternate with one in the ordinary position. At g is further shown a corbelling with a course set on e edge. But likewise also could certain bricks be progected in the same position as the course on edge, and then be connected by a simple or stepped covering, or by bricks set against each other in gable form as at h. Likewise are arches also usable for small corbellings as indicated at i for the entire body.

Stops at moulded angles.

To corbels are allied the stops that change from a chamfered, polygonal or moulded part into a rectangular one, or from a m moulding to a polygon, and at a larger scale on simple forms of doorways, or at a smaller one at the lower or upper end of a pier, jamb of a window or doorway, etc., in a sense serving as a substitute for the plinth or capital. They occur in wooden construction on moulded girders, beams, framework, in brief everywhere that a connection of two similar timbers, thus as the plate rests on the ends of beams and the beams lie on the wall.

In large size this became actual corbellings and may be executed io each of the modes mentioned. A simple example of such . form in brickwork is found on a hexagonal stair tower of the church of S. Egidius in Lübeck, whose upper story changes to a square, indeed only by a series of ordinary courses set in the direction of the sides of the square and thus extending on the surfaces of the hexagon.

At a smaller scale especially the upper and lower endings of moulded edges come in question, that may receive a simple and geometrical or a richer and ornamental treatment.

The geometrical forms even in great simplicity may occur in quite manifold shapes (Figs. 713 to 723). Those in Fig. 713 a

moulding may pass into the square by a movement toward the angle as indicated by the lines a 1 b, c d, but in elevation by a horizontal band or by oblique ascending lines (Fig. 714), or this may occur by the most varied curves (Fig. 715), so that the different members diminish to the point and join there; thus further in the same Fig. 713 are possible the indicated parallel course of the mouldings toward the side of the square, and indeed again by curves, by oblique or horizontal lines (Fig. 716), whereby the actual or a distorted profile appears on the sides. Instead of the same a different or reversed profile may intersect the moulding (Fig. 718). The appearance of a cut-off profile will be avoided if the moulding extends to the angle and returns on itself according to Fig. 717.

A different ending results from the penetration of a plane by the moulding, first included. This may be inclined diagonally as from b to e in Fig. 713 (See Fig. 719), or it may be arranged to rise steeply at one side with a horizontal edge at the other (Fig. 720), or even forming an edge less inclined (Fig. 721). The change may be formed by two planes forming a gable (Fig. 722) or an ogee curve (Fig. 723) is suited for chamfers or simple mouldings. The same arrangements may occur if the transition is to be made to the polygon. Here belong the late Gothic plinths executed in Figs. 574-581, whose principle consists in the change from one ground form to another.

Generally treated stops were preferred in the Romanesque and early Gothic times (Figs. 724 to 726), and they form an abundance of ever novel solutions. Finally as a most effective treatment of the angle in the earlier centuries is to be mentioned the inserted angle column, that finds manifold uses on piers and the stepped angles of portals, and in consequence of the limited projection and of the peculiar onesided development of capital and base has produced forms suited to the purpose.

FORMS OF PLANS OF CHURCHES.

1. Church with single aisle.

Direction of the church from west to east.

Already from the first centuries Christian churches of every ground plan were arranged with their principal axis extending from west to east.

For the eastern location of the choir are rules dating from Christian antiquity, whose innate reasons are found in Kreuser (Note 1) and Otte, Note 2), and still at the present day are as valid as in the beginning, and are also again generally followed in modern times. If it would accordingly be absurd to d desire to strengthen those innate reasons by external ones, we cannot refuse to emphasize the unfavorable results of neglecting them.

Note 1. Christian Church Architecture.

Note 2. Manual of Archaeology and Ecclesiastical Art.

However greatly the tendencies of men diverge in every worldly respect, yet for the Christians without difference of creeds, their one tendency toward the triune God is expressed in the same direction of all churches toward the East. The reasons t that since the 16 th century led to variations are all comprised in one, that the eye is accustomed to symmetry and might feel shocked by an oblique location, which the course of the streets opposes to the orientated church. If we yield for the moment to this injury to the trained eye, yet it cannot be assumed that the same eye will be less sensitive to the entire effect of a city, than to the view of a street or square. But manifestly in spite of all magnificence of railway stations a and manufactories, yet the churches by means of their bodily and monumental grandeur determine the character of the general view. let one once view one of those beautiful cities that has retained the splendor of their old churches like Lübeck, Nuremburg and Mühlhausen, and then thing of these churches suddenly displaced in their locations and diverging in different directions, and seek to present the image of confusion and lack of harmony which must thus result.

Less glaring indeed, yet just as objectionable are the impressions, that one may receive actually in those cities where the number of their old churches has been increased by new ones, almost without exception inferior to the old in dignity and

artistic importance, thereby perhaps even expressing a negative scorn against the location of the old indicated arrangement.

And is it then the impression produced by the divergence of the churches in the interiors of the cities a satisfactory one? Is there not something even comic in it, if as sometimes occurs, when the doors of two churches on either side of the same street invite one to enter?

Might one therefore not leave to the non-Christian religious societies, whose temples are not yet great, the divergent directions or those toward each new grand East ? (Several years since occurred the case, that in the erection of a new church the building officials desired to recognize a passing railway as such), to retain for the Christian church the old sacred direction eastward?

The divergence of the longitudinal axis from the East line occurring on so many mediaeval buildings is explained by the change in the location of sunrise according to the time of year (Note 1), as well as sometimes the slight divergences of the direction of the nave from the choir, where both parts join at an obtuse angle, as in Erfurt cathedral and S. Maria Stiegen at Vienna, from the difficulty of establishing the line of the building in the space hemmed in by other structures (Note 2).

Note 1. Journal of Christian Archaeology and Art.

Note 2. Viollet-le-Duc. Dictionary of Architecture.

General ground plan of churches with a single aisle.

Plans with one aisle were employed at all times of the middle ages not merely for simple chapels, but also for parish churches and those of the orders, and they sometimes appear in great connected groups. There may be recalled the domed churches in southwest France (Angouleme, Fontevrault, Souillac, Gensac, etc), that with a span of the dome of 10 to 12 m. mostly form quite imposing interiors, and further reference may be made to numerous single aisled churches of the 15 th century, that with buttresses included in the interior attain mighty widths of 18 m. and more in the clear.

Seldom is the ground form a simple undivided rectangle, yet even in the smallest chapels is expressed at least the choir, whether with a polngonal or circular ending, by the reduction of the width and height of the eastern choir bays, or in an e expensive way by the insertion of a separation transverse aisle.

in the last case may appear side apses and a chevet, and in g generaly may occur in execution all the rich choir plans possible to churches of several aisles (see farther below).

The western termination may be by a gable wall with or without vestibule and stair towers, and it may form a developed arrangement of towers.

Ground form of choir ending.

The usual name of choir was originally only for the part of the presbytery lying next the nave, but is now generally understood for the eastern structure containing the altar, and whose ending is distinguished as the choir ending or head.

As the location of the altar, the choir is in such a high degree the essential part if the whole, that it may very well exist without a nave and even may form an entirety, such as found in many little chapels, where the free space outside for the community is limited and hence replaces the nave. On the contrary the converse condition is inconceivable, since no church can be conceived without an altar, which for the latter and t the space required for the communion there must be divided from the interior if it does not project entirely. Such an arrangement where the space for the altar is not indicated externally, therefore in architectural language is a denial of the sacrament of the altar to those without the church, and therefore is inadmissable in Gothic architecture, since its nature is to be sought particularly in the greatest truth and clearest emphasizing of all conditions. Accordingly must the choir with its ending project from the body of the church and differ from it, if not in material size, yet by the peculiarity of its ground form and the treatment of its elevation.

General form of the polygon of the choir.

The plan of choir endings most commonly occurring on Gothic works are with 5 sides of the octagon (Fig. 727) or of the decagon (Fig. 728), or 7 of the duodecagon (Fig. 729). The latter forms have the disadvantage, that a pier stands in the longitudinal axis of the eastern window is lost. All these polygonal forms originated from the semicircle and first differ in whether they are circumscribed by a stilted semicircle (Fig. 727) or by a semicircle as an exact half of the polygon (Fig. 728). The first has the advantage, that it makes possible an easier transition from the plan of the polygonal vault to that of the

adjacent rectangular bay, while the system of ribs going to the centre of the polygon has an independent termination within the partial polygon, which in every half or smaller portion of the polygon is only possible by moving the keystone into the choir ending, if this is moved from the proper centre c of the polygon to c′ as shown by Fig. 729. But therefore the eastern ribs of the bay with the same height have a smaller span than the western, and the entire arrangement has the character of a mere expedient. For the choir ending with 3 sides of the hexagon the course of the ribs thus passes into that of the cross vault over a trapezoid (p. 27).

According to all the plans mentioned the choir polygon has a width equal to the parallel elongation. But it receives a greater one if the radius of the circle on which the polygonal form is based exceeds half the width of the rectangular bay, the choir ending being formed as 7 sides of the decagon, 6 sides of the octagon, 9 of the duodecagon or 5 of the heptagon. (Fig. 732). This arrangement is found in certain Rhenish and Westphalian churches, thus in S. Peter and Maria in the desert in Soest from the decagon, in the church at Sayn from the octagon, in the minster at Aix-la-Chapelle from the polygon with 14 sides, but is also in the Baltic provinces, and it has the advantage of obtaining a considerable enlargement of area for the choir, and even makes the omission of the parallel elongation possible, while the choir polygon directly adjoins the triumphal arch indicated by a b in Fig. 731; but then causes a particularly simple connection of any side choirs with the high choir. Yet the increased width seems without purpose for the worship in general. Herein must be the reason, why the arrangement found adoption restricted to certain regions in spite of its other advantages.

With the polygonal choir ending is also to be counted further the rectangular one. In large proportions this is found predominant in the churches of the Cistercian order and in English works, but in more moderate dimensions is very common in Westphalian provinces and in Prussia, and finally in naves without vaults in some village churches still in the Gothic style, and which we shall mention only mention in the church of Schwarzenborn and that of Nieste in Hesse. In the last combination must indeed be given the minimum of a Gothic church building. Unfor-

Unfortunately an approximation thereto is rather given in existing conditions rather than in the earlier, anf therefore the study of such poor warks is not without importance. Fig. 733 shows the plan of the church at Nieste.

In the early Gothic works in France the semicircular choir ending is still the rule, and for example is found in the cathedral of Rheims in the way that the sill of the windows terminates the circle, and the windows thus form the transition to the polygonal form. A Gemman example of the same kind is shown by the eastern choir of the cathedral in Bamberg. But by the choir plan with aisle and chevet chapels consideration leads to a regularly forming of the chapels on a polygon of unequal sides.

Little shafts in the choir.

In the angles of the polygon stand the rounds on corbels for receiving the ribs of the vault. Determination od their number and diameter depends on the arrangement of the vaulting system. In the simplest case assumed in Fig. 734, only one round for the diagonal rib finds its place, whose diameter cannot exceed the width of the latter. This may be increased if the projecting side arches also must rest on the capital of the round.

If separate smaller rounds are to be arranged for the side arches, then those intended for the diagonal ribs must be set farther out according to the construction shown in Fig. 734a. It is then common to project the internal face of the wall below the windows of the dotted line 1 1, so that the round supporting the side arches stands on the window sill or the moulding extending around .

Plan of the windows:

After locating the rounds the plan of the windows is to be drawn. With the greater development in width these would entireln occupy the width between the buttresses, but in Germany they mostly utilize only a part of it. For the general effect in the interior as on the exterior, it is then advantageous to allow one thing to predominate, the mass of the wall or the width of the windows. For a small breadth up to 1.20 m, the windows best remain undivided, since too small a width resulting from a division of the windows injures the effect of the glass, and likewise that of the arches and makes the tracery placed in them seem puerile. Yet by the later works have men

become too much accustomed to regard the latter ar an essential part of Gothic church windows, and conversely to attribute far too little importance to the glass. The opposite condition is more suited to the nature of the facts, and causes a better effect as shown by so many early Gothic works. Meanwhile even here much depends on the local conditions. For example where is concerned the construction of any smaller included part of the church as a chapel, sacristy, etc., the small dimensions of such details enhances the effect of the larger ones on the church, and thereby that of the whole. In general must the widths of 0.50 and of 1.20 m be the limits of the breadth of the windows.

The most common division of the width of the window is that by a middle mullion into two panels. The proportion of the width of the mullion to that of the panel is determined by the m material and the limits of working and durability, when the l latter is opposed to a continuing diminution, and further it it is greater on early Gothic works than on those of later periods. Thus in the windows of S. Elisabeth's church at Marburg, it is 11 to 35 mm, in the church at Haina 1 : 3, the church at Wetter 9.5 : 2c, thus about 3/8 to 1/3, and in the later works it rises to 1 : 5.5 = 2/11. The extremely weak mullions of the late period therefore can maintain themselves only bn the iron bars extending through the arrangement of the glazing and give to it a meagre effect not far removed from that of cast iron. Indeed the great public, always inclined to regard it as artistic, is accustomed to find the greatest pleasure in all such very ulender parts, it was perverted by the long period of absence of taste, the preference of deceptive counterfeits to an openly reasonable construction, and thus can one experience in general, that in the works erected at different periods, the latest parts find most admirers. Yet quite otherwise would also here be established the proportion of estimation, if in works entirely executed in the style of early Gothic is found one opposed to that belonging to the late Gothic, yet neither more costly nor novel, and thus the harmonious effect of the earlier could enter into its rights.

The depth of the mullitins exceeds their breadth. With the mullion is connected the glass and iron work of the window, and therefore is generally found a half mullion at th

therefore is generally found a half mullion at the window jamb, tte so-called wall mullion, yet this is absent on certain early Gothic works. Consequently the wall and middle mullions are c connected, at bottom by the sill and at the top by the tracery, in the entire height by iron rods to which the glass is fastened, as well as by the last, thus forming a wall that is placed farther inward or outward en the thickness of the wall or is ordinarily at the middle, requiring the character of such jamb mouldings.

As a rule all windows of the choir have the same size. Yet sometimes in the simpler plans the east window predominates o over the others in form and size, thus for example being divided in two parts with tracery above the mullion, while the latter are single pointed windows. This simple arrangement shows an entirely analagous accenting of the line of orientation, as expressed in these rich choir designs with aisles in the arrangement of elongated chapels of the Virgin, and thereby produces a very happy effect. The east window is sometimes walled up a and decorated externally bn a figure of S. Maria seen afar. (Erfurt and Marienberg.).

The east window as a point for the eye must always have a p particularly dignified colored painting. In many new churches the glaring light of the colorless window hurts the eyes and gives the altar an unpleasant lighting from the rear.

The window is sometimes made so wide that the side arch for the vault coincides with the largest window arch, indeed the enlargement may proceed so far that the outer splay of the jamb cuts into the mass of the buttress, so that the wall mullion directly joins the latter and the arch of the jamb starts from its face.

Increased upper thickness of the wall.

In the last manner may also be increased the thickness of the upper part of the wall, if concentric with the window arch or excentric thereto, an arch indicated by the dotted line in our Fig. 734 extends between the buttresses or rests on them by c corbelling. Examples of this kind are shown by the choir of young S. Peter and S. Thomas in Strasburg, a bay from the former being shown externally in Fig. 738, as well as in S. Elisabeth in Marburg. This increase in the thickness of the wall may

be required by various reasons, both b: the design of the roof and to make the gutters more easily, (in which case it extends above all bays of the vaults and is especially necessary when no side arches projecting inward exist; ,and hence the necessary thickness of the wall is taken for the support of the compartments),as well as for the base of any superstructure, but in the last case it may be limited to one bay. On the eastern bay of the cathedral at Erfurt is it found with rich mouldings in the last sense.

Place of the wall with regard to the buttresses.

A happy effect results when the wall within the limits determined by the width of the buttresses is placed farther outward, so that the head of the latter is expressed inside by the angle a b c, as shown in Fig. 734b, which either receives the otherwise moulded side arch on a capital or can continue unchanged in it. Such an arrangement, by which is reduced the projection of the buttresses, is found in the nave of the church at Wetter (Fig. 737), in the nave of the Minorites church at Duisburg (Fig. 735), and in the choir and transpets of the church at Haina. In both the latter examples the wall beneath the window sill is set out to the outer face of the buttresses, so that this projects above the corbelling from the face of the wall at the height mentioned.

A combination of this arrangement with that previously mentioned results when to the internal angle a b c in Fig. 734b c corresponds the external angle d e f, so that the side arch extends through the thickness of the wall and also bears the roof structure, while the thickness of the wall is reduced between the piers e d, b c, i.e., in connection with them and as under the last connecting arch. Here the pier e d f is not visible on the floor, but the parapet wall of the window may set back in the face d i, and either receive inside the entire thickness d a, or remain with less thickness as indicated by the dotted line.

In the first case the great thickness of the wall makes possible even with limited dimensions of the whole, the arrangement of the blinds and cupboards serving for various purposes. Otherwise the projection of the upper wall beyond the lower part, therefore the thickness g k, or if the conditions are reversed the g k becomes the lower wall, the thickness g i

beneath the window sill is spanned by an arch, and in this way the arrangement of the galleries is allowed in the most varied manner, as will be explained later. A varied treatment of this arrangement is found in the archbishop's chapel at Rheims, where the wall between the buttresses is set out so far as to p permit passages to be formed between the lower parts of the latter and the face of the wall.

On the church of S. Catherine in Oppenheim the wall is further placed entirely at the outer face of the buttresses, so that it gives a space beneath the window sills of the side aisle. On many French cathedrals as at Paris, Amiens, Rouen, Meaux, and also on the choir of the Frauen church at Bamberg, etc., with the exception of the last, these include the entire height of the side aisles between the buttresses in chapels built later, and on certain German churches of the late period with the arrangement of aisles of equal height, the wall is placed flush with the outer face of the buttresses, so that in the interior are formed deep niches as in Fig. 736, S. John's church at Riga (from the drawing by A. Reinberg at Riga).

Therefore while according to the Oppenheim form the buttresses are visible externally above the chapels, they appear in French works first above the side aisle, thus in relation to the buttress system. On the late Gothic hall churches mentioned, they do not appear externally. Aside from the dry effect of the exterior, the last arrangement mentioned must already be termed the least fortunate possible for the ground, since it omits the projection of the buttress so advantageous in structural respects. It becomes still more imperfect, if the thickness of the pier extends to the jamb of the window, thus a also that the projection of the pier inside is omitted. Certainly by such an arrangement the depth of the pier can be reduced by so much,as the greater width increases the resistance. But the ratio of the two dimensions to the resistance causes, that the cubic volume as greater and hence the arrangement is disadvantageous in principle. Only in such small dimensions t that the thickness and width of the buttress are small in regard to the conditions of the material, can such an arrangement be right, for comparison of the advantages and defects of the external and internal projection was made a statical investigation first for the external buttresses and then for internal

buttress projections, for the same church with one aisle of 14 m span and 7 m length of bay with 20 m height of wall. This g gave as mass of the abutment for one bay (including the wall panel) for equal resistance in the first case 124 and in the second 156 cu. m. of brick masonry. The masonry required is thus in the proportion of 4 to 5. But then it must not be overlooked that with internal projection is opposed to the greater volume a certain advantage by increasing the internal area.

A great diversity in regard to the forms of abutments is shown by the Minorites' church in Duisburg in its various parts. (Fig. 735). In the polygonal choir is here found the usual arrangement of rounds with buttresses projecting externally, which then in the parallel parts of the south side of the choir is then strengthened by segmental wall piers projecting inside. Also in the nave these internal piers assume a rectangular plan, become deeper as they reduce by the same volume the external buttresses, while in the north side of the choir the face of the wall is placed in the outer face of the buttresses, and w with the smaller width of the choir the internal piers again assume a weaker form before the thicker wall.

This varied arrangement likewise allows the utility of the buttress to very clearly appear. For while on the north side a part of the wall to a height of about 48 ft. contains 8256 cu ft, the volume of the corresponding portion at the south side amounts to only 7488 ft. In this the projection of the buttresses and the deduction of the window openings are not made, which are lacking on the north side. Still greater would be the difference in volume, if the foundations were taken into the account.

Combination of the choir with a nave of equal width.

Side of the folygon falling in the longitudinal direction.

The side b i of the polygonal choir facing in the longitudinal direction differs from the others by the posipion of the western buttress, which is perpendicular to that direction. The round at i is corresponding to this buttress, if the choir polygon forms on the west another vaulted bay, then has the f function of supporting besides the diagonal arch of the choir vault, that forms the only load of the other rounds, also the diagonal arch of the added bay and the dividing cross rib i s. Therefore must the pries of these ribs either be strongly con-

condensed , the diameter of the round be increased, or finally 3 rounds are placed at i. In the two first side arches also spans from i to b, and the middle line of the window placed in the bay i b coincides with the face arch, but therefore not w with the middle of the part of the wall between the buttresses on the exterior. The external part of the wall is unsymmetrical and somewhat shorter than the other sides of the polygon. This especially in the simpler designs has nothing disturbing on the exterior, since generally the care peculiar to modern architecture not observing the bilateral symmetry of Gothic architecture is foreign to it. (Frequently these last sides are intentionally made longer than the others).

But also allow that symmetrical wall surfaces are produced internally and externally. (Fig. 734, right). If the internal pier is formed by lines perpendicular to the face of the wall at the points n and o, the walls themselves being set farther out and the internal piers being connected by the sides arches q r, then the windows in the middle inside and outside, but the distance from the round s to the angle q of the pier bearing the side arch becomes greater than that from the round to the angle r. Accordingly it seems indicated in a certain sense, to use the remaining breadth between s and q for setting the diagonal arch, that accordingly can rest either with the side arch on the corresponding part of the pier, on a corbel or round projecting before the surface. In the first way we come to the form of an internal wall pier, which either remains rectangular or can be rounded to a circular arc, as in the choir of the Minorites' church in Duisburg (Fig. 735), but in the latter way in the plan of separate rounds for each rib shown in the right half of Fig. 734.

But by this transfer of the round t to the east and the arrangement, strictly taken disappears the equality if the internal sides of the polygon. If these must remain alike, then must the round supporting the diagonal rib be exactly at the point indicated by the angle of the polygon, so that b u equals b c, and hence the round bearing the cross rib with the entire buttress must be moved to the west in the same proportion. Also we cannot omit here the remark, that it was far less for us to do for this equality, than to show on this example how easy it the principle of Gothic architecture devotes itself to give the

most suitable expression of all conditions.

The equality of the bays between the buttresses results of itself, if a nave joins directly the round standing at i, which is wider than the choir, and is separated from the latter by an arch nearly or quite corresponding to the thickness of the wall, so that only the diagonal rib rests on the round standing in the angle of the polygon.

The adjacent bays.

But as a rule the choir polygon will be extended by one or more rectangular bays of the same span, and in such a shape g gives the simplest ground form of a chapel or church with a single aisle. The length of this bay may either equal a side of the polygon or exceed it.

The number of rectangular bays depends on the length the chapel must have as well as on its proportions. It is probable for the length to predominate and be at least twice the width, but it better equals twice the diagonal or thrice the width, and xs also the effect of the whole substantialln gains if these lengths are produced by a greater number and not by a greater length of bays.

Western ending of single-aisled churches.

Gable with buttresses.

The western termination in the simplest cases will be formed by a straight gable wall, so that in the angles formed there stand the rounds to receive the ribs, whose arrangement is brought to harmonize with that of the other piers, but stands in the most exact connection with the positions of the western buttresses.

The latter are either perpendicular to the two wall surfaces or are diagonal, i.e., in the line bisecting the angle. Two b buttresses at right angles may either form extensions of the faces of the walls, as in the left half of Fig. 734, or be set back from them, so that the corner freely appears between them, as in the right half of the same Fig. The first is simpler. B But the relation of the round to the buttresses set over in t the arrangement shows externally that the window is set still farther from the middle of the bay, than is the case for the side b i of the choir polygon. The second arrangement differs according to the distance by which the buttress is set back from the angle, and even offers the possibility of placing the

buttress exactly according to the round, thereby removing all irregularities in the interior and exterior. At u in Fig.734 is this plan developed from the assumed placing of a round at s for each rib. As in the left half of the same Fig., if all ribs rest on one round i, then the buttresses stand farther apart and the dotted line p will be the middle between them. Yet farther opens the angle between them for the corner formed by the wall surfaces, if the buttresses extend inside. Still a cean arrangement is to be mentioned, according to which a r round standing at u with another of the same diameter bearing both the diagonal rib and a side arch rib, and then the receding of the western wall at x could disappear, ro that this would retain the thickness x y.

The buttress set diagonally corresponds to the combined thrust of all ribs joining the western corner of the vault, here indicated by the diagonal. Strictly taken, for unequal sides of the bay the buttress mustleave the direction of 45° and that of the diagonal rib, just as with the arrangement of two buttresses at right angles that extend west could be weaker than the other. However this reference to the plan of the bay is taken in the direction of the buttress, but rather by increasing its length, which is usually determined so that the front corner lies in the plane of the other buttresses. But the arrangement of these western buttresses on single-aisled churches is also subject to substantial modifications by the proportions of the western gable wall.

Gable without buttresses.

The western wall requires an increased thickness, first on account of its greater free length, but then for the loading caused by the gable and sometimes by a bell turret placed on it. By this increase in thickness as well as by the load it will be placed in condition with reducing it, to resist the t thrust of the vault acting in the longitudinal direction. Therefore in certain cases the buttresses here in a western direction may be omitted, and there results at once a form, constructed in the right half of Fig. 734, so that the internal s side arches are included in the thickness of the wall.

But this thickening can be produced by moving the gable wall farther to the west, whereby in a sense the gable wall is moved to the line of the western buttresses, and therefore the exter-

external pier is changed into an internal one. This internal will then be connected with the one lying opposite by a cross arch (Fig. 739), which gives the western wall the required increase in thickness and a broader base of the gable and the added turret. The strengthening will be more complete if instead of one, about three cross arches are arranged that rest on intermediate piers, whose arrangement then has a considerable effect on the western end (Fig. 739a).

We can indicate here merely the endless diversity of which these arrangements are capable, and we shall particularly return to it in treating the elevation of the gable end. Meanwhile from what is said will already appear what advantages are to be derived for the treatment of the western part from towers, portals, steps, gallieries and passages.

Since for the oblong form of bay the compartment of the vault thrust acting longitudinally is relatively smaller, then is f found on certain economically constructed works, especially on certain Franciscan churches as in Fritzlar and Treysa, then on those of the Carmelites, the so-called brothers' church in Cassel, the western buttresses at the corners of the gable wall are omitted, even when the gaple wall is not made thicker As the thrust at the corner is less than on the continuous wall (Figs. 366, 367), a certain reduction of the abutting mass at the corner is justified (down to 3/4 the thickness elsewhere, p. 136); yet for other reasons men did not like to weaken the corners.

On the churches mentioned the omission of the buttresses may be permissible by other reflections. For especially in Fig. 740 the width of the window is so small, that from the window jamb to the gable wall remains a short length of wall, so that this distance a b may be regarded as an internal buttress, it being assumed that the ashlars and springing of the rib in which passes the thrust of the vault from the arch is so intimately connected with the wall a b, that a displacement of it is impossible. Men appear to have counted on this resistance with great security, for on the church mentioned at Treysa, and further on the transept at Wetter belonging to about the middle of the 13 th century, all buttresses are generally omitted at the corners, where the gable wall was regarded as an internal buttress against the thrust in the direction of the width. But the con-

connection mentioned, and on which depends the security of the construction, alone depends on attaining a length of bonded a ashlars in the direction a b. The adhesive force of the mortar cannot be counted on, since generally all construction is risky that counts on a tensile resistance of masonry.

Hence his boldness has had the worst results on the church in Wetter, indeed in spite of the excellent quality of the mortar, the gable wallu of both transepts have yielded about 10 insa and have entirely separated from the side walls of the transepts. But that in Wetter the consequences mentioned were not caused by the sinking of the foundations results from their excellent condition proved by an investigation made. Fig. 741 shows the plan of the outer bay of one of these transepts with dimensions inserted. The diagonal ribs are semicircles, the c compartments being vaulted with rubble.

But the acceptance of the entire inseparability of the wall led to the churches with two aisles at Fritzlar (Fig. 756) and at Cassel, also to the omission of the buttress on the gable wall corresponding to the thrust of the dividing arch, when men openly counted not only on the resistance of the entire length of the wall, but on the sliding out of that portion of the wall from its plane where nearest to the thrust of the arch. This oversight has been punished in both cases, and the result last mentioned has occurred.

Stair towers at the gable.

The need of access to the attic or the gallery formed in the thickness of the wall led to the necessity of stairs, that could find their place in the thickness of the wall increased for this purpose, as shown below, or in projecting stair towers. But the latter found at the western end a partcularly suitable place, and could be connected with the buttresses in any manner, or where these were wanting, could independently flank the angles, or they might be a continuous transverse connectiong to afford sufficient resistance to replace the buttresses.

The construction of these stair towers will find a fuller t treatment at the proper place, but here will first be explained only the different arrangements of their plans. It was merely so as to be properly combined with the arrangement of the buttress, or to afford in the best way an abutment for the thrust of the vault, and also that the plan was convenient for exit

and entrance. Thus according to Fig. 742 it could adjoin the buttress, even so that the interior space cut into them, and then according to the size of the buttress it was either in t the same plane, projected beyond or receded from it; or it might stand in the angle between them as in Fig. 743; or if the buttress stood diagonally it found its place at the outer end, as on the transepts of the church at Friedberg (Fig. 744); or lays within the nucleus of the buttresses, so that they project from the surfaces of the stair tower (Fig. 745). Likewise could they be combined with the above arrangement and a strengthened gable wall similar to that in Fig. 745, whereby the western buttress is omitted.

The arrangement of such a stair turret may bring with it, t that the intended space for the window in the bay concerned suffered a restriction. In such cases either the width of the window can be reduced as Fig. 746 shown in plan, even so that the arrangement elsewhere of triple or larger windows here becomes single or double as in the western bay of the church in Friedberg, or the window concerned has entirely the same form as the others, so that the stair turret covers a portion of it as shown in elevation in Fig. 747. The last arrangement particularly deserves the preference if the window must occupy the entire space between the buttresses as on the cathedral of Cologne.

It is also to be note, that an anxious care for symmetry and parallelism in such plans is least in place, and while it injures the access and suitability in many cases, the picturesque effect of the whole is reduced. Then the latter may even be secured if only one stair turret is arranged, just as the direct fulfilment of the purpose en simpler conditions. Even on such works that give no evidence of economical construction in other respects, as for example the now ruined church of the monastery of Obin in the Lausitz, is found this arrangement. More monumental indeed is the effect of the western end, if at each angle is found such a turret flanking the structure, as on the S. Chapelle in Paris, and it then approximates the ground form of the double western towers peculiar to the greater churches. But such transfers and such happy effects can be won, yet certain dangerous germs of caprice, and are better avoided in our time, where the strictest adherence to the well understood purpose

is the first requirement.

The size of such stair towers must be in harmony with the dimensions of the whole, but are still arranged according to the purpose, which requires as a minimum a clear width of about 1.5 m, which increases in larger works to about twice this size, as in the stair towers attached to the towers of the cathedral of Paris.

The thickness of the wall is according to the construction, as the tower exists for itself or must resist a thrust acting on it. In the first case and especially with the polygonal form of the exterior, which causes a considerable strengthening of the corners with the round interior, even with the continued transverse connection made by the steps, a very small thickness suffices. Thus on the church of S. Maria in Marburg is found a hexagonal stair tower where the thickness of the wall is but 6 ins. at the middle of the sides.

Such stair towers are sometimes built inside, as in the south transept of S. Severi in Erfurt and the western part of S. Maclou in Rouen. Then then serve exclusively to make accessible the rood screen or some other gallery, yet as a rule owe their origin to a later alteration. Likewise are sometimes found straight stairs with landings ascending in the interior, as in the cathedral of Rouen and in simpler form in the church of the monastery of Haina.

In the western gable wall is generally found an entrance. A Also here must the dimensions harmonize with those of the whole, and first of all must be avoided excessive size, when on the other hand the need already lixes a minimum. For the various designs of portals, reference is made to the section concerned.

Combination of the choir with a wider nave.

The triumphal arch.

The simplest separation between choir and nave in churches with single aisle is made by a greater width of the latter. T The choir polygon extended by one or more rectangular bays then opens by the so-called triumphal arch a b into the nave, whose eastern cross wall serves as an abutment for this arch.

The symbolic meaning of this arch expressed by the name is, that it forms the entrance to the place where the triumph of Christ over death is celebrated. Aside from this significance, which prescribes for the arch found here a certain distinction

in size and form, the strengthening is necessary for several reasons in structural respects. Thus the width c d must oppose the thrust of the rib c e, and the same width in the development of the arch forms the abutment for the thrust of the separate courses of the compartment, at at least when such is lacking, bears the part of the wall up to beneath the arch dividing the nave. Therefore if one thinks as shown in Fig. 749, there stands at a a single round and a cross rib extends from a to b, then to attain the purposes mentioned, also to turn ribs from a and b to c and also over the sides a d, etc., either smaller side arches are found, or the compartments rest on the corresponding parts of the wall in horizontal lines according to the mode of a tunnel vault. In any case must such an arrangement have the advantage, that makes closer the otherwise loose connection between choir and nave, and unites the vaults of both parts in one system. If then a gable wall terminates the nave at the east, against which the roof of the choir abuts, then there must be turned above the vaults the arch supporting the gable wall, like the dividing arches of many late Gothic churches.

More simple than the treatment in Fig. 749 is still the arrangement of a cross arch from a to b (Fig. 748), whose width as well as that of the pier at a depends on the direction of the diagonal rib in the adjacent bay of the choir vault.

Choir ending in a half decagon.

Fig. 748 also shows the choir ending in a half decagon. The peculiarity of this form in regard to the plan of the system of vaults has been explained above. The thickness of the wall and pier could be the same as in the choir with a half octagon, except that in this case is necessary an enlargement of the p pier standing before t, since the adjacent bay by the dircetion of the rib C r transfers almost the entire weight of its vault to this rib and the cross rib, therefore exerting a far greater thrust, than was the case in the octagonal choir ending.

In Fig. 748 two parallel bays are added to the choir with the same length of sides as for the polygon. But this equality will be avoided if the choir passes directly into the nave, since then the inequality of the panels between the buttresses, that results from the greater width of the bays of the nave, better expresses the actual relations of the whole.

However carefully men thought in mediaeval works to oppose a

corresponding resistance and even to those directions of vault thrusts resulting from certain abnormal plans, and did not aven fear certain irregularities, as shown by the church at Immenhaus near Cassel. Here the choir is wider than the middle aisle, and the springing of its ribs is higher. In Fig. 750 a b indicates the face of the south side of the choir, m o the middle aisle and so the south side aisle, whose vaultinm and dividing arches of wide span therefore have not found the necessary abutment at c in the thickness of the pier supporting the triumphal arch. Hence beneath the beginning of the diagonal rib of the choir is arranged an internal buttress a b d e. On the other hand to resist the thrust of the choir vault at a, the upper thickness a f of the wall suffices on account of the small length of the bay.

The triumphal arch may rest on projecting piers extending f from the ground or corbelled piers, or be corbelled below its springing line, or finalln according to the later method extend between the faces of the choir walls. Put the last arrangement has the disadvantage, that the triumphal arch and the adjacent side arches of the choir vaults become excentric.

To the entire arrangement of unequal widths of choir and nave belongs an uncommon flexibility, that is particularly suited to limeted requirements and means. It widens the room for the community, leaves the choir as free as possible, and makes prominent a separation in its equally advantageous for the ethical importance and the picturesque effect. It especially enters into its rights, where limited conditions lead to the square form of the choir, and plan that without this reduction omits all special accenting of the choir, and must generally produce a certain monotony, but which would be avoided by the greatest simplicity.

A rich anf grand example is presented by the already mentioned church of the monastery of Orbin, whose plan shows a choir comprising 5 sides of the octagon extended by a rectangular bay, that opens by the triumphal arch into a nave consisting of 3 bays, whose width is somewhat more than the diagonal of a square described on the width of the choir. There the axis of the choir lies in the extension of the nave.

Addition to width at one side.

The last arrangement is sometimes found changed, so that the

added width is made only at one side of the nave, as on the Minorites' church in Duisburg represented in Fig. 735. Such irregularitees may first be referred to local conditions, as the plane surface of the north side here indicates tha addition of a structure, yet however affords also certain advantages for convenience in use, since then in this case on the wall surface a b is thereby afforded space for the parish altar, for which half this width would not have sufficed, that would have resulted from a continuous axis.

Plan of transverse aisle.

Instead of increasing the width of the nave, the interior may be enlarged by the arrangement of a transverse aisle, that intersects the nave forming a middle aisle before the triumphal arch with the extension of the choir.

The entire ground form results in the simplest way by the development of the six sides of a cube, the eastern being changed into a polygon (Fig. 751), and it first leads to the arrangement of a square or halved cross vaults, which can then be changed into squares of the transepts, so that in the middle of their sides can be assumed a pier, from which extends a rib in the form of a half arch, to the crown of the vault, so that the vault is divided into 7 parts as on the transepts of the church at Wetzlar, and further by the arrangement of intermediate ribs in the middle square (Fig. 66).

Just as the arrangement of oblong cross vaults comprises in a sense an emancipation from the sjuare form, so will its application to the cross-shaped church will lead to extending the middle square to east and west by a varying number of oblong bays, likewise adding on the north and south one or more bays. The proportions of the different bays may then be the same or different, according to whether the entire ground form or the single bay is made the basis, so that in the first case the proportions of the bay result from the subdivision of the whole, in the other from the addition of bays. In Fig. 752 is assumed the first system.

As a rule the middle bay is enclosed by stronger ribs corresponding to the triumphal arch. If they are here required with less certainty by the construction, they are always of great utility. They offer to the rather unequal stresses in the courses of the compartments a safe abutment, whose thus expresses that

the greater length of the middle square contrasted with lesser ones of the other bays seems to demand a more definite evidence for it. They further afford to the construction over the vault, thus first for the roof or a central tower the required basis, a necessity that can only be expressed in the interior indeed, if the vaults of the middle square are raised above the others, as in the interior of the tower or roof, as in S. M.clou in R Rouen. On the magnitude of the loading therefore depends the dimensions of the arch and of the wall piers strengthening the angles and supporting them.

If in Fig. 752 the width of the arch is made equal to the t thickness of the walls and the arch is constructed with two concentric layers, then the angle pier is built accordingly, whose plan is represented in Fig. 752a, while a b c in the same Fig. shows the arrangement of the round. With a richer form in which for each rib and each layer in the great dividing arches is arranged a separate round, there results the plan represented in Fig. 752b, where a b c is again that of the other rounds. The considerable projections arising in this case are reduced by the corbelling of the outermost parts, thus here the rounds, while the rectangular nucleus either extends from the ground or rests on a column, or may be itself corbelled, or further be assuming a different form of the nucleus, that of a circle or arc, from which the rounds may be corbelled.

The entire plan of the cross church has the advantage over the widening of the nave shown in Fig. 748, of a more organic and united development, the effect in both interior and exterior is richer and more varied. There the ground form itself presents already in a happier way the abutment of the thrust of the middle vault of wide span, while the side walls of the transepts themselves aid in placing the buttresses, and thus make possible the addition of a central tower without special strengthening.

Geometrical relations in dimensions of the plan. Ratio of a abutment to width of span.

Men have often attempted to discover from tradition and measurements definite geometrical relations in the parts of old buildings in plan and elevation, and desire to imagine in them the secrets of the master.

The repetition of equal or similar parts, the constant regular

laying off of lengths, as well as many geometrical divisions that may be derived from the regular hexagon or octagon, from the ratio of the side of the square to its diagonal, etc., may contribute much to make clear and expressive the impression of an art work, and is sufficiently known and escaped the lod masters just as little as the modern. Men have even believed in the middle ages and especially in the late Gothic, that such obtaining of dimensions was practised with diligence.(Further on this see the later systems of Geometrical Proportion).

But the desire to conclude from this, that an entire architectural work in general and detail was restricted within an ever recurring web of circles, is rather risky for even the l later works, but for the creations of the early time contradict their own evidence. Just as the art of the time reaches its noble bloom, that like no other is it free from the restraint of patterns, and yet created the work from its internal nature with powerful strength from case to case.

Width of the abutment according to the rules of experience.

There come into consideration geometrical relations on account not only of the architectural expression, but also in reference to statical requirements, especially in the relation between span of the vault and the thickness of the wall or pier, which by its importance comes into the foreground. We h have then accustomed ourselves in daily practice of vaulting to fix the width of the abutment as a fraction of the span ((for example, 1/3, 1/4, etc.), and it is natural to establish similar values from experience also for vaults of churches, except that here the relations are less simple. So long as the results of statics are not available for the practician or are not sufficiently convenient, there must serve as substitutes in f fact for such starting points, as modern masters have therefore justly held of sufficient importance to establish suitable rules. Some of the most useful are as follows.

Hoffstadt developed in his Gothic A B C the dimensions of w walls and buttresses on the ground of some manuscripts belonging to the latest period from the choir polygon, where he assmed for the thickness of the wall and width of the buttress 1/10 of the clear width of the choir, and for the projection of the buttress from the face of the wall the diagonal of a

square constructed with this dimension. (In Lacker's instructions. See the collected writings of A. Reichensperger, Leipzig. This projection is made twice the thickness). The total width of the buttress was made by Hoffstadt 1/4 (more exactly 0.2414) of the span.

2. Viollet-le-Duc gives in his Dictionary of Architecture ((IV, p. 63) a method apparently employed in the 16 th century, according to which three equal parts were laid off in the arch (R S = (M = M T, Fig. 753) and the distance of the point of division from the vertical erected at the end, thus M N gives the width of the abutment, which is to be laid off outward at T. For the semicircle this amounts to 1/4 the span, for the pointed arch according to its steepness, to 2/9, 1/5 to 1/6. As a limit of the application the height of the abutment is designated as 1 1/2 the span.

3. Hase introduces a similar but more complete method. He determines the width of the buttress likewise by dividing the cross section of the vault in three parts (Figs. 753, 753a), but then for each 4 1/2 m height of abutment adds 15 cm to this. If the buttress of this width consists of heavy natural stone, it might support a vault of light cut stone or strong bricks, but if it be of ordinary bricks, it would suffice for a vault of light bricks. These dimensions are suitable for churches of one aisle if the wall aids somewhat; for a church with three aisles (proportions of width of nave about 2 to 1), then are ample for the stress in the middle aisle as a basis; in any c case they also suffice for the external buttress that receives the flying buttress of the middle aisle. Rectangular bays are assumed with sides in the ratio of about 3 : 3. If the bays a are more nearly square, the abutment is to be correspondingly strengthened.

4. In the earlier editions of this manual is finally given a definite construction first for choir vaults, that takes the thickness of the vault into consideration (Fig. 754). The rib arches over O A will be turned as revolved at A C in the plan, the depth of the rib and the vault lying there is added as A A_1, so as to form the greater revolved arch A_1C_1. At the bisecting point B of this is drawn a tangent to cut the ground line at D. Then the length A D is the width of this buttress. It gives for the semicircle 1/5 to 1/4 the span, notably less for pointed

arches according to their rise, for high pointed arches (rise over 2/3) the method can no longer be applied, since the abutment would be too narrow. The thickness of the buttress and of the wall can be determined by a similar proportion as in Hoffstadt's construction.

The width of the buttress found thus for the choir ending is also assumed to be sufficient for the straight extension of t the choir, it being assumed that the length of bay (b c in Fig. 755) is not much greater than the side of the polygon. Indeed the bays b and c must receive a greater part of the vault than the pier a of the choir, but thereby they have the advantage that they receive the thrust of the side arch, while it falls at a in the direction of the pier and a resultant is borne on the buttress. For longer bays, especially if square, a strengthening of the abutment is necessary.

For long bays is recommended a strengthening for the wall exposed to bending outward, and if the thickness of the wall is to be made in proportion to the length of bay, the ratio of 1 : 6 up to 1 : 8 may be assumed as an average.

If the given rules are compared, there is found a tolerably great harmony among them. Testing them by comparison with old works or by static calculations, it is recognized that for conditions not too unfavorable they suffice very well. But as their originator expresses with great decision, they must serve only as approximate starting points, and must frequently experience very considerable alterations in special cases. As the best of the given methods must be designated the third of C. W. Hase. If we should also add a fifth rule on the ground of static investigations (see above), that would be the following, allied to Hase's rule.

Rule based on static investigations.

5. With the average rise of the vault A B in Fig. 753a (lying between the base line and the keystone) is constructed a pointed arch (or semicircle) and it is divided in three equal parts according to the method of Fig. 753, to obtain the width of the buttress (M N in Fig. 753). Instead of this may be assumed directly for the simicircle 1/4 of the span, for a low pointed arch (rise about 2/3) 2/9 span and for a higher pointed arch (to 60° or with rise of 5/6) 1/5 to 1/6 of the span. To this is added 5 cm for each m in height of the abutment below the

springing of the vault. The abutments of small vaults under a about 5 m span require a further addition of 20 to 30 cm.

Buttresses determined thus and executed in heavy natural stone can easily support light vaults of like material (for example sandstone compartments 15 to 20 cm thick); with execution in bricks of average weight can they receive compartments of 12 cm of ordinary and not too heavy brcigs, which must be made 3/4 to 1 brick thick for a span of over 8 or 10 m. For vaults of porous bricks or pumice stone the brick abutments c can be made about 5 per cent narrower and for cut stone abutments about 10 to 15 per cent less.

Assuming square bays which without the aid of the wall are supported by buttresses of the usual form (thickness between 1/2 and 1/3 of the total width at base and slight diminutions upward, about an average inclination of 1 in 20). If the bays are rectangles, according as the length of the rectangle is little or more varied from the square, occurs a reduction of the projection of the pier of ; to 15 per cent. If the moderately perforated wall substantially aids, a further reduction of about 10 per cent or more is permissible.

For churches with one or two aisles the application of this rwle is very simple, the span is taken as a basis, to be measured between the faces of the side arches (not in the clear between the projections). For three-aisled churches it depends on the stability of the middle piers and the transfer of the thrust over the side aisles (see Figs. 350 to 355), whether the buttress is determined according to the span of the middle aisle or to the span intermediate between the middle and side aisles. To determine the abutment only according to the side aisle is seldom allowable.

For braced basilicas with not too flat an inclination, light flying buttresses are the prescribed means applied for strengthening the supporting piers of the flying buttresses, if the span of the middle aisle anf also the height of the abutment are considered. Yet for these important piers such rules should be relied on too much, but always the resultant of the thrust should be obtained, when the weight of the pier etc. is combined with the thrust of the flying buttress (in accurate construction being at most equal to the thrust of the vault increased by a part of the wind pressure acting on the other side. See

p. 165 .od also . section of the basilica.

The correct determination of the abutment is indeed to be regarded as the most important question in all theory of mediaeval construction. Errors in this point are very dangerous on both sides; excessive dimensions increase the usually to low estimated cost, insufficient dimensions not only endanger the duration of the building but also human lives.

If one desires to take into account all the many circumstances, such as the rise, form and thickness of vaults, weight of building materials of vaults and abutments, form and relative heights of the latter, special additional loads of vaults and walls, wind etc.; properly in consideration, the best rules c can no longer suffice, it is then either a trained feeling for construction, or where this is left in the lurch, the calculation of the data. Both are indeed not so very different from each other, what is termed feeling is nothing more than experience based on a reasonable judgment of the most important things in question, the static investigations presuppose just the same logical judgment, that is aided in the less clearly visible points by further aids (theoretical considerations).

Just in the constructions here examined it depends far more on correct basal assumptions, and on the more or less accurate execution of the calculations, simplification and approximation of the latter, that makes the final result a few per cent in error, do not injure the building like large errors in the basal assumptions.

It seemed indicated by the importance of the matter to insert an entire section (p. 122 to 170) in this nev edition of this manual on the relatio of the pier and buttress, and the very simply executed determination of their dimensions. How differently the abutment may result according to circumstances will be shown by a glance at Tables II to IV (p. 150 to 152), the data there will even exhibit still greater variations, if the Tables were extended to other cases, for example the common r relation of wall and buttress, the effects of additional loads on the vaults or abutments.

If the dimensions of the abutments of historical examples are collected, this diversity founded in the nature of the matter rudely appears, and aside from exceptional forms, the thickness of solid walls without piers varies between 1/7 to 1/4 (mostly

1/5 to 1/6), the width of the buttresses between 1/7 to 1/2(m (mostly 1/4 to 1/3), and the thickness of the wall between the buttresses between 1/6 to about 1/14 (mostly 1/8 to 1/10) of the clear span of the vault.

2. Churches with two Aisles.

General plan.

The arrangement of the singleAisled church within certain l limits is connected with the span of the vaults. Indeed certain works of the kind are of unusually wide span, like the cathedral of Alby and the Dominican church of ghent, th.t measure about 19 and 16 m between the wall piers, as then would be possible the erection of vaults in structural respects over still greater widths. But the advantages of such a construction are very doubtful. The extremely important increase in height thereby required increases the volume of the interior in a way m making it difficult to fill the interior by vocal or instrumental means, makes the greater cost of erection, and demands enhanced richness of architectural and decoative treatment to overcome the chilly effect of the width. Thus it goes with excessively wide streets and broad squares on which the most pompous buildings can yet produce no effect. The frequently mentioned church of the monastery of Oybin measures 10.8 m in the nave and must represent about the maximum advisable for a single-aisled plan, beyond which the division in several aisles and first into two appears more suitable.

Plans with two aisles exhibit exceptional forms aside from one of the two following systems.

According to one the choir continues the main aisle with the same breadth, which is adjoined by a narrower side aisle at one side; according to the other both aisles are equal and are separated by a middle row of piers, whose axis falls in the extension of the choir.

Side aisle at one side.

The first plan is found exclusively in the churches of the mendicant orders, is especially common in the provinces of Hesse, on the churches of the Franciscans at Fritzlar (Fig. 756), at Treysa, the church of the Carmelites (brothers) at Cassel.

If already limited space may have contributed to the adoption of this arrangement, as in the places mentioned may be indicated by the lack of windows in the wall of the main aisle,

yet in many restorations are seen peculiar advantages for the preaching that occurred therein, that the pulpit received a location on the entirely closed wall and opposite the open aisles, particularly favorable in respect to acoustics.

The closed wall surface will then be animated by the arrangement of the interior, by hall piers connected by arches, whose purpose is to avoid the obstruction of the buttress externally. On the Franciscan church in Fritzlar, whose plan is shown by Fig. 756, between the piers with openings for this purpose is placed a gallery at the north side.

If the added building does not have the entire height of the principal aisle, as would be the case in cloisters, then above the junction of its roof the buttresses were allowed to project externally as on the Minorites' church in Duisburg (Fig. 735), and thereby the effect of the plain surface of the wall became more varied, since to support the projection of the upper projecting wall must again be turned archer in the interior.

Two similar aisles.

When in regard to the side aisle the entire arrangement coincides with the church of three aisles, the second exhibits the plan with the middle row of piers, also the axial connection of the choir with a hall substantially comes back to that shown in Fig. 748, except that the width of the hall is here divided into two aisles.

In the Organ for Christian Art (9 th year, No. 19), the advantages of this plan are emphasized, that even consist in this, that the reduced spans of the vaults facilitate their construction, much smaller heights, require less thickness of walls and buttresses, thereby producing no insignificant economy in cost, while at the same time the middle row of piers, for which only minimum dimensions are necessary produces no difficulty in execution worthy of consideration, but thereby the picturesque effect of the interior is enhanced in itself and by its connection with the vaulting system of the choir, and at the same t time produces an advantageous division of the interior with a middle passage. The arrangement of the latter then allows the pier to extend to the ground free from the seats avoided with difficulty, so that the entire system was established as particularly applicable to ordinary needs.

The ratio of the width of the nave to that of the choir can

vary, so that the width of the choir varies between the widths of one and two aisles.

As different solutions of the added choir may be mentioned the parish church at Paierbach in lower Austria, whose choir is moved sidewise toward the middle, the little Romanesque cemetery church at Schönna in the Tyrol, which has a semicircular apse before each of the two aisles (also see S. Nicolas' church in Soest, the church at Girkhausen, etc.), and the side building of the parish church at Enns (Note), whose choir occupies the entire width of both aisles, but is divided in three parts by four columns set in a square. Moreover the numerous churches with two aisles, that are scattered over nearly all provinces of northwest Europe and Esthonia, offer ever new and varied solutions.

Note. Also see three in Atlas of ecclesiastical monuments in the Austrian capital.

Dimensions of walls and piers.

External wall.

The thickness of the outer walls and buttresses depends only on the thrust of one aisle, and therefore it is to be determined just as for similar churches with one aisle and equal span of vaults as on p. 148 and 273, thus for half the internal width. At most the wind pressure on the greater roof surfaces of churches with two aisles demands an increase in certain cases.

Middle piers.

The dimensions of the middle piers depend on whether these only support the vaults, or also receive a part of the load of the roof. If the variations of loading are neglected, then the thrust of the vaults neutralize each other for equal widths of aisles and piers. Then the piers are only affected by the vertical load resting on them to crush them, whereby only a small section is required, which is easily found by calculation.

For example if on the pier a in Fig. 759 there meet four square vaults of 7 m span, then the surface of the vaults v x y x rests on the pier, that has an area in plan of 7 × 7 m 49 sq. m, and assuming a unit weight of 450 kil per sq m (see Table on p. 13, column Vb), 49 × 450 = 22050 kil load on the pier. If the pier consists of bricks laid on lime mortar with an allowable pressure of 7 kil per sq. cm, then the area of the pier = 22050 divided by 7 = 3150 sq. cm, consequently there would be

required a round pier of 68 cm diameter.

If the pier consists of ordinary limestone or sandstone, for which is allowed 16 kil per sq. cm, then is needed an area of only $\frac{22050}{16}$ = 1378 sq. cm, for which is calculated a diameter of 42 cm. At the lower part of the pier must be added its own weight to the loading, and thereby the pressure is smowhat increased, therefore to the calculated diameters of 68 or 42 cm must be increased according to the height of the pier. Otherwise the pressures assumed in the calculation of 7 kil for bricks and of 16 for cut stone are to be regarded as very moderate for good construction and good materials.

What small dimensions are made possible by the use of still stronger materials are shown by the granite columns of the Brief chapel in Lübeck, and Artushof in Danzig and the limestone columns of the refectory of S. Martin des pres at Paris. Thus the old works exhibit in all their parts the most accurate consideration of all conditions of statics and of the strength of materials. It would have been childish for an architect of those times to make a pier larger than necessary. In many modern works have men sought to produce a certain effect of earnestness, strength and dignity, sometimes even by the use of excessive dimensions of piers, permitting themselves to rise above such considerations by the feeling for free art. In all cases the way is singular and the effect frequently differs from that intended.

With the dimensions found above agree approximately those of the church in Bornhofen (Fig. 758), where the diameter of the pier measures 50 cm for 5.71 m width of aisles. There the distance between piers is certainly less than the width of the aisle, but the piers are still loaded by the framework of the roof. On the contrary in Namedy the width of the aisle amounts to 3.53 m, the distance between piers in the clear is 4.42 m. With reference to the greater load thereby shown, the diameter of the pier is 38 cm.

For very slender proportions it can be recommended to provide by walls on the cross arches a certain transverse stiffening of the external walls (see the statements respecting this on p. 165 to 169). A similar stiffening lengthwise would be executed over the arch separating the aisles.

Walls over cross and dividing arches.

Such extra walls especially come into consideration, if the roof construction partly rests on the middle piers. If the distance between the piers is not greater than the distances between the main trusses of the roof, then the middle post of the roof will stand exactly on the middle of the pier, that is to have a wall above sufficiently high, that the cross and necessarily also the dividing arches may have a certain stiffness. It is then to be considered that upper walls that do not extend to the ridge above steep pointed arches must rise obliquely to the crown ., so that the latter may not be restricted in height.

When with wide spacing of the piers the distances between t the trusses would be too great, then in the middle between each two piers is inserted another truss, whose supports must stand exactly on the crown of the dividing arch, and in no case load the spandrel of the arch unequally. In this case the wall on the dividing arch must naturally be carried above the crowns; that it may not be too heavy, it is made at most 1 1/2 or 2 b bricks thick, and openings may even be left in the proper places.

The middle supports of the roof will transfer a part of the thrust of the wind, which has to be met by strengthening the middle pier or by strengthening the external walls by stiff cross arches and walls (see p. 165). That the upper walls increase the load on the middle pier and the thrust on the external walls must be properly considered.

Junction of choir and aisles.

The greatest difficulty arises for plans with two aisles from the organic connection desired between the choir and the aisles. To continue this division by the aid of the dividing middle s supports to the triumphal arch presents for most cases an unsatisfactory solution. It is found in the little church at Uezküll on the Duna built about 1200 (the oldest Baltic province. Fig. 747).

If this division of the triumphal arch is to be avoided, then it is necessary to divert the cross rib from the last pier from this direction to one, that will find its support on the triumphal arch itself or on its side piers. This purpose can be attained in different ways, the choice of these being determined by the conditions of the plan.

Triangular bay before the triumphal arch.

1. The chevet form is shown by the church at Bornhofen (Fig.

758). Here the dividing arch divides before the eastern pier into two arches a b and a c. Thus are formed three bays before the triumphal arch, one taking the form of a trinagle and two that of a trapezoid. The diversity of the vaulting bays thus produced may lead to a form of the eastern pier differing from the others. For it increases both the number of the ribs meeting on this pier as well as the area of the vaults loading them, which is enclosed by the figure d e f g h in Fig. 758. This increase of the load will first lead to a strengthening of the pier in question, which with regard to the great number of ribs and of their different directions can be obtained by an addition of a round on the eastern side.

Likewise at the points b and c on which at least a dividing arch and two dividing ribs meet, there is either added to the piers of the triumphal arch, or these piers may themselves receive a form suited to receive these different arches.

Since further the plan of the vault at this place corresponds to that shown in Fig. 741, a strengthened triumphal arch is unnecessary, and therefore in Bornhofen it is replaced by a cross rib.

The ribs meet against the triumphal arch.

2. In Fig. 759 the bays are square and the widths of the aisles equals that of the choir, so that the rib a b from its crown b can be divided into two b c and b d extending down to the piers of the triumphal arch. The diagonal rib e f would then continue from f to the intersection with the rib b c at g on the same arch, but from the crown e of the triumphal arch must rise in an arch like the h f g i and form over e h a side arch, so that the system of ribs will assume about the form shown in Fig. 759a in perspective. Thus the junction at one side for the ribs g e and k e with the triumphal arch makes it necessary to strengthen it.

3. Allied to that just mentioned is the arrangement in the church of Namedy as shown by the plan in Fig. 760. But variations from it result by the ratio of the width of the chapel to that of the nave and consist in this, that the ribs b c and b d into which divide the dividing arch a b do not reach the piers of the triumphal arch, but adjoin its sides at a height determined by the line of the arch as shown in the cross section of Fig. 760a, so that over c e and d f the side arches over c e

and d f assume an arch line rising from the point f to d, which in a way results from the intersection of the swelled line of the compartment with the surface of the wall, which gives the entire arrangement almost the stamp of an expedient.

Great influence have these different forms of plan on the elevation. For while in Fig. 759 the equal spans of the arches in the choir and nave prescribe equal heights for them, the unequal spans in Namedy produce a greater height of the arch in the choir, hence with an equal height of the crowns is a lower one of the springing line (Fig. 760a). According to the arrangement of Bornhofen on the other hand, the continuity of the system of vaults leads to an equal height of springing lines in choir and nave, therefore either to a drpressed form of the arch in the choir as in Fig. 758a, or to a greater height of its crown.

Junction with the western wall.

At the west the dividingarch in the simplest case rests on the gable wall, indeed either as in Figs. 758 and 760 on a corbel placed over the middle doorway or on a round extending to the ground. The last arrangement would then lead to two doorways, which are then either placed at the middle of the aisles, or could be moved nearer to the middle pier, while the former either shows a continuous strengthening of the western wall w with regard to the thrust of the dividing arch or a buttress is placed on the arch over the doorway, and therefore requires a corresponding projection of the jambs of the doorway from t the gable wall as shown in Fig. 761.

But further the system of vaults might also join the western wall in the same way as at the choir and with a triangular bay, and this arrangement could be connected in a suitable way with the western tower. The same system then leads in its application of the longer sides to the resolution of the entire area of the plan into triangular vaulting bays (Fig. 762).

The execution of the vault in the areas a b c d remaining at the angles etc., allows the most varied arrangements, the nearest of which consists in this, that the sides a b and b c inclose the vault by two half side arches, or the triangle a b c is separated by the rib a c from the triangle a c d, and each of these triangles receives its separate system of ribs.

Likewise is shown by Fig. 764, from a to c a cross arch can

be turned below the springing of the rib instead of a rib, on which then rests a wall extending in the same direction, so that on the exterior from the right corner b is formed a transition to the obliquely placed side a c. In all these cases c could be omitted the buttress at b. Yet this would be necessary when the bay a b c d is vaulted as a square, and the diagonal rib is turned from d to b.

Further differences would result according to the system of ribs (p. 28), or by the arrangement of a net vault.

Thus further the arrangement of triangular bays could be limited to the eastern and western bays and otherwise be connected with the rectangular bay (Fig. 763).

Halls of secular buildings.

It is in the nature of the matter, that the arrangements shown may be applied in entirely the same manner to the erection of the different halls serving for secular purposes, indeed ✱ that by correct understanding of the manifold needs to which they must correspond, there must even result many different f forms. Magnificent examples of this kind are found in many places. Here belong the great refectory of the castle at Marienburg, the hall of the Artushof at Danzig, and the new school in Prague, a great number of the most diverse monastery interiors in Haina, Maulbronn, the refectory of S. Martin des pres at P Paris, as well as the existing and numerous hospitals in France, so many examples of which are given by Verdier's work, so frequently mentioned. Not all rooms given have two aisles, but the structural advantages before represented are also based only on equal spans of the different aisles, so that they make possible small dimensions of piers also in the plans of hall churches with approximately equal widths of aisles, like the Wiesen church in Soest, and the nave of Erfurt cathedral.

For a moderate size of interiors results the plan of a middle pier, which again in regard to the form of the pier, as well as for the plan of the system of vaults, is capable of an endless variety and may be adapted to the most diverse proportions of rooms, both retaining the simple cross vault, whose bays assume a form corresponding to the shape of the room; as with a and richer system of ribs. For the latter mode of treatment are given sufficient starting points in what has just been said of churches with two aisles, according to which the arrangements

may easily be made in irregular interiors.

As an exampleof a particularly graceful form of this kind we give also in Figs. 765 aod 766 the plan and cross section of the chapter hall of the Monastery of Eberbach-on-Rhine, whose style is indeed far from the nobler forms of early Gothic, but the rich and bold effect of the 16 ribs rising from the middle pier is made apparent. As Fig. 766a shows, those of the latter which enclose the three triangular bays are larger and are profiled differently from the diagonal ribs subdividing them.

3. Plans of Churches with three or more Aisles.

General proportions of the different parts.

Churches with three aisles first differ in whether a transverse aisle is arranged, or whether the side aisles accompany the middle aisle to the beginning of the choir, and further according to the proportions of the widths of the aisles to each other and to the length of bays.

Square bays.

As already represented on p. 8, the difficulty of vaulting rectangular bays in the Romanesque period led unwillingly to the construction of uniform division in bays according to the mode of Fig. 767, II, but rather instead of these to make for each square middle bay two square side bays of half size as in the mode of Fig. 767, I, When the vaulting of the rectangle offered no further mystery, the uniform division of bays came to the front, although a particular charm was not wanting to the square division with intermediate piers retained in many early Gothic works. The proportion of the width of the middle aisle to those of the side aisles then is 2 to 1, yet small variations occur according to the locations of piers and the faces of the wall, as indicated in Fig. 768.

In the left half are assumed the intersections of the different axes as centres of the wall piers found according to the shape of the detached piers, so that the part which on the former determines the width of the dividing arch, on the latter gives that of the window wall, while the part of the body of the pier remaining within the window wall forms the rounds g h i. In the right half of the same Fig. on the other hand is placed at k the round of the dividing rib,and the direction of the sama as axes of the diagonal ribs m n in the side aisle is assumed, by which the wall is set farther toward the exterior.

At o is then placed a larger round for the dividing arch o p, and at t, r, etc. are smaller ones for the cross ribs, so that the width of the side aisle between the rounds is greater than on the other side. Still more would this be enlarged, if in t the intersections of the axes in the side aisles are placed only single larger rounds, or even if the members were only corbellod out above. From this it follows first, that the proportions in widths just given as fundamental for the entire construction are a basis without being too absolute.

However the basis of a geometrical system, like the squares arranged beside each other in Fig. 768, is not to be taken too abuolutely. Thus the first variation from it results in the same Fig. by increasing the crossing piers and the dividing arches enclosing the middle square beyond the cboss ribs, while the clear distances between piers remain equal.

Rectangular bays.

When the equal division in bays was continued by the use of oblong bays in the aisles, the proportions of the three distances between piers, widths of side and middle aisles, could be varied in manifold ways, by which either in the middle or side aisles or in both resulted rectangular bays (Fig. 767, II, III, IV).

In some early works is even expressed the proportion 1 : 1 : 2, resulting from the division in squares. With almost complete accuracy is found this in the church of S. Elisabeth at Marburg. In the church at Haina the proportion of the distance between piers to the widths of side aisles from axes of piers to the face of the wall measures 1 : 1, while the doubled distance goes from e to u in Fig. 768. In the church at Frankenberg is the proportion 15:16:29, in that at Wetter is 1:1 to the face of the wall, but where it is to be noted, that in the plan the inner buttress is set farther outward; the same distance then extends from the middle of the pier to that of the bay. In S. L Laurentius at Alnweiler the proportion of the distance between centres of piers to the distance to the external face of the wall is 1:1, and the width of the aisles between axes is determined by the diagonal of the square described thereby. In the church at Friedberg the proportion of the distance between piers to that of the side aisle measured to the middle of the wall is 1:1, and the diagonal of this width extends from c to

u, referring to Fig. 768.

In the minster at Freiburg the diagonal of the distance between piers extends to the middle of the thickness of the walll and the diagonal of one bay of the side aisle measured between the middle of the piers gives the width of the nave. A similar ratio is also containeg by the cathedral of Regensburg. In the cathedral of Rouen the three distances are approximately as 5.5:11. 5.In1all works mentioned are employed the most varied relations of the three distances, but there is a more or less decided predominance of the width of the middle aisle, which in Rouen is more than twice the width of the side aisle, while in others occurs an increasing equality of the three measures.

In the Kreutz church at Breslau, and Wiesen church at Soest, the church at Volksmarsen, the church at Immenhausen, the proportion of the distance between piers to the width of the middle aisle is approximately 1:1, the square bays are produced, whilh the widths of the side aisles increase from half this length to the diagonal of the half. In the choir of the church of S. Sebaldus at Nuremburg all three distances approximate e each other, while in the nave of the cathedral at Erfurt the widths of the side aisles predominate, which then approximate the distance between piers and the decreasing width of the middle aisle. Accordingly even all possible profortions are found, The choice of them depends on the system of construction adopted and aids in determining the character of the work.

Choir and middle aisle.

The arrangement of the choir dominates the whole in the simplest as well as the most complex arrangements. First the width of the choir equals that of the middle aisle. Yet this equality can already be varied, according as the inner face of t the wall of the choir gives the axis of the piers of the aisle, like the left half in Fig. 759, or the dividing arch forms the direct continuation of the thickness of the wall or of the window arches as shown by the other half of the same Fig. The last arrangement becomes necessary for the arrangement of an elevated middle aisle. Decided variations in width of choir from t that of the middle aisle are found to be caused by special conditions as in Erfurt, while the aisle joining the choir structure at the west is joined by towers dating from an earlier t time, so that the space remaining between them, which corresponds

to about half the choir gives the measure of the middle aisle.

Transverse aisle.

The width of the transepts according to the nearest scheme equals that of the middle aisle. Meanwhile are also found numerous exceptions from this. Thus theeir width in Erfurt is determined by that of the side aisles. In the cathedral at Regensburg this remains about half the width of the piers less than that of the middle aisle. Thus the cathedral of Rheims has a width of the three aisles of the transepts that equals that of the three aisles of the nave, but the side aisles of the former are wider and therefore the middle aisle is narrower, while in Chartres the side aisles of the transepts are narrower than t those of the nave. All such variation from the normal arrangement find their explanation partly in local conditions of the entire space, partly in the system of the entire plan. But they are nowise isolated in the works mentioned, but require substantially different dimensions. Thus for example in Chartres t the widths of the side aisles of the nave gives the distances between piers in the transepts. In the church at Friedberg the width of the transepts equals the distance between piers, so that these are only expressed by the greater depth of the vaulting bays adjoining the middle square.

Number of bays.

The number of vault bays depends on the relative length of the entire church, the length taken by the choir, and the proportions that the bay must have to the system assumed.

Generally it is in the principle of Gothic church architecture, that the length shall dominate the width, and that the greater length is produced rather by the number of bays than by enlarging them.

Distance between piers.

In Catholic churches by the close placing of the piers is a more expressed separation of the aisles for the needs of the worship, more favorable for the services at the same time at different altars. Such must be the case less in Protestant churches, whose direct need requires a wide area connected with a choir. Yet one should also take care there to consider the visibility of the altar from all seats like the galleries in the theatre. The arrangement of piers is a structural necessity in every wide interior, particularly if vaulted that can be av-

avoided only by the greatest sacrifices in material respects, as by excessive height, if the whole is not to fall into an impossible flatness. Therefore one may go so far that the design of churches with a single aisle, as reason permits, or prefer wider spacing and piers with several aisles. Executed works show that the widest play is afforded here; in merely obtaining a wide and unbroken interior, as opposed to the present Catholic form of architecture, the endeavor is to be made that this form of building may also be an esthetic and an ecclesiastical one.

Eastern termination of the side aisles.

This will be simplest by the transverse aisle, or when such is wanting, by straight walls flush with the triumphal arch, which then may be opened by windows.

Rectangular side choirs.

If such side altars are arranged, these find their most appropriate place at these eastern walls, and then will be separated by the altar steps from the area allotted to the congregation, and therefore in a sense form a projecting side choir. It is accordingly next to extend the ending of the side aisles beyond the triumphal arch by one or two bays eastward, thus f first arranging a rectangular side choir, that is separated f from the high choir by walls between the piers a b in Fig. 770.

This termination is either complete, when the walls extend up to the cross arch a b, or they reach only a certain height corresponding a bout to the height of the window sill, above which the arched openings a b either remain open or can be divided by mullions and tracery. The church of S. Blasien in Mühlhausen, whose plan of the choir anf transverse aisle represented in Fig. 770 shows such a treatment, where the wall a b has the height of about 12 ft. The connection with the high choir will be more complete, if the arched openings extend to the f floor as in S. Stephen's in Vienna. Meanwhile to indicate at least a separation, an arrangement may be found here like that where the first bays of the cathedral of Meaux are separated by the choir aisle, consisting in that the pier under the proper dividing arches are connected by cross arches, on which rests the arched tracery forming the upper opening. Fig. 771 exhibits the system of this arrangement in perspective. The arrangement of the plan of Fig. 770 could then exist just as

well without the transverse aisle, and then the bays eastward from the transverse aisle would represent only a continuation of the side aisles, just as here they intersect the transverse aisle.

Round or polygonal side choir.

Meanwhile Romanesque art had already adopted the plan of semicircular side apses at the places concerned, which then passed into the polygonal form in Gothic art and allowed the most varied treatment. In the simplest cases these are five sides of the octagon or extended by a rectangular bay, and adjoin t the walls of the high choir.

Between the eastern ending of the side choirs and the nearest buttress of the choir resulted an interval, that remained ofen, or if size permitted, it could serve for the arrangement of a stair tower (left half of Fig. 772). But with small dimensions of the whole this space was too small, even inaccessible itself and it is then better either to wall up the angle or extend t the east wall of the side to the face of the choir buttress, or rather to form an extension of it, whereby according to the proportions of the whole the sides of the polygon falling in the length may be longer than the others, just as such a form of the polygon itself on the high choir is found, as on the cathedral of Regensburg.

The triangle a d b in Fig. 773 may also remain open and thereby by means of a rib turned from a to b, or according to the resulting shape it forms a partial rectangle with the side e f of the octagon. The last arrangement however makes difficult the plan of an eastern window, whose arch will either be excentric against the side arch or must assume the entire width f d. On the intersection of the walls dividing the choir is true what was said above, only the arches concerned and according to the scale of the side ab of the polygon or of the corresponding rib are to be limited to the span b h, or are to be turned so low that the rib a b can pass over them. Fig. 773a.

Another form of the side choir results, if instead of the s side h i in Fig. 773 the diagonal d i of the bay concerned becomes the base of the polygon, so that the sides of the polygon project beyond the face of the side aisles or rather beyond the point i; right half of Fig. 772. Such an arrangement is found on the church of S. Katherine at Oppenheim so that it

rests on the diagonal of the half hexagon. The same arrangement can be formed with any polygon, and according to its choice t the east window can be placed in the axis of the side aisle. Yet is to be mentioned the plan of a side choir for a high choir, as such is found on the Wiesen church at Soest (Fig. 774). Here the choir formed of 7 sides of the decagon has two side choirs formed of 5 sides of the decagon.

Side choirs with transepts.

The need of determining the width of the side choirs by that of the side aisles is less compulsory if a transept is arranged, whose vaulting plan either has no connection with that of the side aisle, or consists of a greater number of bays. In t the first case the side choir could be placed at the middle of the vault of the transept, in the last case being arranged on a bay of it, so that between the side and high choirs remains an open space.

With greater length of the transept there are sometimes found several side choirs at its eastern wall, which may be formed as polygonal as in Frankfort,or rectangular as in the cathedral at Erfurt. Those last mentioned occupy the entire length of the northern transept, and are so divided that on each bay meet two separated by a pier, and which with less depth are covered by cross vaults, but remain without windows. The arrangement of rectangular side choirs side by side is especially peculiar to Cistercian churches.

Church with three aisles and without transepts.

In Fig. 772 is given the plan of a simple three-aisled church, whose nave for sake of simplicity is divided in but two bays. The choir consists of 5 sides of the octagon and is extended by a rectangular bay at whose sides lie side apses.

The side choir at the left is likewise drawn with 5 sides of the octagon, the angle between it and the nearest buttress is utilized by the plan of a winding stairs, which requires so much space, that its clear circular opening possibly at no place cuts beyond the middle line of the thickness of the wall and also remains a little distant from the middle of the buttress.

At the right side is shown an obliquely placed choir polygon, whose base is placed in the direction u w, and not as would h have seemed more natural in the direction s v, for in the last case the buttress would have been weakened too much bn the first

side of the polygon.

As the left side of Fig. shows, a sacristy can be added to the side choir as a low structure, or have any other place on the choir.

The bays of the aisles are assumed to be rectangles in the middle of squares in the side aisles. The round pier with 4 rounds is made so large in this case, that its plan is circumscribed by a square formed by the width of the dividing arch. (The members of the vault develop from it as previously mentioned on Fig. 427). Over the last piers of the choir intersect the dividing and triumphal arches, and these piers can be strengthened accordingly, or receive 4 rounds as in the Fig. For the development of the members of the vault from the choir piers, Figs 772a and 772b give two solutions.

For completion, xx the middle tower is attached to the western end, whose breadth extends to the outsides of the dividing arches, the thickness of its walls may equal a fourth of its width. The external square of the tower projects from the face of the western wall according to an arrangement not rare, so that a cross arch extends from x to y, with a width equal to the w wall of the church. The reason for this plan is generally more on the forms of the towers will be given below.

According as the interior of the tower serves as a vestibule or an extension nf the middle aisle, the entrance doorway lies in the eastern or western wall of the tower.

Any possible side entrances can be in the axes of the side aisles in the western wall, or at about the middle on the north or south side, or lie at both places according to the size of the whole. A careful observance of symmehry, so that for a side entrance at one side a similar one must be opposite at the other, is here least appropriate. The location of the doorways is arranged according to the course of those entering, and ends of the streets, and therefore they are omitted or made smaller on such sides from which few or no persons are expected.

Dimensions of walls of choirs.

The thickness of the walls depends on the nature of the vaults, the heights, and generally on the entire development of t the cross section; see above the more definite statements alone under (Abutments. If one desires to have a rule of thumb, it may be assumed, that the dividing arches in basilicas of aver-

average height as well as the clearstory walls borne by them are bout as thick as the walls of a single-aisled church of l like span, and further that the external walls of the side aisles according to the width of the latter, are detemmined rather by the span if the vaults of the middle aisle, and also a about equal the dividing arches. To make a general statement concerning the buttresses is always somewhat risky with the very varied development of cross sections of churches. (See the rules of construction of single-aisled churches on p. 273 and the widths of buttresses on p. 122 etc.). The same are true for the dimensions of crossing and middle piers.

The sides of the pier endings have the same thickness as the walls of the aisles, but much thinner walls can be given to t the low side apses. Yet if here attention also is paid to the support of the roof framework or the formation of the window jambsto make the walls very thick, at least the buttresses can be much reduced or be entirely omitted as on the Wiesen church at Soest.

3. Arrangement of plans of churches with three or more aisles.

Five-aisled churches.

The enrichment of plans of choirs by aisles and chevets (p. 298) leads quite early in the great cathedrals to five aisles in the eastern half. If these are all extended beyond the transepts to the western end, thus originates the proper five-aisled church. But there is also found a five-aisled nave connected with the arrangement of a single choir, as on the S. Maria church at Mühlhausen and Se Severi at Erfurt.

Concerning the development in height and the connected function of the heights of piers separating the side aisles may be distinguished three systems.

1. The side aisles have equal heights and the row of piers supports only the vaults and also a part of the roof load. This case occurs when with a high middle aisle the flying buttresses extend over the side aisles in a single arch, as on the cathedral of Paris and the minster of Ulm. The importance of the intermediate pier corresponds to those of churches with two aisles ((see them), if the spans of the aisles are equal so that the thrusts are neutralized, and then the dimensions of the pier can be made quite small, being dependent only on the vertical loading.

Thus in S. Severi's church at Erfurt the ratio of the size of this intermediate pier to the width of the side aisle between axes of piers = 1:10. In S. Maria at Mühlhausen they are greater, but still the piers are substantially weaker than the main piers, about in the proportion of the side to the diagonal, and moreover it appears to have originally differed from the present one, and these intermediate piers seem to have been i intended or built for the heavy roof loading. A particularly small proportion of dimensions is stoll exhibited by the row of piers later added to the side aisles of the minster of Ulm.

2. the side aisles likewise have equal heights, but the piers have additional loads, since the buttresses are in double spans from the clearstory walls to the intermediate piers on that f row of piers and thence are turned farther to the external buttresses.

3. There is found a reduction in height from the middle of the adjacent side aisle and although fromthat to the outer s side aisle. Like that over the middle dividing arches, there rises over those between the side aisles an externally visible wall, that at the height of the junction of the roof has a triforium and over this a clearstory. The thrust of the vaults of the inner side aisle is conducted by special flying buttresses to the buttresses standing on the outer walls.

The plan with side aisles of equal heights is the usual one and is found in Cologne, Paris, and in plans at Amiens, Chartres and Rheims, where the parallel elongation becomes five-aisled by the system of chapels. The graduated or stepped heights are carried out in the cathedral at Bourges, and is applied to the proportions of the choir chapels and aisle in Beauvais and S. Quentin, but in a more prosaic form on the cathedral of Milan and other Italian works.

It would be useless to undertake an accurate comparison of the effect of side aisles of equal and of stepped heights. Both are properly developed from correct principles, and if the effect of the latter is more surprising, especially of the outer triforium and the windows of the middle aisle are seen through the dividing arches, they afford a particularly rich and varied view, when this arrangement appears as the proper result of an elevated middle aisle, on the other hand by the equally high side aisles is formed a combination of the basilica and the h

hall church, and adopts the advantages of the latter an the m more free and airy form, which the vaults of the side aisles acquire.

The separate parts of the five-aisled plan are so nearly allied to those of the three-aisled church, that then have a common treatment with those in the Chapters on choir endings, pl plans of towers etc.

Polygonal plans of aisles.

In the Early Christian and Byzantine periods of art there c came into the most magnificent use, as shown in the minster at Aix-la-Chapelle, S. Vitale and many other works in the East a and West, the central form for the church proper as already almost dropped in the Romanesque period with the exception of s some Italian churches, until at the boundary of the beginning Gothic a new and more developed system of construction finds a corresponding expression in the exceptional forms of S. Gereon in Cologne.

It is striking that with the further development of Gothic art it disappeared, with the exception of little chapels, and this is chiefly to be explained, by that the actual proportion between choir and nave was reversed in a manner, and a predominant importance was assigned to the latter. Thus such a concentric ground form requires the accenting of the centre, and produces that already by the great effect of the ribs rising to the middle keystone, but this even disturbs the united effect of the principal object and thus splits the line of vision. This effect is to be compared to that of a form where the principal group is contrasted with a side group of equal importance, or to a perspective at the same angle on two streets of equal importance. But to make the direction toward a centre diminant, were it possible as in a circus to arrange a radial treatment or to assume the believers to be in a concentric movement. This disadvantage could indeed be obviated by the arrangement of t the choir, but the conflict no less exists in the nature of the whole. Then the arrangement of each important need for room requires besides colossal dimensions particularly in height, or the provision of a concentric aisle; therefore it could be transferred from this form with advantage to the cross plan with side aisles or to any rectangular form. Fig. 775 exhibits such a transformation of a polygonal church into a cross church with

equal area and length.

But in smaller dimensions the polygonal form must yet have certain special advantages, that may be comprised in these, t that it permits an organic development of a greater width of aisle from the width of the choir, and so produces a certain relation between both, that meanwhile is also formed by the a arrangement of the vaults of a rectangular ground form of the aisle.

The polygonal nave may be spanned either by one vault, or it may be divided according to the number of sides into a certain number of triangular bays that proceed from a middle pier. In the last case the arrangement comes to that shown in Fig. 765., but in the first to the ordinary plan of the choir. Generally with the adoption of an internal row of piers may be easily developed other varied combinations. However all such plans w will demand an imposing height, but better an elevation of the middle space, in order to present externally a pleasing appearance.

Just as now the centralization is removed by the choir plan, so can this with advantage also be done by a western front building, that may be given to a vestibule, or is added to the internal area. But it must be termed as reversed, when for the p plan similar on both sides, projecting structures are placed on the sides, that may contain a sacristy or other subordinate room, since for these rooms is required an unsuitable similarity to the choir, as well as on the other hand an opening of these rooms to the inner area entirely removes the character of the polygon and leads to the cross forms.

Better than for an actual church is the polygonal form suited to the plans of all such chapels in which the separation between nave and choir does not occur. Here belong all baptisteries, chapels for the dead, grave chapels, etc., that also especially recur as baptisteries in the periods of mediaeval art.

4. Transepts of Churches with several Aisles.

Transepts with one aisle.

Plan of the transepts.

The bays lying next both sides of the middle square and forming the transepts either remain in the line of the side aisles or project beyond it.

The first arrangement is found in the cathedral of Regensburg (Fig. 776), in the cathedral and in S. Severin at Erfurt, andit

essentially lessens the effect of the cross plan, particularly with equal heights of the aisles.

Projecting transepts may either form square bays (Figs. 777, 778), as in the cathedral at Magdeburg or Halberstadt, or may consist of several oblong bays arranged beside each other (Fig. 779). The ending may either be in the usual way with a straight gable wall as at the west end, or it may be made like that of the choir b: a semicircle or polygon, as on S. Elisabeth in Marburg (Fig. 780), the church at Frankenberg, the Kreus church at Breslau, the cathedrals of Noyon and of Soissons.

Form of the crossing piers.

What then concerns the form of the crossing piers, it is nearest as already frequently stated and finally exhibited in the analagous shape of the pier supporting the triumphal arch in Fig. 772b, to develop its form according to the arches resting thereon, also increasing it above the dimensions of the aisle piers. According to the proportions of the cross section may this strengthening be more or less, and in some circumstances it can be entirely omitted. The similarity of the aisle and c crossing piers thus produced is found on many and indeed important works with equal heights of aisles, like the church at W Wetter, S. Maria at Mühlhausen and S. Blasien there, yet here with the modification that the crossing pier has eight and the aisle piers have four rounds. The causes of this similarity, which at the first glance is somewhat surprising, are the following.

The aisle piers must have sufficient dimensions to resist t the excess of thrust of the bays of wider span; Fig. 781 shows part of the church at Wetter; A is the crossing pier, B the a aisle pier, whose dimension must therefore resist the excess of thrust of the vault of the middle aisle acting in the direction B b. Since then the thrust acting in the directions A c and A d on account of the greater area of the middle square are unequally greater than those acting in the contrary directions, then the pier A must be shoved sidewise if not aided by the form of the cross section. Figs. 781a and 781b show the cross section according to B b and A B or B d.

If the dimension of the pier in the first Fig. is made too small, then a fracture occurs at a in the arch of the side aisle and the crown b would rise, so that its fall would follow.

But if in the direction of the cross section in Fig. 781 the same effect results pfrom the excess of thrust the arch of wider span, then the breaking of the dividing arch or the rise of the crown would be prevented by the wall built on it, whose weight is increased by the weight of the roof construction, a and hence even here would be ensured the stability of the pier.

In other words the overload on the crossing pier causes, that the line of support in it becomes more nearly vertical below, but it is to be considered, that the overload must be laid more on the narrow than on the wide arches. (See the effect of a w wall on arches in Example II on p. 157). Thus it is indeed possible, that the much more strongly loaded crossing pier is equally capable of resistance as the aisle pier of the same dimensions. Naturally the pressure on the material in the resisting pier must not exceed the allowable amount.

Corner pier with cross arch io transept.

When the transepts project beyond the outside of the side a aisles, as in Fig. 782, there results a peculiar condition for the corner pier of the side aisle opposite the crossing pier at A. For this could then only be formed by a portion of the crossing pier, when connected with the opposite pier by an arch similar to the dividing arch. But since for the plan of the latter all grounds are wanting, it is a simple cross rib, and hence under it is only necessary a weaker round d instead of the corresponding portion of the pier. Therefore result various relations represented in Figs. 783 and 785.

In Fig. 785 we have assumed a subdivision of the entire pier according to the arches resting on it with rounds only at the rectangular angles. There is assumed a form of the dividing a arch with two concentric rings, so that the aisle pier a consists of 12 rounds and the crossing pier formed from it has 16. For the opposite corner pier we retain provisionally the same form, and likewise for the wall pier of the side aisle that of the aisle pier a, so that the proper wall is omitted and the width of the dividing arches determines that of the window wall. Accordingly on the corner pier c the two rounds d and e remain without utility. Consequently they must be omitted, while with unchanged positions of the rounds f and g the round for the cross rib is set back at i, so that the form of the wall pier f i g differs from the other subdivision of the piers. In this

way it is possible to place the cross arch h in the middle before the longitudinal axis of the wall.

In the church at Wetter appears the transformation of the c corresponding wall pier d made in a somewhat violent manner in Fig. 781. The ground form is that of the aisle pier and comes to its full development according to the side aisles, where it was to be employed. But according to the transverse aisle the superfluous arc is cut off abruptly by a continuation of the face of the wall, and only one round is placed to receive the cross and diagonal arches.

To a peculiarly ingenious arrangement has this irregularity of the arches led in the church of S. Ouen in Rouen, indeed i in the bay adjoining the middle square, since the transepts a are accompanied by side aisles. For two diagonal ribs extend from the corner piers of this bay, so that the entire bay is divided in five parts, and thus all space between the rounds of these ribs forms a plain surface. Fig. 784 illustrates only the principle of this arrangement without claiming any accuracy, since it was executed from a hasty sketch without measuring. The same principle applied to the location of the rounds in F Fig. 783 would require for both the hexapartite subdivision, or the pentapartite division for the bay of the transept adjoining the middle square, if the side aisles continue beyond the transept.

Most simply is solved the corner pier, if one avoids allowing the cross arch to fall in the extension of the middle of the wall. Then the necessary rounds are placed beside each other, by which the cross arches are moved nearer the middle aisle.

If the window jambs accordingly directly join the rounds, t then they would externally be nearer the corner on the transept, than the windows in the side aisle.

From the various cases mentioned above it appears how imperious is every change in the condition of the arches that makes itself felt, and its effect is only transferred tnd not removed. Thus this is shown in the changed form of the transept rounds in Fig. 783, and in Fig. 784 in the plan of the vault and in Fig. 785 the width of the window pier. Indeed the last irregularity would be scarcely noticed, the window were narrower, b but it essentially exists. Yet it must be noted, that in the fewest cases the windows extendong over the entire surface if

the wall remain open in their full width in the angles of the crossing, but as a rule are rather closed by buttresses or stair towers as on the cathedral of Cologne. In the before mentioned church of S. Ouen is found in this angle a buttress set diagonally. With the exception of stairway towers all such arrangements are only intended to lighten the arrangement of the flying buttresses, as will be shown below. If the thickness of the upper wall is increased by an arch turned between the buttresses over the window arch, it is next to allow the projection of the part k to receive it in the angle of the transept.

Corner pier without cross arch in the transverse aisle.

When the transepts have vault bays projecting sidewise, like those of square form as in Fig. 778, generally no cross arch rests on the corner pier a. With equal height of the aisles, then to limit in height the side arch, excentric from the dividing arch, as Fig. 770a shows, but it produces no tasteful e effect. By a bisecting rib (Fig. 777) the effect can be substantially improved, and one is inclined to seek in this place the principal impulse to the hexapartite vault. The arches require to be inserted only on one side (Fig. 778, right half). Any windows in the side walls of the transverse aisle would also naturally be excentric.

With the arrangement of the lower side aisles the excentric condition of the lower dividing arches will be less disturbing, and also the windows to be inserted above could again be moved to the middle.

Transepts with side aisles.

There result certain special conditions for the form of plan of the crossing piers, as well as of the wall piers found in the angles of the crossing.

Plan of the angle piers.

Fig. 786 exhibits the scheme of such a part of the crossing in which the last named piers a and b shown in pig. 786a receive form from the number and size of the arches resting thereon. Moreover the depth of the window jamb and of the external arches can also be increased externally in regard to the development of the elevation.

With aisles of equal height the window arches would have to exert a counter thrust against the cross and diagonal arches a and b (Fig. 786a) of the vault, therefore their position and

form are to be arranged accordingly.

That the arrangement of a raised middle aisle be required in a buttress over this corner pier to receive the two flying buttresses turned against the upper pier of the middle aisle, whose middle line meets that of the cross rib a. The simplest means for strengthening is the arrangement shown in Fig. 786a, where the windows are set back farther or were closed almost to the middle by two buttresses c d, d e standing in the angle. In this case the gutters or roof galleries can be carried above the side aisles around a corbel before the buttress.

Further the thrusts of the two flying buttresses meeting at a right angle would be opposed by a buttress set diagonally, by which the windows are again open. The buttress then has the form dotted in Fig. 786a and the roof gallery continues through it.

All the arrangements of the buttersses could be avoided and the thrusts of the flying buttresses from these angle piers be transferred by a second arch to the nearest buttresses m and n in Fig. 786, so that the latter could be met sidewise by the second flying buttress mentioned, and the buttress system intersect at the angle pier a, whereby then the thrust of the nearest window arches increases the resistance of the buttress struck sidewise by a piece of wall remaining beside the window. Such an arrangement is found in the church of S. Ouen in Rouen.

Further difficulties result for the plans of the augle piers enclosing the middle square from the required arrangement of side arches with a raised middle aisle.

Crossing piers with cross arches and without side arches.

Strictly taken the side arches would only be required by the junction of vault compartments to closed wall surfaces, in order to obtain the curved line in which the joining occurs. The latter is produced by the junction of the compartments with cross or dividing arches by the external lines of these. Therefore are they necessary on raised middle and transverse aisles, but not on the side aisles and just as little on the cross arches enclosing the middle square, unless the intended plan of a central tower makes necessary a strengthening of the latter.

Accordingly the crossing piers will be constructed so that a round stands under each arch and three are beneath each dividing arch, and hence results for these the form seen in Fig. 786b, where G G is the cross arch enclosing the middle square

and S S denotes the dividing arch. The sides a b and c d of t the rectangle described around the pier will be greater than the others and have one more round for the dividing arch.

As shown by the Fig. this form brings the disadvantage with itself, that the middle lines of the dividing arch and the cross arches enclosing the middle square separate, as well as a second, that the rounds e and the side arches have the same d dimensions as those of the diagonal ribs f. These disadvantages must lead to retaining the concentric form.

Thus if we reduce the number of rounds on the sides a b and c d by one each, there results the following arrangement.

First the rounds of the side arches can be taken from the d dimensions of the dividing arches, i.e., can rest on the latter. Then if the upper window of the middle aisle fills the entire side of the bay, then can its arches be at the same time side arches (see a in Fig. 846), and also be strengthened by an addition b cut from the edge of the compartment, that takes its development from the compartment. In any case is thereby set back the upper window wall in a manner injurious to the development of the elevation (as will be shown later). It is therefore preferable to allow the rounds of the side arches to project from the face of the dividing arches, and this can be done in two ways. Either these stand on the capitals of the rounds of the diagonal arches enlarged for this purpose, or they are corbelled farther below from the capitals under the dividing arches.

According to what is said here about the form of the regularly subdivided pier will this case be readily solved by the adoption of any different ground form.

Crossing piers for cross and side arches.

If the cross arch es enclosing the middle square are to have s side arches, they require five shafts (Fig. 786c). Compared to Fig. 786b, on the sides a b and c d the number of shafts must be increased by one each, and on a c and b d by two each. d Accordingly the dividing arches must have the form given in Fig. 423 from the minsters of Strasburg and of Freiburg, i.e. of three arches next the side aisle and of two next the middle aisle, unless two are entirely superfluous in the side aisles, when no actual rounds are added, which then stand only under the compartment.

Meanwhile also retaining the usual similar form of dividing arch on both sides, there is constructed a form of crossing pier perfectly corresponding to the parts above it, if the superfluous rounds in the side aisle are replaced by a rectangular increase of the body of the pier, in the angle of which t the round of the diagonal rib finds its place. Fig. 786d shows this last arrangement occurring in the cathedrals of Soissons and Chartres, by which the piers receive a ground form corresponding to the angle of the central tower above as well as to the face of the compartment, and a very useful strengthening.

We have therefore believed more readily that the solving of these conditions must be more accurately developed, than this has not been done everywhere in a happy manner, as in the crossing piers of the cathedral of Rheims the arrangement of such superfluous shafts is not found to be avoided.

5. Plan of the Choir of Churches with more Aisles.

Addition of several side choirs.

The ground form of a simply formed principal choir has been described already for the single-aisled church (p. 259 etc.), and likewise the addition of side pier in an eastern or diagonal direction has found an explanation on p. 286 (Figs. 772 to 774). This treats rather the addition at each side of the main choir more as a side chuir, so that a simple arrangement side by side may follow (Fig. 787) or a stepped arrangement with e extended side aisles (Fig. 787a). Still more animated becomes the plan, if the chapels lie diagonally in the angle of a bay of the side aisle intersecting the transverse aisle (Fig. 788).

That the division in bays is the same in the choir and transept, so that there results an equal length for the sides i k, k b and m n, and the bay b k l m is a square, then the chapels be equal to each other and be symmetrical.

If this assumption be incorrect, when k l thus differs from l m, but l m ∥ m n and l k = k i, then will the dimensions of both side choirs be different, but otherwise each can retain its regular form and its direction.

If the last equality disappears, the lines n l and l i form an angle at l, and thereby the directions of the half polygons become different, unless the pier l is moved back in the line i n, when the bay m b k l loses its form as a parallellogram. In a similar way may yet result further irregularities.

With equal heights of aisles and chapels is produced by the endeavor mostly not too great differences in even developed p plans of chapels, since it is usually possible by proper means to neutralize above the thrusts of the vaults so far as necessary.

With raised middle and transverse aisles on the contrary, the necessity to conquer the thrusts of the vaults leads to peculiar forms of flying buttresses or buttresses.

Thus the points k and m exposed to the thrust of vaults are secured by the flying buttersses k l and m l, but the latter abut against a buttress standing at l and separating the chapels, that receives the thrusts of the flying buttresses in the direction of the resultant.

According to the same system would flying buttresses be turned from i to o and from n to p, .od therefore buttresses are also to be placed at the last points. Their forms must then be such, that the windows in the sides of the chapels are not restricted by them. It is therefore preferable to set them back a small distance beyond the points o and p as given at o.

Further these buttresses can be avoided by the arrangement of a detached buttress. At the points o, p and l would then s stand detached piers and receive the flying buttresses, which extend from u to p and further to q in a double arch, also from i to o and then to q, while those from m and k to l must be t turned in the direction of the resultant l q to reach the pier.

But the flying buttresses i o and n p can be avoided if as shown in the right half of Fig. 788, the chapels are so far reduced in size, that the placing of buttresses at the points t and u becomes possible. Thereby the opening between the chapels can be so much increased, that the arrangement of windows in the walls v w and x y again becomes possible. But at the s same time the span of the dividing arch x z becomes smaller, so far as limited by the pier t r. But this restriction can be avoided by a variation from the regular form of the polygon in the manner, that the face of the buttress forms one side of t the polygon, and the succeeding one abandons its original direction, so that it meets the other in a skilful manner and obtains nearly the same width as it.

All these expedients become superfluous and all irregularities are avoided, when the plans of the chapels consist of rectangles with added triangular enclosings for the octagon as

shown by Fig. 788. Accordingly the walls of the chapels coincide with the buttresses at a and b, and at most flying buttresses became necessary at the points c and d, which find sufficient abutment in the buttress to be erected at e.

On the other hand the fault is peculiar to the last arrangement, that by an orientated position of the altar, as it is retained in the choir chapels of the French system arranged in a radiating direction, the altar to be placed in the eastern chapel comes to stand against the eastern closing wall, and hence the polygonal ending lies at the side end in a rather superfluous position.

The entire form of the plan passes into the central building with entire decision, if also the same system of choir chapel as the eastern angle is added to the western angles between the nave and transepts, as in the Liebfrauen church at Treves (Fig. 789).

As actual or side chapels may indeed not be regarded these western rooms; we rather have it here with a nave imitating the choir plan, and therefore strike a lack in character, which considerably reduces the worth of the otherwise so thoughtfully arranged form of plan. But generally the striking thought of this appears rather on paper, than makes itself felt by a happy effect in the execution. For finally the difference of a church so formed from one of the ordinary plan (Fig. 768) consists only in this, that the diversity of the different views peculiar to the latter gives place to the monotonyprogressing in the mass, since also the endings of the transepts imitate the high choir. However in the entire arrangement must be recognized the highest stage of the development of the central architecture.

Choir plans with choir aisle.

We have treated in the preceding the different forms of the longitudinal church as well as the central building. The richest development of the church plan is that of the French plans of choirs, but this results from the combination of both systems. This combination is to be understood fully, that it is directly formed by the addition of half a central building to a cross shaped church of three aisles, so that the original bay of the central building becomes the high choir, its aisle is the choir aisle, and its original choir becomes the eastern chapel. In this way we pass in the simplest case to the choir

plan represented in Fig. 790, that is modified according to the number of sides of the polygon.

The choir aisle does not participate in the office celebrated in the high choir, it may receive altars and tombs, but first passes to its particular importance, when it permits access to an eastern chapel or to a chevet of chapels.

Functions of the choir piers.

The high choir is therefore separated from the choir aisle by a series of columns or of piers connected by arches. The functions of the choir piers are mort tasteful than those of the nave piers.

If the choir and choir aisle have equal heights, the vault surfaces a b c e f thrust inward and the surfaces a b c d outward, since a b c loads the dividing arches and thus aids in pressing the pier outward. Therefore while in the parallel direction and also in the aisle the excess of the thrush represented by g h k i over that dependent on g k l requires strengthening the piersg and k, the approximate equality of the above surfaces in the polygon allowing this to appear superfluous, a and the dimensions of the pier are exclusively determined by the vertical loading. But even the latter is shown by a glance to be far less than in the aisle, and if e f = l m, amounts to about half the latter.

But that a raised arrangement of the high choir, the thrust of the choir is further transferred by the flying buttresses to the external buttresses, and the dimensions of the pier are determined only by the vertical loading and the need of resistance against the thrust of the choir aisle toward the centre. But this is met by those acting in the sides of the polygon, by dividing arches loaded by masonry, since these extend in the plan in ring form.

Therefore results in both cases the possibility of a considerable reduction of the dimensions of the choir piers from those of the nave piers. But since by the width of the dividing arches as well as by the ribs and rounds of the choir aisle and choir vaults there is required about the same width of the upper area of the pier as in the aisle, then with the smaller dimensions of the choir piers either the capital has a wider projection or the plan of the pier abandons the concentric ground form and with less width must have an undiminished depth.

On the older French works, the churches of Mantes, of S. Leu, the cathedral of Noyon, etc., is necessary an area for setting the upper parts on the slender round piers, whose diameters remain far less than the width of the dividing arches, and only obtained by the bold projection of the square capital, on others like the cathedral of Rouen the projection of the capital is aided by a projecting corbel from its upper edge in bearing the rounds for the choir. Thus by the combination with corbels, as already mentioned in the Section treating of the forms of capitals, there is obtained from the round pier any form required by the entire system.

Plan of the choir piers.

The variation from the round or concentric form of pier occurs, as we have shown in Figs. 425, 426, first by the peculiar arrangement of the rounds. We add further to the examples above that of the choir piers of the cathedral of Beauvais (Fig. 847), where the depth of the form obtained by the pier and rounds is enlarged by the arrangement of a corbel, and thus the picturesque effect of the entire capital is considerably increased.

For here three rounds are placed next the choir aisle but only one next the choir. The capitals of the first have about half the height of the capital of the circular nucleus, the last single round remains without a capital, penetrates the abacus and forms directly above the nucleus a compound corbel, on which again stand three rounds for the diagonal ribs of the choir vault and the arches of the upper window. Therefore the piers in the choir polygon differ from those of the parallel extension of the choir only by the omission on the latter of the rounds beneath the dividing arches.

Since for the present case the arrangement characteristic in the highest degree of that whereby the nucleus of the pier consists of not one, but of two engaged cylinders in depth of smaller and different diameters, which can then have four rounds, two of which conceal the intersection of the cylinders. This arrangement is found in S. Gudule in Brussels, Fig. 791, but in richer development in the cathedral of Cologne. In the cathedral of Coutances both cylinders are further entirely separate from each other, only being connected by a parallel bit of wall, but accordingly are arranged doubled dividing arches, which again are joined together by the projecting stone slabs forming the

floor of the triforium (Fig. 792). We add to this that on the small width of the choir piers, aside from the structural reasons leading to this, already for seeing through toward the choir aisle and the chapels, special weight is to be placed, so much the more the greater the number of sides of the polygon of the choir and accordingly the smaller the sides.

Bays of the choir aisle.

With the longitudinal walls of the eastern chapel then increased the buttress piers standing at the external wall of the choir aisle in the direction of the angle of the polygon (Fig. 790, left), or it formed at the same time this wall, so that the chapel begins with a trapezoidal bay (see Fig. 790, right). In both cases the window is omitted in the first bay. The division into the other bays of the chapel is arranged according to this polygonal ending. Just as well can this eastern chapel also take the square form, likh the cathedral at Auxerre and further as shown on Plates 28 and 32 of the Sketch Book of Vilars de Honnecourt. (Library).

The other polygonal sides of the choir aisle are opened by windows. But here results by the development from the octagon for the sides lying next the chapel a length far exceeding all other arch spans of the choir aisle, which are disadvantageous both for the development of the elevations of the dividing arches as well as for the arrangement of the windows; especially then for the latter, when the entire side is opened by a window, which must have a width exceeding all others.

The nearest means for avoiding this defect lies in the arrangement of a pier in the middle of the side corcerned, and of a bisecting rib r extending to the crown of the vault (Fig. 790, right), where again are led those of the two side arches and also those of both windows. For example this arrangement is found at S. Gudule in Brussels. On the arches opening toward the eastern chapel however this arrangement could not well be employed, rather was given the preference to the arrangement of two heavier columns, whereby then the plan of the vault had to assume in the choir aisle as in the chapel the transformation represented in Fig. 793. The placing of these columns with the intention to divide the span of the dividing arches between choir aisle and chapel, is found on certain older French works, as at

S. Remy in Rheims and Notre Dame in Chalons; but according to the way shown in our Fig. in the cathedral of Auxerre and in yet richer arrangement also in the collegiate church at S. Quentin (Fig. 794).

Another means for attaining the same end lies in avoiding the trapezoidal form of the separate bays of the choir aisle, i.e, in the arrangement of rectangular bays with triangles between them, whereby the polygon of the choir will have twice the number of sides of the choir.

Already the Early Christian central buildings like the minster at Aix-la-Chapelle and the monastery church at Essen, show these in the position reproduced in the left half of Fig. 775, which in certain German works of the 15 th and 16 th centuries has led to the most varied forms of plans of vaults, as in S. Sebald in Nuremberg, the Frauen church in Bamberg (Fig. 804), the Freiberg minster, to which we shall return later. Here further belongs the arrangement occurring in the Liebfrauen church in Worms (Fig. 795), where each trapezoid od the choir aisle is divided into three triangles, so that the side of the high choir forms the base of one triangle and those of the two others lie on the corresponding side of the choir aisle, which is therefore halved by a middle pier.

The side of the polygon of the choir aisle falling in the direction of the length then receives by the position of the cross arch k l (Fig. 790) again a different size, so that all sides of the choir aisle assume a different shape. However all this irregularity is avoided by the last mentioned way of doubling the number of sides of the choir aisle (Fig. 804).

Completed chevet.

By increasing the number of chapels attached to the choir aisle we reach the richest arrangement,

Here are to be distinguished two principal arrangements, according to which the chapels abut against each other and are only separated by the buttresses, or between them is alsc formed a bay of the choir aisle with a window.

Other very important diversities result from the general plan from the choice of the choir polygon.

Choir ending with five sides of the octagon.

For example if we turn to the first arrangement of chapels abutting against each other in the choir ending from the octagon,

then the before mentioned irregularity in the sides of the enclosing wall (Fig. 790) will also be expressed in the chapels, and even more strongly since the chapels placed at the smaller sides of the octagon receive less projection on account of the smaller width. In this case the elongation of the eastern chapel by one or more rectangular bays is to be preferred to three eastern chapels, so that by the smaller magnitudes of the chapels falling in the longitudinal direction originating accidentally as an irregularity becomes systematic. In this manner is arranged the chevet of S. Ouen in Rouen.

This inequality of the chapels diminishes with the increase in the number of sides of the polygon, and therefore becomes much less for the duodecagon than for the octagon. However it is found to be avoided in the most varied way in the works of the middle ages.

Seven sides of the duodecagon.

The nearest means for producing complete equality consists in abandoning the form of the regular polygon for the outer wall of the choir aisle. In Fig. 796 let a b c d be the duodecagon of the choir with centre at i, a k being the width of the choir aisle, so that there results the form of this by the division in seven parts of the arc described from i with the radius i k. Accordingly the chapel walls certainly become equal, but the direction of the diagonal ribs of the high choir no longer continues in that of the cross arches of the choir aisle and the same as the buttresses, but forms a break with them at the angles of the choir polygon, as for example in the church of the monastery of Altenberg and the cathedral of Chartres.

If this irregularity in direction already really makes necessary only a slight addition in dimensions of the upper choir p piers, it is therefore to be regarded chiefly as incomplete.

This will be entirely avoided by the converse procedure, according to which not the high choir but the wall of the choir a aisle is formed as a regular polygon (Fig. 797), whose base is the entire width of the choir and choir aisle with a centre at C. The first angle d of the polygon then results from the intersection of the radius 1 C by the perpendicular erected at e and thus limiting the width of the high choir, the others from the points of intersection of the radius with the arcs described

from C with the radius C d. Accordingly the choir ending is formed as a regular duodecagon, and only the sides d e falling lengthwise and that offosite have lengths greater than the others. From the construction just indicated , that of the choir of Cologne only varies, that the angles of the duodecagon of the high choir as well as of the choir aisle (Fig. 797, right half) lie in the circumference of the circle described from C with G g a and C f, while the points h and b remain the same. Accordingly the first sides of the polygon have a direction changed from t those of the regular polygon, i.e., they no longer fall in the longitudinal direction but converge toward the East.

The increase in the magnitudes of these sides has other advantages besides the equality of the chapels produced thereby.

First the keystone of the choir vault C falls so far eastuard from the line e h, that the ribs turned to it from the piers e and h receive a direction differing from the extension of the choir ribs d C and k C, but thereby one more suited to offer resistance to the total thrust of the other choir ribs.

The second advantage is connected with the functions of the piers h and e, which have to receive the same number of ribs of the parallel elongation of the choir and of the aisle, therefore receiving dimensions equal to them and exceeding that of the c choir piers. With entire equality of the lengths e d, d l, etc., therefore would the spans of the side arches be regarded as unsuitable and form a disproportion to the larger piers, which is happily avoided by increasing the lengths of the sides.

In the choirs of the cathedrals of Amiens and of Beauvais is found this increase in a lesser proportion than is given in Fig. 797. With the problem of the regular polygon for the choir and choir aisle, thus by moving the centre C toward the base line a b there may be obtained any desired proportion of the first sides of the polygon to the others, equal to each other.

The construction in the cathedral of Amiens is found in Viollet-le-Duc, Dict. Vol. II, p. 382. Accordingly the distance from the base line of the choir polygon to the centre x in Fig. 798 was laid off as a definite measure of 2.5 m (about 1/13 of A B), from this centre a circular arc was struck with the entire width of choir and choir aisle, divided in 7 parts, a radius drawn f from each point of division, thus from points 1 and 6 nearest

the ground line being extended beyond the centre to intersect the base line, the width of the high choir being determined by these intersections N and M.

Further then according to the French construction, the intersections of the before mentioned radii with the circle struck from the centre with the width of the high choir determine the other angles of the choir polygon, and those of the centres of the choir piers.

In this construction contrary to that of Cologne (Fig. 797) the ribs C M and M P (Fig. 798) are equally long in plan, and thereby the choir polygon will become more regular, while on the other hand the advantageous removal of the thrust from the longer ribs C e and C h (Fig. 797) disappears.

If in the plan of Amiens a different rise x is taken as a basis, then the given construction would continue the equality of the ribs C M and C P, but the width of the middle aisle would change and indeed a greater x would correspond to a wider, or a smaller x to a narrower middle aisle.

Conversely it would otherwise also be quite possible, first to assume the widths of the aisles and then to determine the corresponding x by trial. By this procedurethe assumption of Viollet-le-Duc would lose its support, that not the builder Renault de Cormont, but already Robert de Luzarches had made the plan of the choir before the erection of the nave. Be that as it may be; in any case must the plan of the choir of Amiens be regarded as masterly.

Five sides of the octagon.

The construction of the choir here developed from the duodecagon or the division of the circle are peculiar to the works of the first size like the three great cathedrals. In more limited proportions the distance of the choir piers from each other would accordingly become too small, and therefore as a rule five sides of the decagon are taken as the choir ending, according to which is shown by Fig. 797, the chapels as well as the bays of the choir aisle without further means of themselwes have equal sizes.

Dimensions of walls and piers.

In the given Figs are deweloped only the skeletons of the forms considered, which in the further execution must receive

the thickness of walls and piers.

For those who adhere to determining the dimensions of the walls and piers not by static standpoints, but according to fixed rules, the following statements may have their place.

In Fig. 797 the widths of the dividing arches are assumed about as great as the thickness of the walls of a single choir, thus perhaps 1/12 to 1/10 of the clear span. The dividing arch consists of two rings according to one of the profiles given in Figs. 422 to 427, and it accordingly determiner the dimensions of the choir piers with reference to the other ribs resting thereon and the rounds. Half the width of the dividing arch can be taken as the width of the cross arches, and the half diagonal of the latter for that of the cross ribs of the vault of the choir aisle, the width of the cross arches is also taken for t the arches turned over the entrances of the chapels, and therefore half of it is laid off at each side of the middle line, accordingly determining in the same way the form of the plan of the supports of these arches, the wall piers with their rounds forming the ends of the walls separating the chapels, with regard to the vault ribs meeting thereon, so that for each rib is arranged a round and the diameter of the nucleus of the pier will about equal the diagonal of the width of the cross rib. The width of the arches separating the chapels from the choir aisle is then to be made only equal to that of the cross arches, if this does not have to bear any walls rising above the roofs of the chapels, but in the last cases it then approaches that of the separating arches, by which the dimensions of the piers must be increased.

The chapels themselves may be formed by five sides of the octagon. They would be separated from each other by the walls placed behind piers 1, 2 and 3, strengthened by the radiating position of the piers. This increased thickness therefore makes possible a very small thickness for the junction with the pier, and the size of the chapels is dependent thereon. The minimum of this thickness will occur when the perpendiculars drawn at the angles of the polygon of the choir aisle to its sides form the internal faces of the wall of the chapels. In Fig. 797 it is better to make the thickness of the junction with the pier equal to the width of the cross arch, so that in the space remaining between the lines 1 m and the rounds n are placed the

rounds for the diagonal ribs and side arches of the chapels. Then the other rounds for the diagonal ribs in the chapels are located according to the regular octagon, the founds for the s side arches are added, and the thickness of the walls of the chapels is made equal to the width of the cross arch, the thickness of the buttresses equal to the diagonal of that measure, and their depth is determined for a simple choir.

The thickness of the great buttresses between the chapels which then have to receive the flying buttresses is determined by the diagonal of the width of the dividing arch of the high choir, and these project about the same distance beyond the ju junction of the walls of the chapel. Accordingly there is given a suitable determination of the width of the chapel buttresses by the circular arc struck from the centre o with radius o p.

The flying buttresses on the choir of the basilica usually receive far less thrust than those of the nave. The high choir vaults exert at the angles of the polygon only a thrust, which is 1/4 or at most 1/2 as great as that of a bay of the middle aisle. For resisting the wind the flying buttress on the choir likewise has less importance, but on the contrary with greater breadth of the windows the side arches intersecting at an obtuse angle produce a resultant outward thrust, which cannot occur at the nave.

Usually the flying buttresses of the choir as well as their buttresses have long not required to be as strong as for the nave. On the other hand one also need not fear too heavy flying buttresses on the choir as much as on the nave, since a great pressure directed inward, that requires in the nave a stiff vault oh a well built cross arch, can be readily received in the continuous polygonal walls. Thereby at most the side arch and its upper wall may be pressed toward the crossing, where a corresponding counter thrust must be found.

Buttresses between chapels.

The small thickness of the chapel walls at the junction with the pier in the cathedrals of Amiens and Beauvais led to the peculiarly spirited arrangement, that the proper buttress that must receive the thrust of the flying buttress, does not begin at the internal face of the side aisle but is set farther out at about g r s t. over the piers in the polygon of the choir

aisle ar set intermediate piers of more concentric plan, that indicate the hexagon on pier 2 extended upward. These last receive directly the flying buttresses and from them are again turned smaller flying buttresses to the proper buttresses g r s t, so that the chapel walls are first loaded there, where they have acquired the necessary strength by their increased thickness.

This arrangement of the buttress system then led further, also to perforate the piece of wall between the chapels by an arched opening, and thus finally produced the arrangement peculiar to the cathedral of Coutances, whereby to the proper buttress are first added the rounds v and w, accordingly having a greater projection, and in the angles of the choir aisle the piers become too detached, between which and the buttresses are now found triangular bays and vaults, that connect together the separate chapels and in a sense form a second narrower choir aisle. The right half of Fig. 797 shows this arrangement.

An increased thickness of the walls separating the chapels and also in the dimensions of the piers standing in the angles of the choir aisle therefore becomes necessary, when the buttresses receiving the flying buttresses begin directly from the choir aisle, and hence the little intermediate arches disappear. They are further required if the arrangement by which the walls or the window wall between the buttresses is placed farther outward, so that passages leading through the latter are found, also employed in the choir chapels, as in the cathedral of Rheims.

This strengthening can be obtained either by a reduction of the chapels or by the arrangement of buttresses with parallel sides between the chapels, as in S. Pierre at Louvain, whereby also the ground form of the chapels differs from the regular polygonal shape (Fig. 800), or to the polygonal ending of it is prefixed a trapezoidal bay in a similar way, as already given in regard to the eastern chapel in Fig. 790.

Various forms of chapels.

This increase in the depth of the chapels can also be obtained by the arrangement represented in Fig. 797, either by prefixing a rectangular bay before the polygonal ending or by lengthening the sides of the octagon next the choir aisle beyond the measure resulting from the regular polygonal form. Further everything said concerning the polygonal form of the choir also applies to

the chapels, and each polygon or any irrational division of the circle would form that ending, so far as the sides have a moderate size.

The form according to five sides of the octagon would be nearest to that according to four sides of the hexagon, which is f found in the choir of the Freiburg minster.

The ending in the half polygon, thus according to three sides of the hexagon, five of the decagonwill for the system of ribs require one of the solutions given in Figs. 728, 729, 731.

But a very peculiar forms then lead those shown in Fig. 799, when the chapels consist of the half polygon without a prefixed rectangular bay. In this case it would be desirable to the system of ribs of the prefixed bay of the choir aisle of the choir concerned with that of the polygonal chapel, as shown in Fig. 801, i.e., the keystone lies in the middle of the arch forming the outer polygonal side of the choir aisle, and from the opposite choir piers a and b are turned diagonal ribs to this keystone, whose thrusts hold in equilibrium that of the chapel ribs. In the choir of the cathedral of Soissons is found this arrangement, which thus by its nature only represents an application of the rib system adopted for the high choir to the chapels.

The peculiarity of this arrangement, that the chapels with the adjacent bays of the choir aisle are connected into one vaulted bay, is found in a simplified way again in certain North German works, and indeed in connection with the high choir composed of five sides of the octagon, as on the church of S. Maria in Lübeck (Fig. 80). This system is first distinguished from that of the choir of Soissons in that the space required for the polygonal ending of the chapels has no choir aisle extendind its full width, but the width is reduced, so that strictly speaking the high choir is directly adjoined by the chapels and a passage results only by openings in these separating walls, therefore as Fig. 802 shows, and the side aisle accompanying the parallel extensionof the high choir only a half width. But there the eastern chapel by its parallel extension projects far beyond t the others, which thereby again extends in the proportion of a widening of the choir aisle like a chapel, as then the polygonal parts are less than half the polygon.

The arrangement of such flatter chapels composed of only three

sides of a decagon before the bays of the choir aisle requiring the full width would therefore represent an adjustment of both arrangements, and thus be counted with the arrangements mentioned on p. 300, by which the inconvenient size of the outer sides of the polygon of the choir aisle would be avoided. [illegible] As actual chapels with altars placed therein the areas so obtained cannot be employed. Therefore those io the cathedral of S. Quentin the chapels shaped according to the entire octagon project, as Fig. 794 shows, so that the columns standing in the outer polygonal side of the choir aisle become at the same time the angles of the octagon, and between the latter anf the choir aisle remain triangular vault bays. The richness of the entire form is then increased, so that the chapels receive less height than that of the choir aisle, and thus over the dividing arches resting on the columns stands a wall perforated by three double windows. But even in this lies the weakness of the construction, since the chapels by means of the low position of their vaults cannot oppose those of the choir aisle, and hence the vaultsof the ribs r r find no sufficient abutment.

Chevet with intervals.

The second of the arrangementr distinguished above, that according to which the chapels abut against each other, but windows are left between them for direct lighting of the choir aisle is the earlier one. Already in certain Romanesque works like S. Godehard in Hildesheim but more frequently in France, appears the plan of little semicircular chapels on those formed by a greater segment and projecting from the external wall of the choir aisle. In Gothic art the walls of the chapels became buttresses for t the vault ribs producing a further division of the bays of the choir aisle. Accordingly the system of this vault changes. Either is retained the trapezoidal form and the vault triangle next the exterior is divided in three parts by two ribs extending to the keystone (Fig. 803), or the trapezoidal form is reversed, so that the larger side is formed by the distance between the choir piers and the smaller by the chapel walls, these trapezoids being then covered by cross vaults (Fig. 803 b). Between each two such bays are inserted two triangular ones as in the cathedral of Bourges, that however is not of polygonal form but is semicircular. The same arrangement applied to the poly-

polygonal form would then lead to shaping the polygon of the c choir aisle to thrice the number of the sides of the high choir (Fig. 803 c).

A simplification, the least for the plan, is found in the system of the Early Christian round buildings, whereby to each of the rectangular bays is attached a chapel, and the triangular sides lying between them are opened by windows (Fig.803 d). On the German works mentioned on p. 300, that exhibit this arrangement of the vaults of the choir aisle the high choir is formed according to the octagon. The chapel wall is there sixteensided with equal sides, if the width of the choir aisle equals the d diagonal of the square of the side of the polygon. But the application of this system to the duodecagon in the choir of the cathedral of Le Mans by the inclination resulting from the last polygon for the sides of the rectangles perpendicular to the sides of the polygon, led again to changing the rectangles into trapezoids slightly narrowed externally, which the chapels adjoin, and between which are found the much smalles bases of the triangles opened by windows. An actual simplification of the structural system was obtained in none of the last mentioned ways, since the continuous straight line through buttresses, choir and keystone was lost, and therefore the necessity appeared to produce the resistance to the vaults of the high choir by two flying buttresses from each pier extendind out to the external buttress. But this increase in buttresses and flying buttresses is already injurious, since the principal object, the high choir, is withdrawn from view. Therefore all such arrangements are better suited for ailes of equal heights.

The simplest solution results when to each space between the piers of the high choir corresponds a trapezoidal bay, and then the bay opening into the chapel alternates with that indepently lighted by a window, so that also with a choir ending from the decagon gives three chapels and two bays with windows as on the cathedral of Rouen.

By the before mentioned division of the vaults of the choir aisle into rectangular bays with the triangular ones lying between them (Fig. 803 d) can also be chapels added to the latter. Such a chevet with the doubled number of the sides of the polygon of the high choir as found in the Frauen church in Bamberg, Fig. 804. Here the vaults of the choir aisle rest on corresponding

heads of the buttresses divided inside into moulded wall piers, and the wall opened with windows is placed back in the face of the latter, so that nine rectangular chapels are formed, whose depths are the widths of the buttresses.

The same arrangement, but with richer subdivision of the plan and with polygonal chapels is then found also on the choir of Freiburg minster (Fig. 805), here the high choir has three sides of the hexagon, and accordingly the choir aisle has six sides of the regular duodecagon, but the transition of these forms into another is effected by a net vault. The polygonal sides of the choir aisle are formed of four sides of the hexagon again with net vaults over the chapels, so that they project two of the sixteen sides beyond the remaining buttresses, with an angle in the middle between them. The chapels are then continued also along the parallel sides of the choir aisle between the buttresses to the side towers adjoining the transepts.

The peculiarities of thes plan are shown in an indeed less c correct mathematically, but freer and grander conception of the plan of the chuir of the cathedral of Paris (Fig. 806).

Here is the high choir formed by a raised semicircle, divided by six round piers at equal distances into five parts and surrounded by double side aisles. The choir aisles are separated by great piers set opposite the choir piers and five smaller piers between them, so that to the five spaces of the choir correspond ten in the choir aisle. Their second circle is again surrounded by a third concentric one, formed by six buttresses opposite the heavier piers and two moulded pries between each pair.

Only the western part adjoining the parallel extension is divided by one moulded pier instead of two piers in the divisions of this circle made by the buttresses. Between the six buttresses mentioned then lie the annular chapels belonging to a later rebuilding. The choir is also surrounded by two galleries divided in triangular bays and they indeed correspond to each space between the piers of the choir, three triangles in the first of and five in the second gallery.

According to all systems before mentioned the ground form of the high choir is surrounded by the concentric aisles anf the series of chapels. By the abundant use of triangular bays would be obtained each irrational relation of the ground forms to each

other, i.e., could be solved according to a regular polygon, or according to a choir with five sides of a decagon, square in its general form, or be formed according to an entirely irregular polygon with single or double choir aisle. Such arrangements are indeed not to be sought, yet may seem required in certain cases by local conditions.

Instead of further explanation, we shall refer to two examples indeed belonging first to the last period of Gothic art.

In S. Stephen in Beauvais, the motive of the arrangement of its choir being shown in Fig. 807, the high choir is enclosed by three sides of the hexagon. One of the two side aisles curves around the choir, but the other stops abruptly, so that here results a straight line termination truncated by small oblique sides from which projects only one eastery chapel.

The same ground motive is followed by the plan of S. Germain l'Auxerrois in Paris with only the difference, that all bays of the eastern end open into chapels of certainly very irregular plans, which are kept within the eastery rectilinear termination.

Arrangement of plan between choir and transepts.

According to the simplest arrangement, the chapels form an external projection from the line of the side aisle. Yet as a rule the space between this projection and the transverse aisle is filled, either by the primarily divided or later added continuation of the chapels to the transverse aisle, or by the arrangement of double side aisles to the choir. The continued chapels either have the same polygonal ending as on the choir (Freibugg), or they lie between the buttresses as simple rectangular bays. (Fig. 799, k).

The grandest arrangement is that of doubled side aisles as shown by Fig. 797, found in Germany in Cologne and Altenburg, as well as on the cathedral of Amiens and at Beauvais, also many other French works. These then occur at their sides again before the chapels, so that their eastern buttresses conceal the western window of the latter. Therefore is found as a rule the space resulting between these buttresses and the oblique sides of the polygonal chapels is filled by the plan of a stair tower (Fig. 797), which then is either accessible from the chapel or the eastern bay of the side aisle.

The wall between the last chapel and the side aisle must rec-

receive the side thrust of the chapel ribs o o_1 in Fig. 797. This is so small, that the wall requires no great thickness, s sometimes by filling the corners, and at others the thrust can be opposed by a partial rib k yin the vault of the side aisle.

6. Forms of Plans of Towers.

The purpose of the towers is essentially demonstrative (Note), they are to make known the church afar by the sound of the bells and their tapering shape , and at the same time bring the characteristic peculiarities of the entire form to an enhanced expression.

Note. We leave aside certain recent and more symbolical meanings of towers, because nothing is changed in them from what is here given.

Position of towers.

There follows from the purpose first the requirement of a predominant height, and thus results the second that they shall not be built at random, but at certain main points stand in intimate relation thereto, i.e., rise over certain particularly accented points.

We accordingly have to distinguish:--

1. Towers belonging to the middle aisle.
2. Towers added to the side aisles.

The former find their places:--

a. Over the middle square of the cross shaped churches as central towers.

b. At the western end of the middle aisle.

c. At the north and south ends of the transverse aisle.

d. Over the choir ending.

The second also naturally stand:--

a. Before or on the western bays of the side aisles.

b. Over the end bays of the side aisles accompanying the transverse aisle.

c. Over the eastern bays of the side aisles before the beginning of the choir polygon.

d. In the angles between the nave and transepts over the corresponding bays of the side aisles.

From these result the following naturally occurring cases.

1. The ordinary arrangement of one western tower.

2. That of two towers before the middle of the transepts as on S. Stephen in Vienna.

A combination of these two first arrangements with each other nowhere occurs within our knowledge.

3. The arrangement of a central tower.

A combination of the last with one of the former or with both the former arrangements is found only on works of the transition style, and was formed so that the middle tower surpassed the o others in size, but it then always has the fault that in a rectangular elevation one tower conceals one of the other towers.

4. The arrangement of two western towers.

5. The combination of these with a central tower.

6. The arrangement of six towers at the ends of the nave and transepts in combination with a central tower, as found on the cathedral of Laon and was intended at Rheims.

7. The combination of the last arrangement with two other towers before the beginning of the choir polygon as intended at Chartres.

With the two last arrangements must therefore be combined that of the central tower, so that one is found to dominate over the great number of competing towers.

Central tower.

The arrangement of a central tower, which is more rare in general but is commonly found in France and England, requires in the plan only the strengthening of the crossing piers treated on p. 295, and therefore must be the most economical of all when a transverse aisle exists. Substantially increased will be the effect by small stair towers in the vicinity of the central tower or at the angles of the transepts as on Notre Dame at Dijon.

Tower over the choir.

The arrangement of a tower over the choir ending does not harmonize well with its polygonal form, and is found only over rectagular choirs on certain buildings of the late period, thus in extremely picturesque form on the two aisled church of Niederasphe in upper Hesse. It cannot further be combined with the arrangement of a transverse aisle, since the tower would seem to be removed from over the crossing to the wrong place.

A single western tower.

The western end of the middle aisle offers the only single location existing on the church, if we except the arrangement of

a crossing tower and that last mentioned, and therefore it will combine here a symmetrical form of the whole with the economically preferable single tower. Further the width of the middle aisle offers a great base for the proposed tower and therefore allows a greater development in height than would be possible for one over the narrower side aisles. Herein lie the advantages of the single western tower. On the contrary the disadvantage is peculiar to it, that at least the four of the side facades produce an excessive detached effect; this fault appears strongest on single aisled churches, where the tower conceals the entire gable end. It is reduced in the measure that the tower is enclosed by the side aisles.

For the relation of the tower to the middle aisle to be conceivable, the width of the latter must determine the square of the tower. But since the necessary thickness of the walls of t the tower or the widths of the arches and piers replacing the latter exceeds that of the divided arches of the nave piers, so here is given greater play, within which that determination is to be understood.

Thus the clear width of the tower corresponds to the clear w width of the middle aisle, or the side of the external square of the tower to the width of the middle aisle with the added widths of the piers, or the axes of the piers may fall in the middle thickness of the tower walls, or finally these may limit the external square of the tower. For all the proportions mentioned could be given a large number of examples.

What then concerns the position of the tower in the longitudinal direction, the usual arrangement is that whereby its entire ground area projects free, and indeed from the internal, or as in Wetter from the external face of the western wall.

The internal area of the tower either, as on Freiberg cathedral, forms an open vestibule and the church portal recedes to the eastern wall of the tower, or it is taken into the church by an arched opening, the portal being placed in the western w wall, a third arrangement would be an enclosed vestibule furnished with doorways at the west and east.

The different developments of these arrangements, according to the height of this lower story of the tower, will be examined with the development of the section and elevation.

The connection with the church is more clearly expressed if at both sides of the tower the side aisles each continue in one bay, so that the tower is built about to the middle, or about as on the church of Frankenberg each of them continue for two bays, so that the western wall of the side aisles is entirely or approximately in the line of the western tower.

Both arrangements last mentioned could be combined with each of the above mentioned uses of the internal area of the tower, but the first could only lead to placing the doorway back in t the middle of the area of the tower, so that half the latter forms the vestibule, but the other half is taken into the interior (Fig. 808). The connection of the latter half with the church will be more complete if it opens sidewise into the adjacent bays of the side aisles, then the eastern angles of the tower being supported by isolated piers as shown in the right half of the same Fig.

If the sides of an entirely included western tower are open, there would result with two bays of the side aisles adjoining the tower a cier standing in the middle of the tower square as shown in the left half of Fig. 809. The omission of this would not harmonize with the remaining system of the tower, and again a dividing rib in the tower vault requiring a pier on the other hand the arrangement of one or more would not harmonize with t the other system of the vaults of the side aisles leads to a b bay adjoining the tower (Fig. 809, right half), or finally it requires a peculiar solution of the vaults of the side aisles corresponding about to Figs. 90 and 90 a, as found in an even more complex way in S. Pierre in Louvain.

Such difficulties are indeed removed, when the system of construction of the entire church is developed from the construction of the tower instead of from the choir. But this reversal of the legitimate proportions contrary tn the importance of the whole, as may be easily conceived, introduces en entire series of other faults.

Therefore the arrangement last mentioned permits the entire opening of three sides of the tower into the church, combined with the arrangement of one western tower in a rather violent manner, while it is to be regarded as the most complete solution of the arrangement of two western towers, and in the latter

is made easier by the small area of the ddimensions of the necessary piers.

Towers before the transverse aisle.

Everything just said of western towers similarly applies to the towers placed before the transverse aisle. Other differences would only result here according as the towers directly adjoin the middle crossing square, or are separated therefrom by a bay corresponding to the width of the side aisle. In the first cases the crossing pier becomes at the same time the inner pier of the tower, and the arrangement takes about the form given in Fig. 810.

Two western towers.

The arrangement of two western towers is best suited to the system of the plan and also most favorable to the development of the western side. The relation of the tower square to the side aisle can be the same as the relation of a single western tower to the middle aisle. The limited width of the side aisle causes this arrangement to be the usual one, whereby the clear width of the side aisle agrees with that of the tower, and therefore with the excess of the thickness of its walls projects at one side beyond the outer face of the side aisle, and on the other side narrows the enclosed part of the middle aisle.

Western towers before doubled side aisles.

A further increase of the tower square results thereby, that its middle lines project beyond those of the side aisles, and therefore the towers have a more important projection externally than from the mere thickness of the wall (Fig. 811). The last arrangement was indeed most common and is found in all periods of Gothic art, from the earliest time (cathedral of Noyon) until the end of the 15 th century (S. Martin's church in Cassel). Indeed it includes a certain caprice and cannot go to the extent, that the walls of the side aisles abut against the middle of t the tower square, and therefore the towers have the same size, which is given them by the arrangement of doubled side aisles.

With doubled side aisles, three arrangements are possible for complete connection of them with the church. For the row of piers between the side aisles may continue in the towers, and there lead to the plan of the four vault bays with intermediate piers in the middle of each square side of the tower with a middle pier at the centre of the area as in Cologne cathedral.

Further this bisection can only go to the intermediate piers of the tower walls, and to avoid the isolated middle column, the internal room of the tower can be spanned by an octagonal cross vault as in the cathedral of Paris. Finally also the bisection of the vaults of the side aisles can be changed into unity by means of a system of triangles before the junction with the tower as in Fig. 812. Likewise a raising of the starting point as in Fig. 90 would be possible.

These different arrangements of the vaults exert a certain influence on the form of the portals of towers. A combination in the mode of Fig. 812 naturally leads to the plan of a portal o opening in the middle. This arrangement is carried out on the cathedral of Paris in the way, that the before mentioned intermediate pier in the middle of the western side of the tower is also the dividing pier of the double doorway of this portal, wherefore in contrast to the otherwise small width of the last pier extends through the entire thickness of the wa,,, while t the arched jambs of the portal project from the face of the wall and extend before the angle buttresses. On the contrary in Cologne cathedral the bisection is made and led to the removal of the portal of the tower to the bay adjoining the middle aisle, while the outer bay adjoining the entrance forms a chapel beside it, that opens to the west by a window.

But generally the arrangement of portals in the side towers is not an absolute rule. For example they are wanting on the c church S. Elisabeth at Marburg, the cathedral in Meissen, the church S. Lorenz in Nuremberg, and they are further lacking on the towers of the side aisles of the transepts of the cattedrals of Laon and of Rheims.

We have already charcacterized the complete combination of t the internal rooms of these side towers with the aisles as best corresponding to the entire arrangement. Differing arrangements may here avail as exceptions, as the complete combination with the towers of the middle aisle. On many older towers the lower room is entirely enclosed, and also is the rule in north German brick churches. The middle aisle extends between the two towers almost without exception, since the abutments for a vault between the two towers are given in them, and hence the continuation of the middle aisle to the line of the western towers through the entire plan seems required. An omission of this outer bay

of the middle aisle, as found on the western side of Friedberg church according to the original plan (Fig. 813), whereby the towers enclose an uncovered forecourt a, leads only to saving a small piece of vault, but on the contrary to the loss of a very useful room and to a divided form of western entrance.

Since the side towers of narrow side aisles were frequently made wider, middle piers before wide giddle aisles of single aisled churches may have less than the breadth of the middle aisle in a converse sense (Fig. 814). The wimth can be made as needed, leaving sufficient space for passage in the lower room of the tower.

In such a plan as given in Fig. 814, the junction with the church can be narrower, if the tower is utilized as abutment of vaults by the course of the ribs, where the triangles a b c in Figs. 57 or 58 are vawlted and the buttresses at the angles bc can be omitted.

By means of corbelling then occurs the possibility of making the lower ground area of the tower rectangular instead of square, and indeed by strengthening the western wall by two buttresses, from which project at both sides the corbellings supporting the walls of the tower (Fig. 814 a, section). Such forms can finally pass i into the higher corbelled gable here.

Unsymmetrical arrangements.

Since in the preceding all arrangements designated as symmetrical are so only in regard to the western facade, on the contrary becomong unsymmetrical for the north and south sides, then also for the western facade simple works may also depart from symmetry, thereby in many cases producing caterial utility and a very picturesque general effect. Justifying reasons therefore must well be found in local conditions.

Such unsymmetrical plans result when the tower has a position with only a symmetrical effect when doubled, or in small dimensions is placed on the intersection formed by the buttress and the adjeacent wall.

In Germany is indeed generally symmetry is as sacred to the "cultured public" as dogs and cats were to the Egyptiann, and anything opposing it can scarcely be executed.

It appears in England as shown by many modern buildings, that greater freedom is allowed in this respect. Indeed it cannot be denied, that the monumental character is so much diminished by

an unsymmetrical arrangement of the tower as the the picturesque effect gains. Unfortunately the cases are not rare when limited means make only the last attainable.

Walls and piers of towers.

Solid tower walls below.

If we now return to that regular symmetrical arrangement of towers, there then results an essential difference whether the system of construction of towers is calculated on a strengthening of the walls by buttresses extending down to the base or such are lacking. The last arrangement is simplest; the tower square increased by the thickness of the walls projects from t the inner, or as in Wetter from the outside face of the wall of the western facade, and affords the required base for all separate parts extending upward as on the tower of the Frankenberg church (Fig. 815), buttresses already result above the portal story by the offset in the outer face of the wall, without coming in contact with the walls and piers of the church. Even the internal area of the tower could be taken into the church, when the eastern corners would be supported by isolated piers, for which certainly would be required considerable dimensions. But generally hre entire system makes necessary great thickness of the walls, if the tower has a moderate height.

Buttresses carried down externally and internally.

If we now assume the buttresses to extend down to the foundation, the piers must fall in the direction of the divided arches, and it would first produce on an externally projecting tower the conditions shown in Fig. 816.

In the right half of Fig. 816 the tower buttresses project f from the internal side of the western wall as internal buttresses, the dividing arches extend from the buttresses, thus from the points a to the nearest piers, and the spaces between the buttresses and the bays lying next them are spanned by tunnel vaults.

In the left half is then found the arrangement corresponding to the system of the Freiberg minster, whereby the buttresses are placed under the dividing arches, so that only the upper part of their western half is deweloped. If the northeast and southeast buttresses extend from the ground, in both cases the doorway or window opening the western wall must be set farther out of the axis of the side aisle, as occurred in Freiberg in

regard to the western rose window. However this fault would be avoided by strengthening the angle pier d, whereby the tower buttresses could stand on the arches turned from c to d in the left half of Fig. 810.

If we now assume a western tower included between the side aisles as closed walls at both sides, then retaining the system of tower buttresses shown in the left half of Fig. 816 at both sides under the cross arches, as then are at the east beneath the dividing arches or according to the arrangement shown in t the right half are beneath the series of compartme ts, or finally a division into bays is given to the side aisles adjoining the tower, a division entirely different from the others according to that shown in the left half of Fig. 817, i.e. it would in a sense form a transverse structure before the western end of the church, from the middle of which rises the tower.

Both halves of Fig. 817 exhibit the analogous case of the plan of a tower open on three sides. In the right half the nucleus of the eastern pier of the tower is formed by the square of the thickness of the wall, before which projects the buttres extending beneath the arch. Io the left half the system of division in bays about like the arrangement of S. Pierre in Louvain is so changed, tha to it is added as essential elements the t tower piers formed accordinm to the required plans. The shape of the last substantially agrees with that shown in the right half, yet with a considerable width of the tower buttress nearly pruduces a termination of the western bay of the nave, as then the great thickness of this tower pier stamps the arrangements in Figs. 816 and 817 as still incomplete.

In the plans described with buttresses at the sides, the tower fomms an entirely independent whole, which maintains perfect s stability without any aid from the mass of the church. This isolation of the pier would bring the advantage, that the settlement caused by the greater weight of the tower would remain without influence on the construction of the church. Yet this advantage is lost by the bond of the tower masonry with the vault p piers, and therefore to ensure it, the tower piers formed as in Fig. 817 must have the necessary strong piers for receiving the vaults without any bond an separate down to the foundation. But this last arrangement would make impossible the extension of the tower foundations on the side next the church, and thus is not

easily executed.

If according to the first rules of construction the foundations are so dimensioned, that beneath each pier and wall each s square unit if the ground bears only a permissible pressure, e everywhere the same for a yielding soil (p. 189, 146), it is generally unnecessary to count on a greater sinking of the tower walls, indeed for the following reasons. The cause of such would only be in the greater compression of the joints in the masonry of the tower by the greater load. But this compression ceases with the complete hardening of the mortar. Since it is now assumed, that between the time in which the masonry of the tower is carried to the height of the walls, and the further construction of this upper part will elapse sufficient time for the hardening of the mortar, then the sinking can only occur in the upper part not connected with the church. (On a yielding s soil the erection can be proceed, that at no time may the pressures under adjacent parts show too great differences.

Reduction of inner tower piers.

But by the loss of independence of the tower are given the means of a considerable reduction of the mass of the eastern tower piers, indeed on the following grounds. It was chiefly t the arrangement of buttresses for the tower that required this inconvenient size. But now the buttress presents a variation from the vertical direction, thus in a way stiffening the tower, then increasing the bearing area of the foundations at the points, where the effect of the load is concentrated, thus at the corners.

But the last purpose may also be attained as a rule by a considerable depth of such tower foundations already of a strong batter, and as for the first, the endeavor would be just as complete if the buttress is entirely separated from the masonry of the tower, and is only connected therewith by arches at different heights, also as with full connection this is broken above each gallery by passages. But to the service given by such isolated buttresses corresponds perfectly the stress imparted to the inner tower piers by the adjacent dividing arches and the walls thereon, and generally by the entire system of the construction of the church. Accordingly for these inner tower piers yet exists only the necessity, that they afford sufficient area for receiving the upper considerably diminished parts extending

above the roof.

With two moderately large western towers, each corresponding to a side aisle, it will suffice to construct the tower and crossing piers with the size of four dividing arches meeting on them with rounds remaining between them for the cross arches.

For more important dimensions of the tower, thus for the arrangement of one western tower or of two corresponding to doubled side aisles, the inner tower piers would be strengthened, and this would be done by the formation of the arches in three layers, so that the plan of the crossing pier would be made as shown in Fig. 786 d.

Farther back was given the plan and internal elevation of the lower part of the tower portion of the collegiate church of Mantes, which in a particularly clear manner shows, how the stability of the inner tower piers is obtained by the connection with adjacent parts of the building.

Dividing arches receive the buttresses.

A further reduction of the mass of the inner tower piers could be obtained within certain limits by placing the buttresses on the dividing arches.

A similar arrangement, the placing of piers on arches, may f first be represented by that given in Fig. 811, produced by side piers projecting beyond the side aisles. For here according to the projection of the tower, the eastern buttress a of the tower closed the window of the bay of the side aisle if carried up f from the ground. Therefore on the cathedral of Rheims broad arches are turned from the angles of the tower square to the nearest buttresses on the side aesle, thus to b in Fig. 811, which strike the side of the latter, on it being set the tower buttress with length extending beyond the crown of the arch. But this arrangement even requires the extreme width of the buttress, to offer resistance to the thrust so much increased by the load.

But we meet an essentially different condition in the internal aisle piers. None of them is sufficiently strong to resist the thrust of the arch increased by the loading, and therefore it only remains, either to strengthen the piers nearest the tower so far, that this thrust ends in them, or to take into account the resistance of the entire series of arches, then strengthening the corner or crossing pier. Thus in both cases must be added to one of the piers mentioned about what could be taken from

the tower pier, so that no real advantage is to be obtained thereby.

However far the reduction of the inner tower pier must go is in certain cases to be based on calculations, allied to those made for the middle pier (p. 153 et seq.). The load must not exceed the allowable limit of compression of the stone, and the line of pressure under the influence of the vaults of the tower and of the aisle must not take adirection too near the outside, and the foundation must be enlarged sufficienltn that the tendency to sinking of the inner piers is no greater than for the outer ones. If these conditions are satisfied and corresponding consideration is devoted in the progress of the construction to the setting of the masonry, then for a suitable building nothing is to be feared for the tower.

The depth of the foundation walls in the ground will be determined by their gradual increase in thickness, by the limit of frost and by any possibility of the removal of earth around them. To carry the foundations of towers into ground of high resistance much lower than the ground walls of a church is generally useless, but is even doubtful in some cirmumstances.

Thickness of walls.

On the thickness of walls and size of piers nothing general may well be stated, since according to the constructive principle of the whole, since this is made according to the height as well as to the execution of the masonry. An enumeration of these proportions on executed works could therefore be really useful only in connection with a representation of the entire construction, and therefore we limit ourselves to the proportions of the tower at Frankenberg as limits for the thickness of the walls of the lower story of a tower,on which no buttresses are found on the lower story, and the thickness of the walls is 8/14 of the internal span, with those of freiberg minster can be contrasted, where the thickness of the walls is 1/8 of the tower square, w while the arrangement of very wide buttresses aids them. On north German brick buildings, partly on account of the smammel strength of the materials, partly for the massive tower forms thereby required, the proportions at Frankenberg are even exceeded, and for example on the towers of the church S. Maria at Lübeck the thickness of the wall is 3/4 of the clear width of the tower.

Connection of towers with stair towers.

With the towers are usually connected stair towers, which even

become necessary, if access to the upper rooms in the towers is not arranged from the attic over the vaults.

We have here chiefly to distinguish between two kinds of plans, first the usual one of stair towers placed before the exterior, and those entirely concealed and more rarely occurring, where the stairway is taken in the thickness of the wall, as in the tower of Frankenberg church (Fig. 815 a).

With the last arrangement is connected a disadvantage, that they weaken the masonry, while the volume about the stairway is reduced about 9/10, if the steps are included. Therefore in regard to construction it is to be justified in all cases, when the mass of the tower walls and pier consist of rubble, so that the cut stone of the steps and enclosing walls must replace the loss in volume by the goodness of the material and the jointing of the masonry.

Yet in a higher degree is it injurious to the artistic expression of the tower. For exactly according to the predominant extent in half the tower will the stairs, that give access to the most important room in the tower, namely the belfry, forms an especially important arrangement for all that do not fear it. This is indeed the same in every building of several stories, but it is here made evident and must be visible externally or internally. But the internally visible location of massive stairs inside the tower room would restrict the tower wall below and the arrangement of the belfry above, also the space required for swinging the bells inconveniently. Therefore it remains most usual to project from the exterior in order to give its character to the tower, which consists in a contrast to the enclosed structures representing pylons and pagodas, in that it has to obtain the largest possible internal space.

But the combination of the smaller stair towers with the great structure also increases the effect of the latter, and lends it a certain picturesque charm, even when the arrangement varies from symmetry, as then in simpler works the arrangement of the stair tower seldom forms the principal ornament of the whole.

The ordinary arrangements of stair towers are the following.

1. Before the middle of the longitudinal side of the tower, either so that the interior lies outside the face of the wall, or that it cuts into that (Fig. 812 at a).

2. In combination with the buttresses (Fig. 811 at a), either

lying against one, or in the external angle between the two set at right angles, or between a buttress and the wall of the aisle. (Fig. 813 at b).

With a richer resolution of the entire tower structure into a system of piers and arches, such as found in the greater cathedrals, such an unsymmetrical arrangement exerts a certain influence on the whole. Thus the stair towers of S. Gudule in Brussels project from the western side of the tower and its external buttresses, thereby limiting the magnitude of this side of the square by their own width. However the same architectural subdivision is carried out on the restricted sides of the towers as in the longitudinal direction, retaining the full size, so that the middle of the architecture of the western facade falls outside the middle line of the square of the tower. The possibility of this arrangement lies in the horizontal termination of the tower lacking a spire, that finds its upper ending in the belfry flanked by stair towers and opened by two openings for sound at each side, but which would be made essentially more difficult by the addition of a spire necessary to the whole.

On the contrary the spire makes the entirely regular division of the entire tower a necessity, in whose system the unsymmetrically placed stair tower enters in an irregular way, as on the towers of Cologne cathedral, on which just these stair towers form one of the most interesting and richest parts.

The arrangement of stair towers in the angle opened between two buttresses at right angles is sometimes found, so that they have the effect of essentially integral parts of the western facade, thus projecting ar towers at the outer angles, as on the cathedral of Coutances, on which those buttresses form the side walls of the square stone towers, thus giving an entirely independent form.

Then further may also occur as towers all arrangements given in Figs. 742 to 746.

Massive stairs in the interiors of towers are found in the c church of Ahrweiler (Fig. 818), where one of the two rises only to the tower gallery and the section goes to the upper story of the tower. Others variously connected with the development of the elevation with stairs extending to the upper stories can be first described in connection with them.

7. Subordinate Buildings of Churches; Internal Arrangements; Rood Lofts.

Sacristies.

The sacristies required by the Christian confessions have led to many peculiarities in modern churches, when impressed by the necessity of symmetry, they are partly placed in false apses, while the church itself must be contend with a rectangular plan, partly accommodated by a corresponding duplicate of the first requirement.

In the conditions of ordinary parish churches, they are satisfied with one sacristy, while on larger churches like cathedrals, two would be required besides various halls.

In regard to plans suitable for these subordinate structures may be distinguished three sorts.

1. In a room belonging to the system of the church, for example in one or more bays of the side aisle accompanying the choir, first in the choir plan with choir aisle and chevet, in the rectangular bays inserted between the chapels and transepts.

2. As directly external additions to the longer sides of the choir or aven to the polygonal ending.

3. As independent buildings connected with the church by a passage, as indicated by dotted lines in Fig. 819.

The first arrangement according to the modern conception, the monumental effect of the whole being injured by the outgrowth, must also be by its nature least suited, so far as it claims excessive justification for the room in question with the church, and prescribes a very inconvenient height and arrangement of t the windows. So far as known to us, it is found only on certain cathedrals in southern France.

The second arrangement predominates by far, corresponding in the simplest way to the direct needs, and executed at the least cost. It is far removed from injuring the external effect of the church, but enhances its picturesque charm, and we could name a series of churches on which the choir arrangements furnished with just such additions form the splendid parts of the whole.

Only the arrangement of the roof presents some difficulties for the limited height of the church.

According to the simplest arrangement the roof of the sacristy joins a continuation of the choir. Indeed the windou nf the bay of the choir next the sacristy would be covered, thus causing

a defect, which may be changed to an advantage by the use of mural paintings in the surface thus obtaened. A model arrangement of this kind is shown by the church in Wetter, where the lower part of the wall surface is utilized for placing choir stalls, while the mural painting above is S. Maria crowned by two angels, representing at their feet the foundresses of the convent, and filling the space beneath the side arch.

By the arrangement of an independent gable or hip roof over the sacristy with a gutter between et and the church, that extends before the buttresses of the latter, so that a shed roof again rises to the church walls, the windows of the latter can remain open. But this arrangement by the independence of the r roof leads over to third mentionned above, according to which t the sacristy may take any form and size and not be affected by the buttresses of the church. As mediaeval examples of this kind we mention the sacristy of the cathedral of Amiens, which forms an acute angle with the church by which it is connected by a passage, but further the decagonal addition placed before the eastern side of the S. transept of the cathedral at Soissons, now serving as a sacristy but originally a chapel. Quite particularly does this plan appear correct, where it is necessary to connect a greater number of rooms with the church, then leading to the plan of enclosing a rectangular court, the so-called cloister passage opening from it, which the necessary rooms adjoin.

As true models of this kind at a smaller scale can count the buildings added to the cathedral of Paris and of Amiens mentioned by Viollet-le-Duc, while grand arrangements remain still in rich number in the cloisters of many cathedrals, monasteries and collegiate churches.

Principal parts of the internal equipment.

In Fig. 819 we further indicate tehe chief parts of the internal equipment of the church and will here insert the explanation. There belong to this:--

1. The altars placed in the choir, of which the high altar is free and stands at about the centre of the polygon, and the side altars in the east line at the eastern walls of the side choirs, thoseat the right side coming to stand before the angle at a'.

2. The tabernacle b', the little recess about 60 cm wide and 75 cm high, 40 cm deep, closed by a door, which as a rule is in the last bay of the north side, also sometimes is in the northeast

angle of the polygon, or finds its place in the east side of a rectangular choir. Since now a symmetrical arrangement in reference to the whole is impossible, it is superfluous to place it in a certain bay. The interior of the tabernacle must contain a means to protect it from dampness. Before it the everburning lamp finds its place on a metal bracket with a roller for raising and lowering it.

3. The piscina c' in the simplest arrangement being an open niche with a generally corbelled stone basin, usually in the side opposite the tabernacle.

4. The repository d' for the consecrated oil. A closed recess similar to the tabernacle, but smaller.

5. A sedilia e'. A triple seat in a niche in the wall for the celebrant priest and the deacons. The seat as a rule consists of a stone bench not projecting beyond the inner face of the wall, down to which extends the niche. More rarely the niche descends to the floor, so that the seats are placed in it.

The arrangement of such sedilias is also entirely in place in Protestant churches, and would replace the hateful latticed wardrobes that serve as seats for pastors. Where the thickness of the wall is not sufficient, the projection even occurs on the outside of the wall. The minimum height if the niche is fixed at the height of the body.

The place of the pulpit is customary against one of the aisle piers. It can then depend on the magnitude of the church whether it is one of the piers supporting the triumphal arch or is brought forward to one of the middle piers. In single-aisled churches it stands against one of the walls. For such of small width of the choir, as the church at Nieste (Fig. 733), against the very suitable eastern wall of the aisle caused by the difference in widths of the choir and aisle. Entirely nonsensical on the contrary is the usually preferred modern location behind the altar, whereby the speaker is placed at the greatest possible distance from the listeners. This location culminates in the favorite construction in several stories, by which altar, pulpit and organ are combined in one object.

7. The font or baptismal basin also sometimes retains its traditional position in the western bay of the northern side aisle at g', as in the cathedral of S. Martin at Lübeck, between the

western towers or the western bay of the middle aisle. But the last arrangement requires the incessant closing of the doors leading into the bay concerned. The most perfect arrangement is accordingly consists in the arrangement of a separate baptismal chapel, which either as in the church S. Nicolai in Hamburg is added in the angle between the tower and the western end of the side aisle, or results from the elongation of the side aisle to the western facade of the tower, or finally a more isolated location connected by a passage with the western part of the church.

8. The organ came from Byzantium and was probably utilized f first for church purposes in the minster of aix-la-Chapelle, then gradually introduced more widely. Since the 13 th century the great churches frequently had two organs, the smaller one on the rood loft, the larger mostly standing in the western part of the nave. On the best mode of its establishment, the middle ages t therefore left us without exact conclusions, since the few remaining ancient organs were added later to the older churches, so that they chiefly came to accommodate themselves to what existed. The different arrangements of them known to us are the following.

In the minster at Strasburg the organ is found above the third bay of the northern side aisle, so that the works are in an organ chamber projecting externally, and the front including the keyboard is placed in a balcony corbelled out from the dividing arch concerned. Likewise in Ulm, Stendal and Dortmund (still p preserved), its place is found in the northern aisle.

In S. Severi at Erfurt is found on the eastern wall of the north transept a corbelled balcony, on which formerly the organ had its place.

In the Lübeck church of S. Maria the organ stands on the vault between the western towers.

The reasons for one or the other place have already been often so fully explained (Note 1), that it is scarcely necessary to add them here. Chiefly must we give the preference to a location nearer the middle of the church as found in Strasburg, in so far that this best corresponds to better hearing, allows the organist a direct view of the altar, and the organ is in its proper and more sidewise location. With aisles of equal height, this arrangement should be somewhat modified, so that the piers of the bay of the side aisle concerned would be spanned by a lower

vault, about as in the galleries at Ahrweiler and Kidrich-a-R. (Note 2). However it was hard to give up the position common in the last centuries at the western end of the middle aisle, partly from habit and partly since the public finds objectionable an unsymmetrical arrangement. With the arrangement of a western tower may then the wind shutters and generally the works of the organ be placed in the interior of the tower and on the lower vault, the so-called front standing on the arch. x y, or i. Before the organ is the arrangement of a gallery required at least for the singers, when the organist has to direct the singing at the same time. In other cases, especially when the precentor exists, this gallery could be separated from the organ, as by placing it in the side aisle in the opposite bay, thus saving the symmetry. Thus is found in S. Severi in Erfurt such a balcony in the S. transept in the same place as the organ gallery. But the balcony must have its place at the west side, where it can be corbelled out for this requires less space, but larger ones should not be separated by piers. With small length of bays, these piers then stand so near the aisle piers, that it would be better to connect the gallery with them and arrange one intermediate pier, to reduce the stress in the arch and the required height. But in any case is to be given preference here to a stone gallery over a wooden one.

Note 1. Reichensperger. Hints, Organ for Christian art, church decoration.

Note 2. In the collegiate church in Wetter is found in the last bay of the s. side aisle before the transept a vault dating from the end of the 15 th or beginning of the 16 th centuries for receiving the organ, whose diagonal and cross arches are pointed and have only 10 ft. in height to the crown.

In the arrangement with double towers the organ comes between them, and if the towers are wanting at the west end, it then stands on the gallery. But it would also be better to arrange a projection and to place the organ in its upper story. But often the arrangement at the west end is to be regarded as a misfortune for the very reason, that the peculiar expression of its clever character is thereby disturbed, and the west end is only one, but the bays of the side aisle are repeated, and therefore an alteration of them by the organ cannot be injurious.

Arrangement of the rood loft.

The rood loft (lettner, lectorium) denotes a gallery for speaking, that originated from the connection with the pulpit (ambo), at the same time it forms a separation between the choir and the nave.

Its place is found either at the beginning of the high choir under the triumphal arch as at Naumburg, Gelnhausen, Wetzlar, Friedberg, Lübeck, etc., or at the western side of the middle square as in S. Elisapeth's at Marburg, or is placed one or more bays westerly as in the monastery churches of Maulbronn and Haina. Its position is determined by the proportion of the size of the choir to the actual one, depending on the special purpose of the church, thus being a particularly great need of space in the monastery church.

Its extent also depends in a certain respect on its position. Under the triumphal arch it therefore extends only for the clear width of the choir, either on the western side of the middle square or it extends on three sides, adding the area of the transepts to the high choir, and continues across the widths of the arches, is sometimes transferred further west and also crosses the three aisles, finally with the richer choir with choir aisle it is inserted between the piers of the high choir, thus continuing the dividing walls to a certain height, and sometimes is perforated.

If we disregard the original form produced by the combination with the two ambos, no example of which is known in Germany, its simplest design is the wall always furnished with openings, before the middle of which and at the west is an altar for the parish service, over the latter being found a balcony for speaking, that meanwhile for acoustic reasons is not to be regarded as a pulpit for the sermon, but is only used for reading the epistles and gospels.

This gallery for speaking must not be placed on the altar slab, as for example was done in Haina in the earlier restoration (Note), so that the clergy step on that, but it must be formed by a balcony at the east side of the rood screen, thus lying behind the altar and opening into the middle aisle by an arch in the middle of the rood screen.

Note. It is supposed that this peculiar arrangement owes its origin to the desire to use this balcony as a pulpit.

As examples we mention the present arrangement imitated from the original in the church of S. Elisabeth in Marburg, as well as the original arrangement at Haina, which we represent in section in Fig. 822.

Far more complete in every respect is the arrangement of the gallery for speaking on a vault covering the before mentioned altar and serving it is a ciborium, extending from the wall of the rood screen to two isolated columns. For example such is d found in the church at Friedberg (Fig. 821, section) and at Gelnhauses (Figs. 820, 820 a).

According to the former an arrangement affording a small height for the balcony for speaking may be in the middle behind the rood screen and requiring stairs, while with high rood screens and the corresponding arrangement of the choir stalls, the location of the steps may be around one of the piers between which the rood screen is built.

By this is further necessary the arrangement of a connecting passage on the upper surface of the wall, and hence a widening of the latter is required by corbelling, which is then continued also on the other side of the gallery, so that this forms at both sides of it open galleries with railings as in Friedberg.

The endeavor to increase the widths of these galleries then leads to placing before these walls arcades, after the style of the arcades under the parapets of the windows in the side aisle, as on the western rood screen at Naumburg, and further to place the colonnades or arcades at a certain distance from the wall, covering this by cross vaults (Fig. 823).

Accordingly the retaining of the canopied projecting balcony for speaking was superfluous, since the entire rood loft formed it. Meanwhile a reminiscence of it is found in the polygonal form (Fig. 820 a), so that the altar finds its place under the middle bay.

In order to limit the view of the altar as little as possible, in most examples known to us the columns have the smallest dimensions, and the stability against the thrusts of the vault is sought by iron anchors, connecting the spans of the vault together not always directly over the capitals, but better at the height where the thrusts of the arches act.

The rear wall opens into the triumphal arch by two doors at both sides of the la

both sides of the altar (Fig. 820 a), and furtter in certain cases, as in the collegiate church at Wetzlar, by a latticed arched opening made over the altar, whhich arrangement however is impossible wherever at iy Gelnhausen, choir stalls are placed on the east side of the rood screen. Over the rood loft is generally placed a great crucifix, whose arrangement differs accordingtofthat of the rood loft.

According to that simplest arrangement found in Haina, where the speaking gallery opens to the aisle by an arch covered by a gable, the crucifix stands directly in the gable (Foig. 822).

Yet this arrangement becomes impossible when the rood loft t terminates at top in a balcony or gallery.

In the last case are above the gallery between which extends the rood screen, and are connected by a wooden beam on which is placed the crucifix, or the latter is suspended by an iron rod directly from the keystone of the arch over it. In the cathedral at Lübeck is added to that receiving the crucifix the extremely rich wooden construction of the next spaces between piers.

The ornamental execution of tte rood loft varies according to the period of time.

For the simple mode of treatment peculiar to the older examples a specimen is given in Fig. 820, where the entire decoration consists of a relief filling the spandrels above the arch, that represent the rising from the dead, the ascension to heaven, the expulsion to hell, and in the extreme of our Fig. are no longer visible the mouth of hell with the damned.

On the later examples the execution of the architecture is even richer.

The arches are crowned by angular or recurved gables, usually having little suspended arches, between them rising canopies or enclosures of figures, whose tops sometimes rise above the upper gallery, the spandrels filled by tracery, the details ever finer, in brief the whole attains the overrich character, showing itself in its highest bloom on the little tabernacles for the sacrement.

With especial clearness is expressed this transformation on the rood loft of the cathedral at Lübeck, that in the 13 th centwry (Note) was executed in a very simple way in brickwork on four granite columns, and that corresponds about to the plan g given in Fig. ~~822~~ 823. There the sole ornament indeed consisted in

painting those surfaces, that in Gelnhausen are covered by reliefs.

Note. According to the assumptions of Building Director Schwiening at Lübeck this dates from a later time.

But at the end of the 15 th century the entire outside to above the capitals was covered by an overrich but masterly executed paneling, whose arrangement is that indicated generally above, in the style peculiar to the last of Gothic, and which was originally painted in the greatest splendor of color.

8. Different Systems of Geometrical Proportion.

If the proper technical construction not alone leads to the various systems of the entirety and the forms developed from it, but even prescribes the dimensions for certain details, then as we have frequently indicated in the course of this work, there can occur also besides it a second purely geometrical method of construction, which is designed to accurately determine the different dimensions and to establish between them a certain harmonic proportion. (Note).

Note. That one must not value too highly the importance of s such proportions and dimensions has already been emphasized in the proper place.

There is no special peculiarity in such a procedure, no arbitrary discovery of the Gothic period of art, but according to recent investigations it is the transmitted inheritance of preceding centuries. More on hris is contained in the great work of Henczlman:-- "Theory of proportions applied in architecture."

However before and besides Henczlman were developed various other systems for the same purpose, that we must limit ourselves to generally indicating in the following.

The idea lying at the basis of all is to be sought in this, that the effect of every architectural form is obtained in the mass by decision and unity, when the different extremities of it can be circumscribed by a square or triangle of certain harmonic proportions, when further all subordinate divisions, groups and detail forms follow the same law, and accordingly the entire volumes of the whole stand in the like harmonic proportion to each other and to the whole. Before going farther, we must yet add that the following of this law can only be of value, where it is to be seen in perspective, therefore is only to be applied to paints lying in the same horizontal or vertical plane.

The master's rules of Roriczer, etc., preserved to us en the Gothic A B C of Hoffstadt, as well as the system derived from mediaeval sketches and models consist in this, that first the details of the plan were found from the ground form of the same, thus from the square, equilateral triangle or pentagon, indeed by a simple division of the sides or diagonals, and further from the inscribing and circumscribing of the ground form, from setting it diagonally inside and outside, that also the different measures are to each other in the proportion of 1:2:3:4:5 etc., and with regard to the diagonals as 1:√2:2:√8 etc.; also with regard to the diagonals of cubes formed with the same ground form as 1:√3, etc. The relation of 2:√3 further results from t the relation of the side to the altitude in the equilateral triangle. In like manner the dimensions of the elevation are then also obtained from the proportions of the ground form, as generally the above conditions can be fully expressed in this way, when the choice of the magnitude concerned is the correct one. The latter has to be done by a freehand sketch, on whose value the completed work depends on the first place. An example of t procedure is presented by the later indicated Roriczer's construction of a finial.

Appearing substantially different, yet in reality leading to nearly the same results, is the system of Hay given in the English Builder in the year 1861. As shown in Fig. 824, its starting point is the right-angled triangle a b c with equal sides. The hypothenuse b c is then laid off from a to c on the ground line, e c is drawn and then c c is laid off from a to d, c d is drawn and laid off.from a to d, c e drawn, etc., whose arcs with unimportant rectifications have the values 45°, 36°, 30° and 27°. Between these values are interpolated others produced from the rectangle, whose sides are to each other in the proportion of the side of the equilateral triangle to its altitude, thus a i c, then developed from the latter in the same way, no longer illustrated in our Fig., and further are added those found by doubling and halving those already obtained, so that the following scale results.

90°, 80°, 72°, 67 1/2°, 60°, 54°, 51 3/7°, 48°, 45°.
45°, 40°, 36°, 33 3/4°, 30°, 27°, 25 7/10°, 24°, 22 1/2°.
22 1/2°, 20°, 18°, 16 7/8°, 15°, 13 1/2°, 12 6/8°, 12°, 11 1/4°,

by which the different harmonic rectangles are determined, that

limit the various observed points in plan and elevation.

A similar though somewhat more complex system is that established by Henczelman. In Fig. 825 a b c is the triangle obtained from the proportions of the cube, 1: √2: √3. The smaller side b c is then the unit of the work to be constructed; in the Greek triangle it is the width of the cella, and probably that of the middle aisle in the Gothic church. From the triangle a b c by a method analogous to Hay's, by transferring the hypothenuses to the least side are found the triangles a d e, a f g, etc., and by transferring backward a c from the side of the hypothenuse, etca, are found the triangles a k l, a m n, a o p, etc., so that the sides of all triangles form a scale of sizes further formed by their division by two or four, that comprises the volumes and dimensions of the entire work as well as of all details.

In the Discourses on Architecture by Viollet-le-Duc on p. 393 et seq. is developed a system of construction, which like the worgs of this eminent author has the advantage of particular accuracy.

There are employed three different triangles; 1, the equilateral; 2, that made diagonally on the squared base of the normal pyramid formed according to the equilateral triangle; 3, that found in the following manner in Fig. 826. A b c is a right-angled triangle, whose sides as indicated by the dimensions given in the proportion as 3:4:5. At the middle of the base at 2 is erected a perpendicular, whose length = half the hypothenuse, i.e., = 2 1/2, and thereby is found the triangle a e b, namely the normal section of the pyramid of Cheops.

The height of this triangle compared to that taken as the unit of the ground line would be first, √3/4; = 0.86603; second, 3/8 = 0.61237; third, 5/8 = 0.625. (Between the last two is the golden mean = 0.618). In his Dictionary (Vol. VII, p. 535) the same author gives three triangles, two of which agree with those previously explained. These three are:- 1, the right-angled one with two equal sides, whose altitude = half the base; 2, the so-called Egyptian triangle with two equal sides and an altitude = 5/8 of its base (see above); 3, the equilateral triangle.

To follow the systems mentioned and others in their details may be omitted, since wot all are based with sufficient conclusiveness to entirely exclude objections, that the fame of their authors may have contributed something here to them.

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